Design of FIR Filters

- Unlike IIR filter design, FIR filter design is directly performed in the discrete domain.
- Two approaches:

  Window method and Minimax method

- <u>Window method</u>:

  Given a desired frequency response $H_d(e^{j\omega})$;

  Compute the desired impulse response $h_d(n)$;

  Multiply the desired impulse response by a window to yield a finite impulse response $h(n) = h_d(n)w(n)$;

- To ensure the property of linear phase and causality, we can simply force a symmetry property on $h_d(n)$ and $w(n)$. *How?*
- The frequency response of the actual filter is:

$$H(e^{j\omega}) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\theta})W(e^{j(\omega-\theta)})d\theta$$

- The window effects:

  The sidelobes of $W(e^{j\omega})$ cause the ripples in the passband and stopband of $H(e^{j\omega})$;

  The mainlobe of $W(e^{j\omega})$ causes the finite transition between the passband and stopband of $H(e^{j\omega})$.

- Good windows should be such

  The mainlobe width of $W(e^{j\omega})$ is small;

  The peak sidelobe amplitude of $W(e^{j\omega})$ is small.

- Common windows:

  Rectangular: $w(n)=1 \quad 0 \le n \le M$

  Bartlett (triangular):

$$w(n)=\begin{cases} 2n/M & 0 \le n \le M/2 \\ 2-2n/M & M/2 < n \le M \end{cases}$$

  Hanning: $w(n)=0.5-0.5\cos(2\pi n/M)$

  Hamming: $w(n)=0.54-0.46\cos(2\pi n/M)$

  Blackman:

$$w(n)=0.42-0.5\cos(2\pi n/M)+0.08\cos(4\pi n/M)$$

- The spectral comparison of the common windows:

  See Figure 7.22 (a)-(e) and Table 7.1

  Observe the mainlobe width and peak sidelobe amplitude

- Near-optimal window - Kaiser window:

$$w(n) = \frac{I_0\left[\beta\left(1-[(n-\alpha)/\alpha]^2\right)^{1/2}\right]}{I_0(\beta)} \quad 0 \le n \le M$$

$$\alpha = M/2$$

$I_0(.)$: zeroth-order modified Bessel function of the 1st kind

M and β control the mainlobe width and peak sidelobe amplitude (see p474)

- <u>Minimax method – optimal filter design:</u>

$$\min_{h(n):0\le n\le M/2}\left(\max_{\omega\in F}|E(\omega)|\right)$$

$$E(\omega) = W(\omega)\left[H_d\left(e^{j\omega}\right) - H\left(e^{j\omega}\right)\right]$$

- A key property of an optimal filter:

  Equiripple in passband and stopband