Abstract—We study two scenarios of full-duplex (FD) multiple-input–multiple-output cognitive radio networks: FD cognitive ad hoc networks and FD cognitive cellular networks. In FD cognitive ad hoc networks (also referred as interference channels), each pair of secondary users (SUs) operate in FD mode and communicate with each other within the service range of primary users (PUs). Each SU experiences not only self-interference but also interuser interference from all other SUs, and all SUs generate interference on PUs. We address two optimization problems: one is to minimize the sum of mean-squared errors (MSE) of all estimated symbols, and the other is to maximize the minimum per-SU MSE of estimated symbols, both of which are subject to power constraints at SUs and interference constraints projected to each PU. We show that these problems can be cast as a second-order cone programming, and joint design of transceiver matrices can be obtained through an iterative algorithm. Moreover, we show that the proposed algorithm is not only applicable to interference channels but also to FD cellular systems, in which a base station operating in FD mode simultaneously serves multiple uplink and downlink users, and it is shown to outperform HD scheme significantly.

Index Terms—Cognitive radio, full-duplex (FD), interference channels, MIMO, MSE, multiuser, self-interference, transceiver designs.

I. INTRODUCTION

The increasing demand for the improved spectral efficiency with the proliferation of wireless services is calling for powerful communication technologies that deliver increasing data rates and utilize the current spectrum resources more efficiently. In half-duplex (HD) wireless communication systems, namely time-division duplex (TDD) and frequency-division duplex (FDD), a node can either transmit or receive on a single frequency band, but not simultaneously. This incurs significant loss of spectrum efficiency. With full-duplex (FD) communication system, a node receives and transmits simultaneously on the same channel [1]–[29], which can potentially double its link capacity, and increase its spectral efficiency targeted by the next generation wireless communication systems.

There has been a recent growth of interest in the high spectral efficiency gain of a FD radio over a HD radio. In particular, FD relaying has drawn much interest since it is effective in extending the network coverage and improving the link reliability of the network [1], [10], [11]. In addition to relay nodes, small cells which extend the service coverage and/or increase network capacity can be deployed in the FD mode due to low transmit powers, short transmission distances and low mobility. A small cell network where a FD base station (BS) serves multiple uplink (UL) and downlink (DL) users operating in the HD mode has been considered in [2]–[5]. Moreover, cognitive radios, which substantially increase spectrum utilization efficiency by allowing unlicensed users to share the spectrum with licensed users, can be deployed in the FD mode. A FD cognitive radio can transmit and sense the transmission status of other nodes [6], [7]. FD technology is suitable to combat several problems at the medium access control (MAC) layer, such as hidden terminals, large delays, and congestion [8], [9].

The limiting factor on practical implementation of FD radios is the so-called self-interference at the front-end of the receiver, which is caused by the signal leakage from the transmitter antennas of a FD node to its own receiver antennas. The power of the self-interference signal can be 100 dB stronger than the received signal of interest coming from a distant source, which can exceed the dynamic range of the analog-to-digital converter (ADC). Unless this self-interference is canceled satisfactorily, a radio transceiver cannot perform FD operation. Recently several research groups have developed methods for self-interference cancellation. These works include the transmit beamforming (spatial domain suppression) methods in [10]–[12], in which the self-interference is canceled at the front-end of the receiver by a cancellation signal generated from a transmitted signal in the baseband. Promising results from experimental research that demonstrate the feasibility of FD transmission using the off-the-shelf hardware are also available in [9], [13]–[16]. However, due to imperfect self-interference channel knowledge and the hardware impairment in the transmitter chain (amplifier non-linearity, phase noise, and I/Q channel imbalance), the self-interference cannot be canceled completely in practice. Therefore, further optimization
of a FD radio, such as power allocation and beamformer design, must take into account the residual self-interference [17]–[26]. In this paper, we refer to the above optimization problems as transceiver design.

Cognitive radio system is also a promising technology to enhance spectrum efficiency [30], [31]. In cognitive radio systems, a set of unlicensed secondary users (SUs) operate within the service range of licensed primary users (PUs) where the amount of interference from SUs to PUs must be constrained to meet the Quality-of-Service (QoS) requirements for the Pus [32]–[45].

Examples of transceiver design for a single pair of FD radio nodes are available in [12], [17]–[26]. But there has been little work on transceiver design for multiple pairs of FD radio nodes. Recently, the interest on MIMO channels has migrated from point-to-point MIMO and MIMO downlink channel to MIMO interference channels because the latter is inherent in many practical problems [46]. With the increase of wireless devices that share the same frequency and time resources, interference becomes the key bottleneck that limits the performance of communication networks. Studies on the performance of cellular communication systems where each cell causes interference to other cells can be carried out by focusing on MIMO interference channels [47]. In this paper, we propose a joint and iterative transceiver design method for a MIMO FD cognitive interference channel (i.e., an ad-hoc FD cognitive network) and a FD cognitive cellular system by taking into account the limited dynamic ranges of the transmitter and receivers.

Particularly, we consider $K$ pairs of FD SUs exchanging information, while the SUs also provide protection to multiple PUs. The nodes in each SU pair suffer not only from self-interference due to operating in the FD mode, but also from inter-user interference due to simultaneous transmission from all SUs. We first consider the sum mean-squared-error (Sum-MSE) as the objective function to minimize subjecting to power constraints at the SUs and interfering power constraints at the PUs. An iterative algorithm which optimizes the transmit and receiving beamforming matrices in alternating manner is proposed. At each iteration, Sum-MSE decreases monotonically, and is guaranteed to converge to at least a local optimum solution. However, in a multiuser MIMO systems, with an optimization of overall efficiency, the transmission power resource is focused on the good channels, i.e., the good channels are favored over the bad channels, where some users are not even allowed to transmit their signals. In order to avoid this effect, we also consider the problem of minimizing the maximum per-user MSE (Min-Max) subject to power constraints at the SUs and interfering power constraints at the PUs. It is shown in the simulations that while the proposed Sum-MSE minimization scheme achieves smaller total MSE, the proposed Min-Max scheme achieves the same MSE for every user.

Moreover, we show that the proposed algorithm can also be applied to FD cognitive cellular systems. In current cellular systems, DL and UL channels operate either in orthogonal time or frequency domain, resulting in inefficient use of the radio resources. In this paper, we consider a scenario where a BS operating in FD mode communicates with UL and DL users operating in HD mode simultaneously. In addition to self-interference at the BS, the optimization problem is exacerbated by the co-channel interference (CCI) caused by the UL users to DL users. Sum-rate maximization for FD multi-user systems has been investigated in [2], [27]–[29]. However, the CCI is not taken into account in [27], single-antenna users are assumed in [2], and these works [2], [27] and [28] did not consider any transmitter/receiver distortion. In this paper, we take into account the major hardware impairments in practical transceivers. The simulation results show that the proposed FD system can achieve a significant improvement of throughput over HD system.

A. Rationale for MSE-Based Optimizations

MSE-based transceiver designs have been considered extensively due to its good performance and significantly reduced complexity. It has been shown in [48] that minimum mean-squared-error (MMSE) estimation plays an important role in approaching the information-theoretic limits of Gaussian channels. When MMSE receiver is used, MSE-based optimization problems are equivalent to signal-to-interference-plus-noise ratio (SINR)-based optimization problems, since they are related as [49].

$$\text{MSE} = \frac{1}{1 + \text{SINR}}. \quad (1)$$

Therefore, rate-based optimization using $\log_2(1 + \text{SINR})$ can be conveniently transformed into MSE-based optimization, $-\log_2(\text{MSE})$. And as mentioned in [50], the user-wise MSE can be used to approximate the achievable rate of the users when they jointly decode their streams. In particular, when MMSE receivers are employed, the achievable rate of a user is written as the negative logarithm of the determinant of the MSE error covariance matrix, which is tightly related to the user MSE. Hence, minimizing the MSE of a user maximizes a tight lower bound on its rate. With (1), instead of considering each design criterion such as the MSE and the maximal mutual information in a separate way, a unifying framework can be developed. The link between most practical objective functions and the main diagonal elements of the MSE matrix has been established in [49] for point-to-point multicarrier MIMO communications, and this work has been extended to multicarrier MIMO relay communications in [51].

Our extensive literature survey reveals that MSE-based optimization problems have been considered for many communication systems but not for FD cognitive radio systems. This work tries to fill this gap and reveals useful insights into FD cognitive radio systems via MSE-based optimization.

B. Notation

The following notations are used in this paper. Matrices and vectors are denoted as bold capital and lowercase letters, respectively. $(\cdot)^T$ is the transpose; $(\cdot)^H$ is the conjugate transpose. $E[\cdot]$ means the statistical expectation; $I_N$ is the $N$ by $N$ identity matrix; $0_{N \times M}$ is the $N$ by $M$ zero matrix; $\text{tr}[\cdot]$ is the trace; $[\cdot]$ is the determinant; $\text{diag}(\mathbf{A})$ is the diagonal matrix with the same diagonal elements as $\mathbf{A}$. $\mathcal{CN}(\mu, \sigma^2)$ denotes a complex
Gaussian distribution with mean \( \mu \) and variance \( \sigma^2 \).\( \text{vec}(\cdot) \) stacks the elements of a matrix to one long column vector. The operator \( \otimes \) denotes Kronecker product and \( \perp \) denotes the statistical independence. \( \| \cdot \|_2 \) is the Euclidean norm of a vector. \( [A_i]_{i=1,...,K} \) denotes a tall matrix (or vector) obtained by stacking the matrices \( A_i \), \( i = 1, \ldots, K \).

**II. System Model**

In this section, we describe the system model of a FD MIMO cognitive radio system, in which \( K \) pair of SUs communicate simultaneously within the service range of \( L \) PUs as seen in Fig. 1. Let us denote the set of SU pairs with \( \mathcal{K} \triangleq \{1, \ldots, K\} \). The signals mentioned below are defined in complex baseband. We consider MIMO wireless channels, where all SUs are equipped with multiple antennas, and exchange information simultaneously with their pairs in a two way communication. We assume that the SU nodes in \( i \)th link have \( N_i \) and \( M_i \) transmit and receive antennas, respectively.

We also take into account the limited dynamic range (DR), which is caused by non-ideal amplifiers, oscillators, ADCs, and digital-to-analog converters (DACs). We adopt the limited DR model in [17], which has also been commonly used in [21]–[25]. Particularly, at each receive antenna an additive white Gaussian “receiver distortion” with variance \( \beta \) times the energy of the undistorted received signal on that receive antenna is applied, and at each transmit antenna, an additive white Gaussian “transmitter noise” with variance \( \kappa \) times the energy of the intended transmit signal is applied. This transmitter/ receiver distortion model is valid, since it was shown by hardware measurements in [52] and [53] that the non-ideality of the transmitter and receiver chain can be approximated by an independent Gaussian noise model, respectively. The SU \( i^{(a)}, i \in \mathcal{K}, a \in \{1, 2\} \) receives signals from all the SU transmitters in the system via MIMO channels. \( \mathbf{H}_{ij}^{(ab)} \in \mathbb{C}^{M_i \times N_j} \) is the desired channel between node \( a \) and \( b \) of the \( i \)th SU transmitter-receiver pair, where \( b \in \{1, 2\} \) and \( b \neq a \). \( \mathbf{H}_{ii}^{(aa)} \in \mathbb{C}^{M_i \times N_i} \), \( a \in \{1, 2\} \) denotes the self-interference channel of the SU \( i^{(a)} \). \( \mathbf{H}_{ij}^{(ab)} \in \mathbb{C}^{M_i \times N_j}, (a, b) \in \{1, 2\} \) denotes the interference channel from the transmitter antennas of the SU \( b \) in the \( j \)th pair to the receiver antennas of the SU \( a \) in the \( i \)th pair, \( (i, j) \in \mathcal{K} \) and \( j \neq i \). All the channel matrices are assumed to be mutually independent, and the entries of each matrix are independent and identically distributed (i.i.d.) circular complex Gaussian variables with zero mean, independent real and imaginary parts, each with variance \( 1/2 \). Particularly, each entry of all channel matrices has a uniform phase and Rayleigh magnitude, which models a Rayleigh fading environment [54].

The transmitted data streams of size \( d_i \) at the SU \( i^{(a)} \) is denoted as \( \mathbf{d}_{i}^{(a)} \in \mathbb{C}^{d_i}, i \in \mathcal{K}, a \in \{1, 2\} \), and are assumed to be complex, zero mean, i.i.d. with

\[
\mathbf{E} \left[ \mathbf{d}_{i}^{(a)} \right] = \mathbf{0}_{d_i \times 1}, \tag{2}
\]

\[
\mathbf{E} \left[ \mathbf{d}_{i}^{(a)} (\mathbf{d}_{i}^{(b)})^H \right] = \begin{cases} \mathbf{I}_{d_i} & i = j \text{ and } a = b, \\ \mathbf{0}_{d_i \times d_j} & i \neq j \text{ or } a \neq b. \end{cases} \tag{3}
\]

The \( N_i \times 1 \) signal vector transmitted by the SU \( i^{(a)} \) is given by

\[
\mathbf{x}_{i}^{(a)} = \mathbf{V}_{i}^{(a)} \mathbf{d}_{i}^{(a)}, \quad i \in \mathcal{K}, \ a \in \{1, 2\}, \tag{4}
\]

where \( \mathbf{V}_{i}^{(a)} \in \mathbb{C}^{N_i \times d_i} \) represents the transmit beamforming matrix, and \( \mathbf{x}_{i}^{(a)} \) is assumed to be Gaussian distributed with zero
mean and covariance matrix as

$$\mathbb{E} \left\{ x_i^{(a)} \left( x_i^{(a)} \right)^H \right\} = v_i^{(a)} \left( v_i^{(a)} \right)^H.$$ 

We consider a FD MIMO interference channel between SUs that suffers from self-interference and interference from other pairs. Thus, the SU $i^{(a)}$ receives a combination of the signals transmitted by all the transmitters and noise. The $M_i \times 1$ received signal at the SU $i^{(a)}$ is written as

$$y_i^{(a)} = \sqrt{\rho_i} h_i^{(ab)} \left( s_i^{(b)} + c_i^{(b)} \right) + \sqrt{\eta_i} h_i^{(an)} \left( s_i^{(a)} + c_i^{(a)} \right)$$

$$+ \sum_{j \neq i} \sum_{c=1}^{2} \sqrt{\eta_{ij}} h_{ij}^{(ac)} \left( y_j^{(c)} + c_j^{(c)} \right) + e_i^{(a)}$$

$$+ n_i^{(a)}, \quad i \in \mathcal{K}, \quad (a, b) \in \{1, 2\} \text{ and } a \neq b.$$ (5)

Here, $n_i^{(a)} \in \mathbb{C}^{M_i}$ is the additive white Gaussian noise (AWGN) vector at SU $i^{(a)}$ with zero mean and unit covariance matrix, and it is uncorrelated to all the transmitted signals. In (5), $\rho_i$ denotes the average power gain of the $i$th SU transmitter-receiver pair, $\eta_i$ denotes the average power gain of the self-interference channel at the $i$th SU pair, and $\eta_{ij}$ denotes the average power gain of the interference channel between the nodes at the $i$th and $j$th SU pair.

In (5), $e_i^{(a)} \in \mathbb{C}^{N_i}$, $i \in \mathcal{K}, \quad a \in \{1, 2\}$ is the transmission noise at the transmitter antennas of the SU $i^{(a)}$, which models the effect of limited transmitter DR, and closely approximates the effects of additive power-amplifier noise, nonlinearities in the DAC and phase noise. The covariance matrix of $e_i^{(a)}$ is given by $\kappa (\kappa \ll 1)$ times the energy of the intended signal at each transmit antenna [17]. In particular, $c_i^{(a)}$ can be modeled as

$$c_i^{(a)} \sim \mathcal{CN} \left( 0, \kappa \text{ diag} \left( v_i^{(a)} \left( v_i^{(a)} \right)^H \right) \right),$$

$$c_i^{(a)} \perp x_i^{(a)}.$$ (6)

In (5), $e_i^{(a)} \in \mathbb{C}^{M_i}$, $i \in \mathcal{K}, \quad a \in \{1, 2\}$ is the additive receiver distortion at the receiver antennas of the SU $i^{(a)}$, which models the effect of limited receiver DR, and closely approximates the combined effects of additive gain-control noise, nonlinearities in the ADC and phase noise. The covariance matrix of $e_i^{(a)}$ is given by $\beta (\beta \ll 1)$ times the energy of the undistorted received signal at each receive antenna [17]. In particular, $e_i^{(a)}$ can be modeled as

$$e_i^{(a)} \sim \mathcal{CN} \left( 0, \beta \text{ diag} \left( \Phi_i^{(a)} \right) \right),$$

$$e_i^{(a)} \perp u_i^{(a)}.$$ (7)

1We assume that each SU uses a Gaussian codebook, since the Gaussian inputs are theoretically optimal, and are capacity achieving [54]. Since Gaussian inputs cannot be realized, discrete modulations/constellations are used in practice, but the performance of discrete constellations are far from that of ideal Gaussian inputs. Interested readers can refer to [55] in which the precoder designs under discrete constellations have been considered.

2Since the SU receiver cannot differentiate the interference generated by the PUs from the background thermal noise, the noise vector $n_i^{(a)}$ in (5) captures the background thermal noise as well as the interference generated by the PUs. This assumption is also adopted in [32]–[37].

3The channel state information (CSI) of the self-interference channel can be acquired by using pilot signals. Since the pilot signal of a FD node is echoed back to itself, and the received power of this echoed-back pilot signal is very high (due to small distances between transmit and receive antennas of a node), the self-interference channel can be estimated with high accuracy [1].

4In practice, even if the self-interference is suppressed to some extent using analog and digital self-interference cancellation techniques, due to transmitter ($c_i^{(a)}$) and receiver ($e_i^{(a)}$) distortion, the self-interference cannot be canceled completely resulting in residual self-interference. Depending on the strength of the residual self-interference, optimal transmit strategies for HD systems, can depart from optimal. If the residual self-interference is not well managed, it can still prevent us from exploiting the benefits of FD systems.

5Note that (12) is approximated under $\kappa \ll 1$ and $\beta \ll 1$, which is a practical assumption [16], [17]. Therefore, the terms including the multiplication of $\kappa$ and $\beta$ are negligible, and have been ignored in the approximation.
The received interference signal at the \( l \)th PU from SUs is expressed as
\[
y_i^{PU} = \sum_{i=1}^{K} \sum_{b=1}^{2} \sqrt{P_{li}^{(b)}} \mathbf{G}_{li}^{(b)} \left( \mathbf{x}_i^{(b)} + \mathbf{c}_i^{(b)} \right), \quad l = 1, \ldots, L, \tag{15}
\]
where \( \mathbf{G}_{li}^{(b)} \in \mathbb{C}^{N_i \times N_l} \) is the channel between the \( l \)th PU and \( i \)th SU, which is modeled similar to \( \mathbf{H}_{ij}^{(ab)} \) discussed in Section II, and \( \mu_{li}^{(b)} \) is the average power gain of \( \mathbf{G}_{li}^{(b)} \). Using (15), the power of the interference resulting from SUs at the \( l \)th PU can be written as
\[
I_i^{PU} = \sum_{i=1}^{K} \sum_{b=1}^{2} \mu_{li}^{(b)} \text{tr} \left[ \left( \mathbf{G}_{li}^{(b)} \left( \mathbf{V}_i^{(b)} \right)^H \right) \left( \mathbf{G}_{li}^{(b)} \right)^H \right] + \kappa \text{tr} \left( \left( \mathbf{V}_i^{(b)} \left( \mathbf{V}_i^{(b)} \right)^H \right) \left( \mathbf{G}_{li}^{(b)} \right)^H \right) \tag{16}
\]
Note that underlay cognitive radio systems enable SUs to transmit with overlapping spectrum with PUs as long as the QoS of PUs is not degraded. This is managed by, e.g., introducing some interference constraints that impose upper bounds on the total aggregate interference induced by all SUs to each PU. The choice of this upper bound (or threshold) is a complex and open regulatory issue, which can be the result of a negotiation or opportunistic-based procedure between PUs (or regulatory agencies) and SUs [32]. Both deterministic and probabilistic interference constraints have been suggested in the literature [30], [31]. In this paper, we will consider deterministic interference constraints as assumed in [32]–[34]. Particularly, we assume that the PU imposing the interference constraint, has already computed its maximum tolerable interference threshold.

Channel estimation between SUs can be accomplished using standard signal processing techniques via training through the pilots and feedback [38]. Channel estimation between SUs and PUs is more challenging, because PUs are unlikely to cooperate with SUs. But, if the primary system adopts the TDD scheme, by exploiting the channel reciprocity, channel between SUs and PUs can be acquired at the SUs by overhearing the transmissions between PU transmitter and receiver pair [38]–[42]. If TDD scheme is not feasible, blind beamforming techniques can be employed [38]. Other methods to acquire the CSI knowledge between PUs and SUs is 1) through environmental learning [43], [44], 2) by exchange of CSI between the PUs and SUs through a band manager, which mediates between the two parties [39], [40], [45], and 3) the primary system can cooperate with the secondary system to exchange the channel estimates [38].

### III. Sum-MSE Minimization

We take Sum-MSE as the performance measure to design the transceivers. Upper limits on both transmit power of the SUs and interfering power at the PUs are considered. Using the following definition for a stacked matrix \( \mathbf{V} : \mathbf{V} = [\mathbf{V}_1^T, \ldots, \mathbf{V}_K^T]^T \) with \( \mathbf{V}_k = [\mathbf{V}_1^{(1)}, \mathbf{V}_2^{(2)}]^T \). Sum-MSE optimization scheme is formulated as follows
\[
\begin{align*}
\min_{\mathbf{V}, \mathbf{R}} & \quad \sum_{i=1}^{K} \sum_{a=1}^{2} \text{tr} \left[ \text{MSE}_{i}^{(a)} \right] \tag{17} \\
\text{s.t.} \quad & \text{tr} \left[ \left( \mathbf{v}_i^{(a)} \left( \mathbf{v}_i^{(a)} \right)^H \right) \left( \mathbf{H}_{ii}^{(ab)} \right)^H \right] \leq \rho_{i}^{(b)}, \quad i \in \mathcal{K}, \quad b = 1, 2, \tag{18} \\
& I_i^{PU} \leq \lambda_i, \quad l = 1, \ldots, L, \tag{19}
\end{align*}
\]
where \( P_{li}^{(b)} \) is the power constraint at the \( i \)th SU transmitter, and \( \lambda_i \) is the upper bound of the interference allowed to be imposed on the \( l \)th PU.

Note that the Sum-MSE function (17) is not jointly convex over transmit beamforming matrices \( \mathbf{V} \) and receiving beamforming matrices \( \mathbf{R} \) (since they are coupled), but is component-wise convex over \( \mathbf{V} \) and \( \mathbf{R} \). Since it is not jointly convex, we cannot apply the standard convex optimization methods to obtain the optimal solution. Therefore, we will employ an iterative algorithm that finds the efficient solutions of \( \mathbf{V} \) and \( \mathbf{R} \) in an alternating fashion. Particularly, we update the transmit beamforming matrices \( \mathbf{V} \) when the receiving beamforming matrices \( \mathbf{R} \) are fixed, and then using \( \mathbf{V} \) obtained at the previous step, we update the receiving beamforming matrices \( \mathbf{R} \). The iterations continue until convergence or a pre-defined number of iterations is reached.

Under the fixed transmit beamforming matrices, the optimal receive beamforming matrices at the SU \( j^{(a)} \) is MMSE receiver filter which can be expressed as
\[
\mathbf{R}_{j}^{(a)} = \arg \min_{\mathbf{R}_{j}^{(a)}} \text{tr} \left[ \text{MSE}_{j}^{(a)} \right] = \sqrt{\rho_{j}} \left( \mathbf{v}_{j}^{(b)} \right)^H \left( \mathbf{H}_{jj}^{(ab)} \right)^H \times \left( \rho_{j} \mathbf{H}_{jj}^{(ab)} \mathbf{v}_{j}^{(b)} \left( \mathbf{v}_{j}^{(b)} \right)^H \left( \mathbf{H}_{jj}^{(ab)} \right)^H + \mathbf{S}_{j}^{(a)} \right)^{-1} \tag{20}
\]

\[
\mathbf{S}_{j}^{(a)} \approx \rho_{j} \kappa \mathbf{H}_{jj}^{(ab)} \text{diag} \left[ \left( \mathbf{v}_{j}^{(b)} \right)^H \left( \mathbf{H}_{jj}^{(ab)} \right)^H \right] + \eta_{j} \kappa \mathbf{H}_{jj}^{(aa)} \text{diag} \left[ \left( \mathbf{v}_{j}^{(a)} \right)^H \left( \mathbf{H}_{jj}^{(aa)} \right)^H \right] + \beta \rho_{j} \text{diag} \left( \mathbf{H}_{jj}^{(ab)} \mathbf{v}_{j}^{(b)} \left( \mathbf{v}_{j}^{(b)} \right)^H \mathbf{H}_{jj}^{(ab)} \right)^H + \beta \eta_{j} \text{diag} \left( \mathbf{H}_{jj}^{(aa)} \mathbf{v}_{j}^{(a)} \left( \mathbf{v}_{j}^{(a)} \right)^H \mathbf{H}_{jj}^{(aa)} \right)^H + \kappa \text{diag} \left( \mathbf{v}_{j}^{(c)} \left( \mathbf{v}_{j}^{(c)} \right)^H \right) \mathbf{H}_{jj}^{(ac)} \right)^H + \mathbf{I}_{Mj} \tag{12}
\]
TABLE I
MSE-Based Transceiver Designs

1) Set the iteration number \( n = 0 \) and initialize \( V^{[0]} \).
2) \( n \leftarrow n + 1 \). Update \( \left( R_i^{(a)} \right)^{[n]} \) using (20).
3) Update \( \left( V_i^{(b)} \right)^{[n]} \) by solving (27)–(30).
4) Repeat steps 2 and 3 until convergence or a predefined number of iterations is reached.

Under the fixed receive beamforming matrices, the optimum transmit beamforming matrices are found as follows. Using epigraph form and introducing slack variables \( \tau_i^{(a)} \), (17)–(19) is rewritten as

\[
\begin{align*}
\min_{\bar{V}, \bar{r}^{(a)}} & \sum_{i=1}^K \sum_{a=1}^2 \tau_i^{(a)} \\
\text{s.t.} & \quad \text{tr} \left( V_i^{(b)} \left( V_i^{(b)} \right)^H \right) \leq P_i^{(b)}, \quad i \in \mathcal{K}, \quad b = 1, 2, \\
& \quad \lambda_i, \quad l = 1, \ldots, L, \\
& \quad \text{tr}(\text{MSE}_i^{(a)}) \leq \tau_i^{(a)}, \quad i \in \mathcal{K}, \quad a = 1, 2.
\end{align*}
\]

(21)

(22)

(23)

(24)

To solve the optimization problem (21)–(24), we need to write \( \text{tr}(\text{MSE}_i^{(a)}) \) and \( I_i^{PU} \) in vector form. As shown in Appendix, the vector forms of \( \text{tr}(\text{MSE}_i^{(a)}) \) and \( I_i^{PU} \) can be written as in (25) and (26), respectively, given at the bottom of the next page.

Using the vector forms, the optimization problem (21)–(24) can be rewritten as

\[
\begin{align*}
\min_{\bar{V}, \bar{r}^{(a)}} & \sum_{i=1}^K \sum_{a=1}^2 \tau_i^{(a)} \\
\text{s.t.} & \quad \left\| \text{vec} \left( V_i^{(b)} \right) \right\|_2^2 \leq P_i^{(b)}, \quad i \in \mathcal{K}, \quad b = 1, 2, \\
& \quad \| \alpha_l \|_2^2 \leq \lambda_l, \quad l = 1, \ldots, L, \\
& \quad \| \mu_{(a)} \|_2^2 \leq \tau_i^{(a)}, \quad i \in \mathcal{K}, \quad a = 1, 2.
\end{align*}
\]

(27)

(28)

(29)

(30)

Since the objective function (27) is linear, and the constraints (28)–(30) are second-order cones, (27)–(30) is a second-order cone programming (SOCP) problem (56), and can be efficiently solved by standard SOCP solvers with polynomial complexity using interior point methods [57].

A. Discussion

The algorithm for the Sum-MSE optimization problem (17)–(19) is given in Table I. Since the proposed Sum-MSE algorithm monotonically decreases the total MSE over each iteration by updating the transceivers in an alternating fashion, and the fact that MSE is bounded below (at least by zero), it is clear that the proposed Sum-MSE algorithm is convergent and is guaranteed to converge to a local minimum. Since the Sum-MSE optimization problem is not jointly convex, the proposed algorithm is not guaranteed to converge to a global optimum point. Therefore, good initialization points should be selected to ensure a suboptimal solution with a good performance. In the simulations, we use right singular matrices as initialization [46].

Assuming the same number of transmit antennas, receive antennas, and data streams at each node, i.e., \( N_t = N, M_i = N, \) and \( d_i = N, \) the complexity of the proposed algorithm is computed as follows. The update of \( R_i^{(a)} \) requires \( O(8N^3K) \) for the matrix multiplications inside the inverse, \( O(N^3) \) for the inverse, and \( O(2N^3) \) for the matrix multiplications outside of the inverse. The complexity of computing the optimal transmit beamforming mainly depends on solving SOCP problem. The number of iterations required to solve a SOCP problem using interior point methods can be computed according to [56]. For a SOCP with \( n \) variables and \( m \) conic constraints with dimension \( m_i \) each, its complexity scales by \( O(n^2 \sum m_i) \). Therefore, the update of \( V_i^{(b)} \) requires a total computational complexity of \( O \left( 8K^3(N^2 + 1)^2(N^2 + 2LN^2 + N^2(6K - 1) + 1) \right) \).

The proposed algorithm requires a central scheduler, which coordinates the calibration of channel matrices, collects all channel matrices, and then computes and distributes the beamforming matrices of all links. In a cellular system, the BS can take up the role of the scheduler. In an interference channel (or an ad-hoc network), the scheduler can reside at any node in the network. In a dynamic environment, the scheduler can be adaptively elected among the eligible nodes in the network [58]–[60]. The election can be done based on the capacity of a node, the status of a node, and the location of a node, etc. The research of the scheduler election issues is important but beyond the scope of this paper. We assume that a scheduler is available for the network within the time scale of interest.

IV. EXTENSIONS

A. Min-Max MSE Minimization

Unlike the minimum Sum-MSE transceiver design discussed in Section III, the Min-Max MSE transceiver design ensures each receiver has the same MSE so that it introduces fairness among the nodes, i.e., it guarantees a certain level of fairness among the nodes. The Min-Max MSE optimization problem can be formulated as:

\[
\begin{align*}
\min_{\bar{V}, \bar{r}} \max_{i, b = 1, 2} & \quad \text{tr} \left( \text{MSE}_i^{(a)} \right) \\
\text{s.t.} & \quad \text{tr} \left( V_i^{(b)} \left( V_i^{(b)} \right)^H \right) \leq P_i^{(b)}, \quad \forall (i, b), \\
& \quad I_i^{PU} \leq \lambda_i, \quad l = 1, \ldots, L.
\end{align*}
\]

(31)

(32)

(33)

Similar to the Sum-MSE optimization problem (17)–(19), the Min-Max MSE optimization problem is not jointly convex over transmit beamforming matrices \( \bar{V} \) and receiving beamforming matrices \( \bar{R} \). Therefore, we carry out the optimization procedure iteratively in an alternating fashion.

Under the fixed values of the transmit beamforming matrices \( \bar{V} \), the optimal receiving beamforming matrix, \( R_i^{(a)}, \quad i \in \mathcal{K}, \quad a = 1, 2 \) is linear MMSE receiver given in (20). Under the fixed receiving beamforming matrices \( R_i^{(a)} \), with the introduction of...
an auxiliary variable $t$, which is an upper bound on the square root of $\text{tr}(\text{MSE}_i^{(a)})$ (i.e., $\sqrt{\text{tr}(\text{MSE}_i^{(a)})} \leq t$, $\forall i \in \mathcal{K}$, $a = 1, 2$), the Min-Max optimization problem (31)–(33) can be written as

$$\min_{\mathbf{V}} \quad t$$

$$\text{s.t.} \quad \sqrt{\text{tr}(\text{MSE}_i^{(a)})} \leq t, \quad i \in \mathcal{K}, \quad a = 1, 2,$$

$$\text{tr}\left\{ \mathbf{V}_i^{(b)} \left( \mathbf{V}_i^{(b)} \right)^H \right\} \leq I_{i}^{(b)}, \quad i \in \mathcal{K}, \quad b = 1, 2,$$

$$I_{l}^{PU} \leq \lambda_l, \quad l = 1, \ldots, L.$$  

With the vector forms in (25) and (26), the Min-Max optimization problem of transmit beamforming matrices (34)–(37) can be written as

$$\min_{\mathbf{V}, t} \quad t$$

$$\text{s.t.} \quad \left\| \mathbf{\mu}_i^{(a)} \right\|_2 \leq t, \quad i \in \mathcal{K}, \quad a = 1, 2,$$

$$\left\| \text{vec} \left( \mathbf{V}_i^{(b)} \right) \right\|_2 \leq \sqrt{I_{i}^{(b)}}, \quad i \in \mathcal{K}, \quad b = 1, 2,$$

$$\left\| \mathbf{\alpha}_i \right\|_2 \leq \lambda_l, \quad l = 1, \ldots, L.$$  

Similar to (27)–(30), the optimization problem (38)–(41) is also a SOCP problem [56], and can be efficiently solved by standard SOCP solvers with polynomial complexity using interior point methods [57].

### B. Sum-Power Constrained Transceiver Design

In this subsection, we consider sum-power constrained optimization problem, which will be applied in FD cellular systems discussed in Section V. Note that sum-power constraint is still important and motivated for interference channels under the emerging scenarios such as energy-harvesting-based communication systems, distributed antenna systems, games played by resource-constrained players, and fair comparison in heterogeneous networks [61].

Similar to the individual power constrained optimization problem, the proposed SOCP algorithm can also be applied for the sum-power constrained optimization problem given as:

$$\min_{\mathbf{V}, \mathbf{R}} \quad \sum_{i=1}^{K} \sum_{a=1}^{2} \text{tr}\left\{ \text{MSE}_i^{(a)} \right\}$$

$$\text{s.t.} \quad \sum_{i=1}^{K} \sum_{b=1}^{2} \text{tr}\left\{ \mathbf{V}_i^{(b)} \left( \mathbf{V}_i^{(b)} \right)^H \right\} \leq P_T,$$

$$I_{l}^{PU} \leq \lambda_l, \quad l = 1, \ldots, L.$$  

$$\triangleq \left\| \mathbf{\mu}_i^{(a)} \right\|_2^2.$$  

$$I_{l}^{PU} \triangleq \left\| \mathbf{\alpha}_i \right\|_2^2.$$
where $P_T$ is the total power constraint of the system. Using the vector forms in (25) and (26), the optimization problem (42)–(44) can be rewritten, under the fixed receive beamforming matrices, as

$$\min_{\bf V^{(a)}} \sum_{i=1}^{K} \sum_{a=1}^{2} \tau_i^{(a)} \quad \text{s.t.} \quad \|\text{vec}(\bf V)\|_2^2 \leq P_T, \quad \|\alpha_l\|_2^2 \leq \lambda_l, \quad l = 1, \ldots, L, \quad \|\mu_i\|_2^2 \leq \tau_i^{(a)}, \quad i \in \mathcal{K}, \quad a = 1, 2. \quad (45)$$

Since the problem (45)–(48) is a SOCP problem, it can be efficiently solved by standard SOCP solvers.

V. FULL-DUPLEX COGNITIVE CELLULAR SYSTEMS

In this section, we show that the algorithm proposed for the FD cognitive MIMO interference channel also holds for FD cognitive cellular systems, in which a FD BS communicates with HD mode UL and DL users, simultaneously as seen in Fig. 2. The BS serves $K$ UL users and $J$ DL users simultaneously. The BS is equipped with $M_0$ and $N_0$ transmit and receive antennas, respectively. The number of antennas of the $k$th UL user and the $j$th DL user are denoted by $M_k$ and $N_j$, respectively. The number of data streams transmitted from the $k$th UL user (to the $j$th DL user) is denoted by $d_{k,j}^{UL}$ ($d_{j,k}^{DL}$).

$\bf H_k^{UL} \in \mathbb{C}^{N_0 \times M_k}$ and $\bf H_j^{DL} \in \mathbb{C}^{N_j \times M_0}$ represent the $k$th UL channel and the $j$th DL channel, respectively. $\bf H_0 \in \mathbb{C}^{N_0 \times M_0}$ is the self-interference channel from the transmitter antennas of BS to the receiver antennas of BS. $\bf H_k^{DL} \in \mathbb{C}^{N_j \times M_k}$ denotes the CCI channel from the $k$th UL user to the $j$th DL user.

The vector of source symbols transmitted by the $k$th UL user is denoted as $\bf s_k^{UL} = [s_{k,1}^{UL}, \ldots, s_{k,d_{k,UL}}^{UL}]^T$. It is assumed that the symbols are i.i.d. with unit power, i.e., $E[\bf s_k^{UL}(\bf s_k^{UL})^H] = \bf I_{d_{k,UL}}$. Similarly, the transmit symbols for the $j$th DL user is denoted by $\bf s_j^{DL} = [s_{j,1}^{DL}, \ldots, s_{j,d_{j,DL}}^{DL}]^T$, with $E[\bf s_j^{DL}(\bf s_j^{DL})^H] = \bf I_{d_{j,DL}}$.

Denoting the precoders for the data streams of the $k$th UL and $j$th DL user as $\bf V_k^{UL} = [\bf v_{k,1}^{UL}, \ldots, \bf v_{k,d_{k,UL}}^{UL}] \in \mathbb{C}^{M_k \times d_{k,UL}}$, and $\bf V_j^{DL} = [\bf v_{j,1}^{DL}, \ldots, \bf v_{j,d_{j,DL}}^{DL}] \in \mathbb{C}^{M_j \times d_{j,DL}}$, respectively, the transmitted signal of the $k$th UL user and that of the BS can be written, respectively, as

$$\bf x_k^{UL} = \bf V_k^{UL} \bf s_k^{UL}, \quad \bf x_0 = \sum_{j=1}^{J} \bf V_j^{DL} \bf s_j^{DL}. \quad (49)$$

We consider a FD multi-user MIMO system that suffers from self-interference and CCI. The signal received by the BS and that received by the $j$th DL user can be written, respectively, as

$$\bf y_0 = \sum_{k=1}^{K} \bf H_k^{UL}(\bf x_k^{UL} + \bf c_k^{UL}) + \bf H_0(\bf x_0 + \bf c_0) + \bf e_0 + \bf n_0, \quad (50)$$

$$\bf y_j^{DL} = \bf H_j^{DL}(\bf x_0 + \bf c_0) + \sum_{k=1}^{K} \bf H_k^{DL}(\bf x_k^{UL} + \bf c_k^{UL}) + \bf e_j^{DL} + \bf n_j^{DL}. \quad (51)$$

where $\bf n_0 \in \mathbb{C}^{N_0}$ and $\bf n_j^{DL} \in \mathbb{C}^{N_j}$ denote the AWGN vector with zero mean and unit covariance matrix at the BS and the $j$th DL user, respectively. In (50), (51), $\bf c_k^{UL}$ ($\bf c_0$) is the transmitter distortion at the $k$th UL user (BS), which is modeled as in (6), (7), and $\bf e_j^{DL}$ ($\bf e_0$) is the receiver distortion at the $j$th DL user (BS), which is modeled as in (8), (9).

Fig. 2. Full-duplex multi-user MIMO system model.
From (50), (51), the aggregate interference-plus-noise terms at the kth UL and the jth DL user are written, respectively as

$$m_{UL}^k = \sum_{j=1, j \neq k}^{K} H_{j k}^U x_{UL}^j + \sum_{j=1}^{K} H_{j k}^U c_{UL} + H_0 (x_0 + c_0) + c_0 + n_0, \quad k = 1, \ldots, K,$$

(52)

$$m_{DL}^j = H_{j k}^D \sum_{k=1}^{K} V_{k j}^D c_{DL} + \sum_{k=1}^{K} H_{j k}^D (x_{UL}^j + c_{UL}) + H_{j}^D c_{0} + c_{DL} + n_{j}^D, \quad j = 1, \ldots, J.$$

(53)

Similar to [17], the covariance matrix of $$\Sigma_{UL}^k$$ can be approximated, under $$\beta \ll 1$$ and $$\kappa \ll 1$$, as in (54) at the bottom of the next page. The covariance matrix of $$\Sigma_{DL}$$ can be defined similarly, i.e., by replacing $$H_{j k}^U$$, $$V_{j}^U$$, and $$H_0$$ in (54) with $$H_{k j}^D$$, $$V_{j}^D$$, and $$H_{i j}^D$$, respectively.

The received signals are processed by linear decoders, denoted as $$U_{UL}^k = [u_{k1, 1}, \ldots, u_{k1, d_{UL}^k}] \in \mathbb{C}^{N_0 \times d_{UL}^k}$$, and $$U_{DL}^j = [u_{j1, 1}, \ldots, u_{j1, d_{DL}^j}] \in \mathbb{C}^{N_0 \times d_{DL}^j}$$ by the BS and the jth DL user, respectively. Therefore, the estimate of data streams of the kth UL user at the BS is given as $$\hat{x}_{UL}^k = (U_{UL}^k)^H y_0$$, and similarly, the estimate of the data stream of the jth DL user is $$\hat{x}_{DL}^j = (U_{DL}^j)^H y_j$$. Using these estimates, the MSE of the kth UL and jth DL user can be written as in (55) and (56), respectively, shown at the bottom of the next page.

The power of the interference resulting from the UL users and BS at the lth PU can be written as

$$\mathcal{I}_{PU}^l = \sum_{k=1}^{K} \left[ \text{tr} \left\{ G_{lk} \left( V_{UL}^k \left( V_{UL}^k \right)^H \right) \right\} + \kappa \text{tr} \left\{ G_{lk} \right\}\right],$$

(57)

where $$G_{lk} \in \mathbb{C}^{N_0 \times M_0}$$ is the channel between the lth PU and kth UL user (lth PU and the BS).

A. Joint Beamforming Design

The optimization problem can be formulated as:

$$\min_{V_{UL}^k, V_{DL}^j} \sum_{k=1}^{K} \left[ \text{tr} \left\{ \text{MSE}_{UL}^k \right\} \right] + \sum_{j=1}^{J} \left[ \text{tr} \left\{ \text{MSE}_{DL}^j \right\} \right]$$

s.t. $$\left[ \text{tr} \left\{ V_{UL}^k \left( V_{UL}^k \right)^H \right\} \right] \leq P_k, \quad k \in S^{UL},$$

(59)

$$\sum_{j=1}^{J} \left[ \text{tr} \left\{ V_{DL}^j \left( V_{DL}^j \right)^H \right\} \right] \leq P_0,$$

(60)

$$I_{PU}^l \leq \lambda_l, \quad l = 1, \ldots, L,$$

(61)

where $$P_k$$ in (59) is the transmit power constraint at the kth UL user, and $$P_0$$ in (60) is the total power constraint at the BS. We use $$S^{UL}$$ and $$S^{DL}$$ to represent the set of K UL and J DL channels, respectively.

1) Simplification of Notations: To simplify the notations, we will combine UL and DL channels, similar to [28]. Denoting $$H_{ij}$$ and $$n_i$$ as

$$H_{ij} = \begin{cases} H_{UL}, & \text{if } i \in S^{UL}, j \in S^{UL}, \\ H_0, & \text{if } i \in S^{UL}, j \in S^{DL}, \\ H_{DL}, & \text{if } i \in S^{DL}, j \in S^{UL}, \\ H_{ij}, & \text{if } i \in S^{DL}, j \in S^{DL}, \end{cases}$$

and referring to $$V^X, \Sigma^X, X \in \{UL, DL\}$$ as $$V_j, U_i, \Sigma_i$$, respectively, the MSE of kth link, i.e., $$i \in S \subseteq S^{UL} \cup S^{DL}$$, can be written as

$$\text{MSE}_i = (U_i ^H H_{ij} V_j - I)(U_i ^H H_{ij} V_j - I)^H + U_i ^H \Sigma_i U_i,$$

(62)

where

$$\Sigma_i = \sum_{j \in S, j \neq i} H_{ij} V_j V_j ^H H_{ij} + \kappa \sum_{j \in S} H_{ij} \text{diag} \left( V_j V_j ^H \right) H_{ij}^H + \beta \sum_{j \in S} \text{diag} \left( H_{ij} V_j V_j ^H H_{ij}^H \right) + I.$$ 

(63)

2) Transceiver Design: Using the simplified notations, the optimization problem (58)–(61) can be rewritten as

$$\min_{V_{i}, U_{i}} \sum_{i \in S} \text{tr} \{ \text{MSE}_i \}$$

s.t. $$\text{tr} \{ V_j V_j ^H \} \leq P_j, \quad i \in S^{UL},$$

(65)

$$\sum_{i \in S^{UL}} \text{tr} \{ V_j V_j ^H \} \leq P_0,$$

(66)

$$I_{PU}^l \leq \lambda_l, \quad l = 1, \ldots, L.$$ 

(67)

The optimization problem (64)–(67) has the same formulation as the optimization problems proposed for MIMO interference channels, and thus under the fixed receive beamforming matrices, we can apply the individual-power constrained transceiver design proposed in Section III for UL users, $$i \in S^{UL}$$, and apply the sum-power constrained transceiver design proposed in Section IV-B for DL users, $$i \in S^{DL}$$. 

VI. SIMULATION RESULTS

In this section, we numerically investigate the MSE-based optimization problems for FD MIMO interference channel and cellular systems as a function of signal-to-noise ratio (SNR), interference-to-noise ratio (INR), and dynamic range parameters $$\kappa$$ and $$\beta$$. The tolerance (the difference between MSE of two iterations) of the proposed iterative algorithm is set to $$10^{-4}$$, the maximum number of iterations is set to 100, and the results are averaged over 1000 independent channel realizations. Since the optimization problems we are dealing with are non-convex, we
need to choose good initialization points to have a suboptimal solution with a good performance. In this paper, we use right singular matrices initialization [46].

### A. Interference Channel

For FD MIMO interference channel, for brevity, we set the same number of transmit and receive antennas at each node, i.e., $M_i = N_i = N$, $i \in K$, and each transmitter sends same number of data streams $d_i = N$, $i \in K$. We also set the same transmit power constraint for each node in the system, i.e., $P_i(b) = N, \forall (i, b)$. We define SNR of the nodes in the $i$th pair as $\text{SNR}_i = \frac{\rho_i N}{\eta_i N}$, and the INR from the nodes in the $j$th pair to the nodes in the $i$th pair as $\text{INR}_{ij} = \text{SNR}_i = \frac{\eta_j N, i \neq j}$. The INR of the self-interference channel at the nodes in the $i$th pair is denoted as $\text{INR}_{si}$. The cognitive radio system is installed within the service range of a primary network having $L = 2$ PUs. For simplicity, we set the same maximum allowed interfering power to the PUs (i.e., $\lambda = \lambda_i, l = 1, \ldots, L$) and same channel gains $\mu = \mu_i(j, l = 1, \ldots, L, i \in K, b = 1, 2$.

Fig. 3 illustrates the convergence behavior of the proposed Sum-MSE and Min-Max algorithms. It shows that the proposed algorithms converge in few steps, and it does so monotonically.

In our next example, we examine the sum-rate performance of the proposed Sum-MSE algorithm under individual and sum-power constraints, and Min-Max algorithm. The sum-rate of the MIMO interference channel can be expressed as

$$I_{\text{sum}} = \sum_{i=1}^{K} \sum_{b=1}^{2} d_i \log_2 \left(1 + \text{SNR}_i(b)\right),$$

where $\text{SNR}_i(b)$ is SNR of the $k$th stream of node $i(b)$ defined in (69) shown at the bottom of the next page. In (69), $r_i^{(b)}$ is the $k$th row of $\text{SNR}$, and $v_i^{(b)}$ is the $b$th column of $\text{SNR}_i$. As it is seen from Fig. 4, Sum-MSE algorithms achieve higher sum-rate than Min-Max algorithm, since they are designed to achieve the minimum total MSE of all the nodes in the system. Moreover, sum-power constrained and individual power constrained problems perform similar to each other, but sum-power constrained problem achieves higher sum-rate than the individual-power constrained problem, because the sum-power constrained problem is more relaxed than individual power constrained, and thus can allocate more power to the node that contributes more to achieve higher sum-rate.

$$\Sigma_k^{UL} = \sum_{j \neq k}^{K} H_{j}^{UL} V_{j}^{UL} (V_{j}^{UL})^H (H_{j}^{UL})^H + \kappa \sum_{j=1}^{K} H_{j}^{UL} \text{diag} \left(V_{j}^{UL} (V_{j}^{UL})^H \right) (H_{j}^{UL})^H$$

$$+ \sum_{j=1}^{J} \left( V_{j}^{DL} (V_{j}^{DL})^H + \kappa \text{diag} \left( V_{j}^{DL} (V_{j}^{DL})^H \right) \right) H_{0}^H + \beta \sum_{j=1}^{K} \text{diag} \left( H_{j}^{UL} V_{j}^{UL} (V_{j}^{UL})^H (H_{j}^{UL})^H \right)$$

$$+ \beta \sum_{j=1}^{J} \text{diag} \left( H_{0} V_{j}^{DL} (V_{j}^{DL})^H H_{0}^H \right) + I_{N_0},$$

$$\text{MSE}^{UL}_k = \left( (U_{k}^{UL})^H H_{k}^{UL} V_{k}^{UL} - I_{d_k^{UL}} \right) \left( (U_{k}^{UL})^H H_{k}^{UL} V_{k}^{UL} - I_{d_k^{UL}} \right)^H + (U_{k}^{UL})^H \Sigma_k^{UL} U_{k}^{UL},$$

$$\text{MSE}^{DL}_j = \left( (U_{j}^{DL})^H H_{j}^{DL} V_{j}^{DL} - I_{d_j^{DL}} \right) \left( (U_{j}^{DL})^H H_{j}^{DL} V_{j}^{DL} - I_{d_j^{DL}} \right)^H + (U_{j}^{DL})^H \Sigma_j^{DL} U_{j}^{DL}. $$

Fig. 3. Convergence behavior of Sum-MSE and Min-Max algorithms. Here, $N = 2$, $K = 3$, $\kappa = \beta = -40$ dB, $\text{SNR} = 20$ dB, $\text{INR} = 10$ dB, $\text{INR}_{ij} = 20$ dB, $\mu = 0$ dB, $\lambda = 0$ dB.

Fig. 4. Sum-rate comparison of the proposed algorithm versus SNR. Here, $N = 2$, $K = 3$, $\kappa = \beta = -40$ dB, $\text{INR} = 10$ dB, $\text{INR}_{ij} = 20$ dB, $\mu = 0$ dB, $\lambda = 0$ dB.
Fig. 5. MSE distribution of Sum-MSE and Min-Max MSE algorithms. The schemes 1 and 2 correspond to Sum-MSE and Min-Max MSE, respectively. For each scheme, the first six bars are the achieved user MSEs and the seventh bar is the sum MSE. Here, $N = 2$, $K = 3$, $\kappa = \beta = -40$ dB, $\text{SNR} = 20$ dB, $\text{INR} = 10$ dB, $\text{INRSI} = 20$ dB, $\mu = 0$ dB, $\lambda = 0$ dB.

The next example computes the MSE values for each node in the system for the Sum-MSE and Min-Max schemes out of one channel realization. We can see in Fig. 5 that the Sum-MSE scheme achieves the minimum total MSE over all the nodes and the Min-Max scheme introduces fairness, by ensuring that the all the nodes have almost the same MSE.

In our next example, we investigate the performance of the proposed Sum-MSE minimization algorithm under transmitter/receiver impairments ($\kappa$, $\beta$) in Fig. 6. As it is seen in Fig. 6, at low SNR values, since the thermal noise power dominates the transmitter/receiver distortion power, the sum-rate achieved under different $\kappa$, $\beta$ are close to each other. But, as the SNR increases, transmitter/receiver distortion power starts dominating the thermal noise power, and the performance of the system is determined by $\kappa$ and $\beta$. Moreover, decreasing $\kappa$ and $\beta$ (decreasing the distortion power) has a diminishing gain on the sum-rate performance of the system.

The next example examines the MSE performance of the proposed Sum-MSE algorithm for various maximum allowed interference at each PU, i.e., $\lambda$. It can be seen from Fig. 7 that as the interference constraint $\lambda$ decreases, the performance gets worse and MSE increases, since a lower interference threshold imposes a more stringent constraint.

\[
\text{SINR}_{ik}^{(b)} = \frac{\rho_{ik}^{(a)}H_{ii}^{(ab)}v_{ik}^{(b)}H_{ii}^{(ab)}H_{ii}^{(ab)}(r_{ik}^{(a)})^H}{r_{ik}^{(a)}(\Sigma_{i}^{(a)} + \sum_{j \neq k} \rho_{ij}H_{ij}^{(ab)}v_{ij}^{(b)}H_{ij}^{(ab)}H_{ij}^{(ab)}(r_{ij}^{(a)})^H)}.
\]  
(69)
B. Cellular System

In this section, we numerically investigate the MSE-based problem in MIMO FD multi-user system. We compare the proposed algorithm with the HD algorithm under the 3GPP LTE specifications for small cell deployments [62]. We consider small cells, since they are considered to be suitable for deployment of FD technology due to low transmit powers, short transmission distances and low mobility [2], [3], [63]. A single hexagonal cell having a BS in the center with $M_0 = 2$ transmit and $N_0 = 2$ receive antennas with randomly distributed $K = 3$ UL and $J = 3$ DL users equipped with 2 antennas is simulated. The cognitive radio system has $L = 2$ PU_s, with the same maximum allowed interfering power (i.e., $\lambda_I = 0 \text{dB}$). The channel between BS and users (both SU_s and PU_s) are assumed to experience the path loss model for line-of-sight (LOS), and the channel between UL and DL users are assumed to experience the path loss model for non-line-of-sight (NLOS) communications. Detailed simulation parameters are shown in Table II.

The channel gain between the BS to $k$th UL user is given by $H_{k}^{UL} = \sqrt{\kappa_k} h_{k}^{UL}$, where $h_{k}^{UL}$ denotes the small scale fading following a complex Gaussian distribution with zero mean and unit variance, and $\kappa_{k}^{UL} = 10^{-X/10}$, $X \in \{\text{LOS, NLOS}\}$ represents the large scale fading consisting of path loss and shadowing, where LOS and NLOS are calculated from a specific path loss model given in Table II. The channels between BS and DL users, between UL users and DL users, between BS and PU_s, and between UL users and PU_s are defined similarly. For the self-interference channel, we adopt the model in [2] and [13], in which the self-interference channel is distributed as $H_0 \sim C\mathcal{N}(0, C_{\text{SI}}^2)$, where $K_R$ is the Rician factor, $H_0$ is a deterministic matrix, and $C_{\text{SI}}^2$ denotes the self-interference attenuation level [2].

The average sum-rate achieved in FD system for uplink (FD-UL), downlink (FD-DL), and the entire system (FD-Sum) is shown in Fig. 8. It is seen that while the sum-rate achieved in FD-UL system always decreases as $C_{\text{SI}}^2$ increases, the sum-rate achieved in FD-DL system decreases until a certain value of $C_{\text{SI}}^2 = -80 \text{dB}$ and increases after that. The decrease in the sum-rate performance of the FD-UL system is intuitive, since as the self-interference suppression capability decreases, a greater amount of self-interference power is added to the background thermal noise. The changing sum-rate performance of the FD-DL system is explained as follows. Since the proposed algorithm optimizes the UL and DL channels jointly, i.e., Sum-MSE minimization of the entire FD system, at low $C_{\text{SI}}^2$ values, the joint optimization scheme slightly reduces the transmit power of the DL channel to maintain a good performance of UL system. But, as $C_{\text{SI}}^2$ increases, the self-interference power starts overwhelming the desired signals coming from the UL users, which reduces the achievable sum-rate in the UL channel. Thus, the performance of the entire system is determined mostly by the DL transmission. Therefore, reducing the transmit power in the UL channel and concentrating on DL channel is more beneficial. And also, as the transmission power of the UL users is reduced, CCI is also reduced, resulting in improved performance in the DL channel. The same observation for sum-rate maximization problem has been reported in [2]. Moreover, the sum-rate comparison of FD and HD systems in UL and DL channels is also depicted in Fig. 8. As it is seen, the sum-rate achieved in FD-DL system is always higher than that of HD-DL system, while FD-UL outperforms HD-UL in terms of sum-rate only when the self-interference is substantially suppressed. The sum-rate gains of the FD system over HD system as a function of $C_{\text{SI}}^2$ is shown in Table III. It is demonstrated that for FD system to achieve a higher sum-rate than HD system, $C_{\text{SI}}^2$ must be at least $-70 \text{dB}$.

<table>
<thead>
<tr>
<th>$C_{\text{SI}}^2$</th>
<th>$-110 \text{dB}$</th>
<th>$-100 \text{dB}$</th>
<th>$-90 \text{dB}$</th>
<th>$-80 \text{dB}$</th>
<th>$-70 \text{dB}$</th>
<th>$-60 \text{dB}$</th>
<th>$-50 \text{dB}$</th>
<th>$-40 \text{dB}$</th>
<th>$-30 \text{dB}$</th>
<th>$-20 \text{dB}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uplink</td>
<td>83%</td>
<td>65%</td>
<td>45%</td>
<td>23%</td>
<td>3%</td>
<td>-16%</td>
<td>-51%</td>
<td>-78%</td>
<td>-94%</td>
<td>-99%</td>
</tr>
<tr>
<td>Downlink</td>
<td>50%</td>
<td>40%</td>
<td>25%</td>
<td>12%</td>
<td>5%</td>
<td>14%</td>
<td>36%</td>
<td>65%</td>
<td>91%</td>
<td>93%</td>
</tr>
</tbody>
</table>

Note that as seen in Fig. 8, at very high self-interference cancellation levels, the DL rate is lower than the UL rate. The reason is that the quality of the DL channel is degraded by the CCI, while the UL channel is not affected by the self-interference at high self-interference cancellation levels. Since the DL service is larger than the UL service in general, in order to give priority to (increase) the DL rate, we can take weighted sum-MSE as the objective function, and give DL larger weights. The weighted sum-MSE problem does not change the proposed algorithm.

If the residual self-interference is high, it is concluded that FD mode is not a feasible choice. If the level of residual self-interference is time varying and can be measured in real time, then this paper also suggests that a dynamic switching between FD and HD modes may have an advantage. By using a multiple timeslot data transmission as in [17], [18], we can exploit both spatial and temporal freedoms of the MIMO links. Particularly, the use of distinct time slots gives the freedom to switch between FD and HD signaling depending on the power of the self-interference channel.
MSE_i = tr[MSE_i]

= tr \left\{ \left( \sqrt{\rho_i} R_i^{(a)} H_{ii}^{(ab)} V_i^{(b)} - I_{d_i} \right) \left( \sqrt{\rho_i} R_i^{(a)} H_{ii}^{(ab)} V_i^{(b)} - I_{d_i} \right)^H \right\} \\
+ \rho_i \kappa tr \left\{ R_i^{(a)} H_{ii}^{(ab)} \text{diag} \left( V_i^{(b)} (V_i^{(b)})^H \right) (H_{ii}^{(ab)})^H (R_i^{(a)})^H \right\} \\
+ \beta \rho_i tr \left\{ R_i^{(a)} H_{ii}^{(ab)} \text{diag} \left( V_i^{(b)} (V_i^{(b)})^H (H_{ii}^{(ab)})^H \right) (R_i^{(a)})^H \right\} \\
+ \eta_{ii} \kappa tr \left\{ R_i^{(a)} H_{ii}^{(ab)} \text{diag} \left( V_i^{(b)} (V_i^{(b)})^H \right) (H_{ii}^{(ab)})^H (R_i^{(a)})^H \right\} \\
+ \beta \eta_{ii} tr \left\{ R_i^{(a)} \text{diag} \left( H_{ii}^{(ac)} V_i^{(c)} (V_i^{(c)})^H \right) (H_{ij}^{(ac)})^H (R_i^{(a)})^H \right\} \\
+ \sum_{j=1, j \neq i}^K \sum_{c=1}^2 \eta_{ij} tr \left\{ R_i^{(a)} H_{ij}^{(ac)} V_j^{(c)} (V_j^{(c)})^H (H_{ij}^{(ac)})^H (R_i^{(a)})^H \right\} + tr \left\{ R_i^{(a)} (R_i^{(a)})^H \right\} . \tag{70}

MSE_i = \left\| vec \left( \sqrt{\rho_i} R_i^{(a)} H_{ii}^{(ab)} V_i^{(b)} - vec(I_{d_i}) \right) \right\|_2^2 + \rho_i \beta \left\| vec \left( \text{diag} \left( (R_i^{(a)})^H R_i^{(a)} \right) \right)^{1/2} H_{ii}^{(ab)} V_i^{(b)} \right\|_2^2 \\
+ \rho_i \kappa \left\| vec \left( \text{diag} \left( (H_{ii}^{(ab)})^H (R_i^{(a)})^H R_i^{(a)} H_{ii}^{(ab)} \right) \right)^{1/2} V_i^{(b)} \right\|_2^2 \\
+ \eta_{ii} \kappa \left\| vec \left( \text{diag} \left( (H_{ii}^{(ac)})^H (R_i^{(a)})^H R_i^{(a)} H_{ii}^{(ac)} \right) \right)^{1/2} V_i^{(c)} \right\|_2^2 \\
+ \sum_{j=1, j \neq i}^K \sum_{c=1}^2 \eta_{ij} \left\| vec \left( (R_i^{(a)} H_{ij}^{(ac)} V_j^{(c)} \right) \right\|_2^2 + \eta_{ii} \beta \left\| vec \left( \text{diag} \left( (R_i^{(a)})^H R_i^{(a)} \right) \right)^{1/2} H_{ii}^{(ac)} V_i^{(c)} \right\|_2^2 \\
+ \sum_{j=1, j \neq i}^K \sum_{c=1}^2 \eta_{ij} \beta \left\| vec \left( \text{diag} \left( (R_i^{(a)})^H R_i^{(a)} \right) \right)^{1/2} H_{ij}^{(ac)} V_j^{(c)} \right\|_2^2 + tr \left\{ R_i^{(a)} (R_i^{(a)})^H \right\} . \tag{71}

in the simulations that the proposed Sum-MSE minimization scheme achieves the minimum total MSE, and the proposed Min-Max scheme almost achieves the same MSE for every user. Therefore, the Sum-MSE minimization algorithm can be used when there are only best effort services in the system, while it is better to use Min-Max MSE algorithm when there are services with QoS requirements because of the fairness it introduces. Moreover, we show that the proposed algorithm is not only applicable to FD MIMO cognitive interference channels, but also applicable to FD cognitive cellular systems. It has been shown in simulations that the sum-rate achieved by FD system is higher than that of HD system under reasonable self-interference cancellation values.

**APPENDIX**

Using (12) and (14), MSE_i = tr[MSE_i] can be written as in (70) given at the top of the page. Applying the vec(-) operation, and the identity \( \| vec(A) \|_2^2 = tr(AA^H) \), MSE_i in (70) can be rewritten as in (71) given at the top of the page. Then using the identity vec(ABC) = (C^T \otimes A) vec(B), (71) can be written as (25).
Using the identity vec(ABC) = (CT ⊗ A)vec(B), (72) can be written as (26).

REFERENCES


