Lab No.2: Frequency Response of a Rational System Function

Objective
In this lab we will use MATLAB to test and display the frequency response of a given form. Using the `plot` function, we will draw the frequency response on the screen and determine how the position of the poles and zeros will affect the frequency response.

Suggested Reading
MATLAB functions, `freqz`, `plot`, `abs`, `angle`, `figure`, `subplot`, `root`, `poly`.

Problem 1 A Simple Low Pass Filter
Consider the system function given as follows:

\[ H(z) = \frac{1 + z}{z - 0.8} \]

a) Make a pole/zero plot for this system function, using the function `pzplot` from the last lab.

b) Make a sketch of the frequency response (magnitude and phase) of \( H(z) \) using your pole/zero plot and the rules for graphically interpreting pole-zero plots. Your plot should show a frequency range from \( 0 \) to \( 2\pi \).

c) Use the function `freqz` to calculate the complex frequency response \( h \), and \( \omega \), a vector containing the \( n \) frequency points (use 128 points). The frequency response is evaluated at \( n \) points equally spaced around the upper half of the unit circle, so \( \omega \) contains \( n \) points between \( 0 \) and \( \pi \). To find the magnitude and phase, use the functions `abs` and `angle` respectively. Graph the magnitude on a y-semilog plot using the function `semilogy`. Also plot the phase response. Check these plots against the sketches created in b).

d) Try varying the position of the pole. Move it closer and/or further away from the unit circle (note: if you move the pole on or beyond the unit circle, you will produce an unstable system). How does this affect the impulse response? How does moving the pole affect the magnitude response?
Problem 2 The Unity-Gain Resonator

A unity-gain resonator is a system with a peak gain of approximately unity at a frequency called the resonant frequency and some amount of attenuation at all other frequencies. An appropriate second order form is:

\[ H(z) = \frac{b_0(1 - z^{-2})}{1 + a_1z^{-1} + a_2z^{-2}} \]

The peak gain of this system occurs roughly at \( \omega_0 \) is the argument (angle) of the poles, assuming the poles are close to unit circle. A pole/zero plot of a resonator is shown in figure 1.

a) Use graphical techniques to estimate and sketch the magnitude response of the system shown in figure 1.

b) Set \( b_0 = 1 \) and implement the system of a). What are the values of \( a_1 \) and \( a_2 \)? (You should be able to use MATLAB function to calculate it). What is the value of \( \omega_0 \)?

c) Try to move the poles/zeros, see how it affects the system response.

d) What value of \( b_0 \) gives the system a peak gain of unity?

![Pole/zero plot](image)

Figure 1: The pole/zero plot of the resonator for problem 2.
The poles are located at \( z = 0.4 + j0.8 \) and \( z = 0.4 - j0.8 \)
Problem 3 An Interesting System Function

Using the system function:

\[ H(z) = \frac{1 - az^{-1}}{1 - a^{-1}z^{-1}} \quad |a| > 1 \]

a) Graph the pole/zero plot of \( H(z) \). Try several values of \( a \).

b) Plot the frequency response of \( H(z) \) for each \( a \) in a).

c) Show analytically that \( H(z) \) has constant magnitude response. Find that magnitude.