Networking by Parallel Relays - Diversity, Lifetime and Routing Overhead
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Abstract—For several decades, research activities in mobile ad hoc networking have predominantly relied on the notion of serial relays where a data packet hops from one node to another in a serial fashion. Recent research activities in signal processing and information theory suggest that parallel relays have a greater potential than serial relays. In this paper, a significant diversity gain of using parallel relays over serial relays is highlighted, the lifetimes of a single-path route of parallel relays and multi-path routes of serial relays are compared, and the routing overheads for each type of routes are also investigated. This study strongly supports that parallel relays have great advantages over serial relays in dense and mobile ad hoc networks.

I. INTRODUCTION
Mobile ad hoc networks (MANET) are important for both military and civilian applications where high mobility of nodes and absence of infrastructure are both a desired feature as well as a major source of challenges. Research activities in MANET have traditionally and almost exclusively concentrated on the network layer, and each node in the network is considered to be a “black box” that is independent from others at the physical layer, e.g., see [1]. However, recent research activities in information theory and signal processing have shown that the capacity between any two nodes highly depends on their neighboring nodes, e.g., see [2], [3]. Therefore, to achieve a higher network throughput, neighboring nodes should not only cooperate at the network layer but also at the physical layer. In a recent work [4], a chain of parallel relays is introduced as one type of cooperations among neighboring nodes, and the notion of parallel relays is shown to be a feasible and advantageous alternative to the traditional serial relays from a networking perspective.

In this paper, we explore further the notion of parallel relays by comparing a single-path route of parallel relays with multi-path routes of serial relays. Multi-path routes of serial relays have been actively investigated within the networking community as a multi-path route can reduce the end-to-end delay between a source node and a destination node [7], [8]. However, a multi-path route causes a larger area of interference than a single-path route. We will show that a single-path route of parallel relays not only has a higher diversity of packet loss rate (i.e., more saving of power) than a route of serial relays in a high mobility environment, but also lives longer than multi-path routes of serial relays when each node has a random lifetime, and also requires less (normalized) routing overheads than multi-path routes of serial relays.

For reference purpose, we will consider a single-path route of serial relays (SP-SR), which was also compared to a single-path route of parallel relays (SR-PR) in [4]. For multi-path routes of serial relays [7], [8], we consider two options: completely disjoint multi-path route of serial relays (CDMP-SR) and partially disjoint multi-path route of serial relays (PDMP-SR).

In order to compare the performances of routes, routing protocols must be specified. For SP-PR, we formulate an on-demand routing protocol. For SP-SR, CDMP-SR and PDMP-SR, the existing on-demand routing protocols will be assumed. There are three major parts in routing: route discovery to establish a route, route maintenance to keep a preestablished route operational as long as possible, and scheduling policy to forward data packets along a preestablished operational route. The routing overhead is the number of packets transmitted in the network to establish and maintain a route. The route lifetime is the time that an preestablished route lasts until it is beyond repair. The diversity of a route is the slope of the packet loss rate versus the averaged signal-to-noise ratio on a log-log scale.

II. ON-DEMAND ROUTING OF PARALLEL RELAYS
For a single-path route of parallel relays (SP-PR), we formulate an on-demand route discovery protocol next, which is different from that in [4] but similar to that in [9].

A. Route Discovery
Stage 1 (flooding): The source broadcasts a route request (RREQ) packet to its neighbors. The RREQ contains the source ID, the destination ID, a lifetime counter and other control information. Each node that receives such a RREQ packet checks whether such a packet was received before. If yes, the packet is discarded. If no, the lifetime counter of the packet is checked for expiration. If expired, the packet is dropped (unless the current node is the destination). If not expired, the packet is updated with the ID of the current node, the lifetime counter is reduced by one, and the packet is rebroadcasted.

Stage 2 (reply): When the destination receives the first RREQ packet, the primary path between the source and the destination is determined by the destination based on the information registered in the received RREQ packet. A route reply (RREP) packet containing the IDs of all nodes on the primary path is generated and sent back along with the primary path to the source. Other RREQ packets arriving at the destination will be discarded. The nodes along the primary path are assigned as Head Relays.

We assume that each node in the network knows its nearest (one-hop) neighbors. Once a RREP arrives at tier \( t \), the Head Relay in tier \( t \) broadcasts a local route request to its nearest neighbors. Each neighbor that receives the request now knows the Head relays in tier \( t − 1 \) and tier \( t + 1 \). If a neighbor has a link to the Head Relays in tier \( t − 1 \) and tier \( t + 1 \), it sends back a reply with its own ID and table of neighbors. The Head Relay in tier \( t \) decides on its Assistant Relays and then broadcasts a confirmation to them. The Assistant Relays in tier \( t \) should have as many possible links to the relays in tier \( t + 1 \). The maximum possible number of relays in each tier is set to be \( N \).

It can be realized that all relays in tier \( t \) knows all relays in tier \( t + 1 \) as the Head Relay in tier \( t \) knows the IDs of all relays in tier \( t + 1 \) from the RREP packet sent from the Head Relay of tier \( t + 1 \).
B. Route Maintenance

For a single-path route of parallel relays, route maintenance required is minimum. In fact, as long as there is at least one relay in each tier that is operational, zero maintenance is required. If all relays in tier \( t \) fail, the Head Relay in the tier \( t - 1 \) should detect it and a route error (RERR) packet is then generated and forwarded to all relays in tiers \( t - 1 \) and \( t - 2 \). The RERR packet eventually arrives at the source. The source can then initiate a new process of route discovery. Note that when a Head Relay fails, another relay in the tier should become the new Head Relay.

C. Scheduling Policy

After a route is established, the source can send a data packet to all relays in tier 1. A data packet hops from a transmitting tier to a receiving tier. A receiving tier of a packet becomes a transmitting tier of the same packet if and only if at least one of the receiving relays has received the packet correctly. If a packet is not received correctly by any relays at the receiving tier, the packet will be retransmitted until it is received correctly or until the link is considered to be broken. The data packet is encoded differently at the expected lifetime of a node. This assumption differs from that of our analysis of route discovery and scheduling policy of parallel relays are available in [4].

III. Assumptions for Lifetime and Overhead Analysis

For our analysis of route lifetimes and routing overheads of SP-SR, CDMP-SR, PDMP-SR and SP-PR, we set the following assumptions.

Assumption 1: The transmission from each node is omnidirectional with radius \( r_0 \). The nodes in the network are uniformly randomly distributed with an average node density \( \rho \). This is a natural assumption in the absence of any prior information on the network topology. Also see [11].

Assumption 2: The length (number of hops) for the primary path in all of SP-SR, CDMP-SR, PDMP-SR and SP-PR is \( L+1 \) (\( L \geq 1 \)). For CDMP-SR, there are \( N \) completely disjoint paths between the source and the destination. For PDMP-SR, there are \( N \) completely disjoint paths between each node on the primary path and the destination. For SP-PR, there are \( N \) relays within each tier. The lengths of the non-primary paths in CDMP-SR are \( H^{(i)} \geq L + 1 \) where \( i = 2, ..., N \).

For the non-primary paths in PDMP-SR, the length of the disjoint (partial) path between ith node in the primary path and the destination is \( H^{(i)} \geq L - i + 1 \) where \( k = 2, ..., N \) and \( i = 0, 1, ..., L \). Note that for a fixed \( N \), there are \( NL \) nodes in CDMP-SR, \( NL \) nodes in SP-PR, and \( (N - 1)L(L + 1)/2 + L \) nodes in PDMP-SR. (Clearly, for a fixed \( N \), PDMP-SR requires more nodes than other routes do.)

Assumption 3: The lifetimes of the source and the destination are excluded from the consideration of the route lifetime. (In [5], the destination node is included as part of a route.) The lifetimes of nodes are denoted by \( X_i \) and assumed to be i.i.d. random variables with the probability density function \( f_X(x) = \lambda e^{-\lambda x} \), where \( 1/\lambda \) is the expected lifetime of a node. This assumption differs from that of link lifetimes as used in [7], [12].

Assumption 4: For the route request (flooding) stage of route discovery, the lifetime (in hop numbers) of a RREQ packet is set to be \( T_f (T_f \geq L + 1) \), i.e., a RREQ packet from a source can only hop \( T_f \) times [10]. For the route reply stage, we assume that no memory of partial route is available at any node and only the destination can respond to a received RREQ [10]. For analysis of route maintenance, the probability that two or more relays fail simultaneously is assumed to be negligible.

IV. Diversity of Parallel Relays

For a single-path route of \( N \) parallel relays at each tier, it has been shown in [4] that the averaged packet loss rate at tier \( t \) is in the order of \( SNR^{-d(t)} \) for large \( SNR \), where \( SNR \) is the signal-to-noise ratio (at each receiving relay) averaged over random channel fading, and

\[
\begin{align*}
  d(1) &= N \\
  d(2) &= 2N - 1 \\
  d(t) &= \min \{ tN - t + 1, N^2 \}
\end{align*}
\]

An alternative (more rigorous) proof of the above result is outlined here. First, we define \( P_b \) as the averaged packet loss rate (PLR) at a receiving node in response to \( k \) transmitting nodes. With use of a full-diversity space-time code, it is known [4] that \( P_b \propto SNR^{-d(t)} \) for large \( SNR \).

We now define \( \tilde{G}_1(g) \) as the averaged probability that \( g \) receiving relays in tier \( t \) lose a packet, and \( G(t)(g) \) as the averaged probability that \( g \) receiving relays in tier \( t \) lose a packet given that there is a transmission from tier \( t - 1 \). Based on these definitions, we can show that for \( t = 1 \),

\[
\tilde{G}_1(g) = \tilde{G}_1(g) = \left( \frac{N}{g} \right) P^0_k (1-P_k)^{N-g} \leq N \leq N. \tag{1}
\]

For \( t \geq 2 \) and \( 0 \leq g \leq N \),

\[
G(t)(g) = \left( \frac{N}{g} \right) \sum_{k=1}^{N} P_k^g (1-P_k)^{N-g} \tilde{G}_{t-1}(N-k) \tilde{G}_{t-1}(N) \tag{2}
\]

and

\[
\tilde{G}_1(g) = \left( \frac{N}{g} \right) \sum_{k=1}^{N} P_k^g (1-P_k)^{N-g} \tilde{G}_{t-1}(N-k) \tilde{G}_{t-1}(N), \tag{3}
\]

where \( \delta(g) = 1 \) if \( g = 0 \), and zero otherwise. Since \( P_k \propto \rho^{k} \), we have,

\[
\tilde{G}_1(g) \propto \rho^{g} \leq g \leq N, \tag{4}
\]

\[
\tilde{G}_2(g) \propto \begin{cases} 
1 & g = 0 \\
\rho^{(N-1)+g} & \rho^{N}, g < N \\
\rho^{N} & g = N
\end{cases} \tag{5}
\]

and for \( t \geq 3 \),

\[
\tilde{G}_1(g) \propto \begin{cases} 
1 & g = 0 \\
\rho^{(t-1)(N-1)+g} + \rho^{N} & \rho^{N}, 0 < g < N \\
\rho^{N} & g = N
\end{cases} \tag{6}
\]

Thus,

\[
\tilde{G}_1(g) \propto \rho^{g} \leq g \leq N, \tag{7}
\]

\[
\tilde{G}_2(g) \propto \begin{cases} 
1 & g = 0 \\
\rho^{(N-1)+g} & \rho^{N}, 0 < g \leq N \\
\rho^{N} & g \leq N \tag{8}
\end{cases}
\]

and for \( t \geq 3 \),

\[
\tilde{G}_1(g) \propto \begin{cases} 
1 & g = 0 \\
\rho^{(t-1)(N-1)+g} + \rho^{N} & \rho^{N}, 0 < g \leq N \\
\rho^{N} & g \leq N \tag{9}
\end{cases}
\]

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In particular, $G_t(N)$ measures the averaged probability of link failure between tiers $t-1$ and $t$. It follows that $G_1(N) \propto \rho^N$, $G_2(N) \propto \rho^{2N}$, and for $t \geq 3$, we have,
\begin{equation}
G_t(N) \propto \rho^{\min(tN-t+1,N^2)}
\end{equation}
which completes the proof of $d(t)$ shown earlier.

Furthermore, a normalized route throughput of SP-PR can be shown to be
\begin{equation}
C(N) = \frac{1}{\sum_{i=1}^{L-1} \frac{G(i+1)}{G(i)} + \frac{G(L)}{G(L-1)(1-P_t)}} \approx \frac{1 - P_t^N}{1 - P_t}
\end{equation}
Assuming $P_{k+1} < P_k$, we have $G_t(N) < P_t^N$ and $G_L+1(1) < P_t$. It is known that the route throughput of SP-SR is $C(1) = \frac{1 - P_t^N}{1 - P_t}$.
Then, the throughput gain $K(N)$ of parallel relays over serial relays is
\begin{equation}
K(N) = \frac{C(N)}{C(1)} > \frac{L + 1}{L - P_t^N} (1 - P_t) \approx \frac{1 - P_t^N}{1 - P_t}
\end{equation}
which confirms a result from [4] but from a different angle. (The last expression in the above equation would be a strict lower bound on the throughput gain if the destination node is excluded from the computation of route throughput.)

V. LIFETIMES OF ROUTES

It is clear that the lifetime of a route depends on the structure of the route. Since the lifetime of each node is exponentially distributed, we define the normalized expected route lifetime as:
\begin{equation}
C = \lambda E(T),
\end{equation}
where $1/\lambda$ is the expected node lifetime, and $C$ is purely determined by the routing structure.

A. SP-SR

As there is only one relay in each hop, the route lifetime is $T_{SP-SR} = \min \{ X^{(1)}, X^{(2)}, \ldots, X^{(L)} \}$. Thus, $C_{SP-SR}(L) = \frac{1}{L}$.

B. CDMP-SR

Most of the recently proposed multi-path routing schemes fall in this category [8]. The lifetime of the whole route is given by $T_{CDMP-SR} = \max \{ H_i, Z_i, \ldots, Z_k \}$, where $Z_i = \min \{ X^{(1)}, \ldots, X^{(H(i))} \}$. If $H(i) = L+1, \forall i$, the cdf of $T_{CDMP-SR}$ is
\begin{equation}
F_{CDMP-SR}(t) = (1 - e^{-\lambda t})^N,
\end{equation}
Thus, the normalized expected route lifetime is
\begin{equation}
C_{CDMP-SR}(N, L) = \int_0^1 (1 - 1 - x^k)^N dx = \frac{H_N}{L},
\end{equation}
with $H_N = \sum_{k=1}^N \frac{1}{k}$ the harmonic number of $N$.

C. PDMP-SR

In [7], a partially disjoint multi-path routing strategy is proposed. This strategy ensures that any relay on the primary path is connected to one or more alternative paths to the destination. Thus, when a relay on the primary path fails, the relay in the previous tier can detect the failure and readily forward packets along some of its alternative paths to the destination. With Assumption 2, the lifetime of this type of routes is $T_{PDMP-SR} = \max \{ Y_1, Y_2, \ldots, Y_{L+2} \}$, where $Y_1 = \min \{ X^{(1)}, X^{(2)}, \ldots, X^{(L+2)} \}$, $Y_2 = Z_2$ and $Y_i = \min \{ X^{(i)}, X^{(i+1)}, \ldots, X^{(L+2)}, Z_i \}$ for $3 \leq i \leq L+2$.

Here, $Z_i$ is the lifetime of the bunch of $N - 1$ alternative paths starting from the Head Relay in tier $(i - 2)$; i.e., $Z_i = \max \{ W_1^{(i)}, \ldots, W_{L}^{(i)} \}$, where $W_l^{(i)}$ is the ith path $(2 \leq l \leq N)$ in ith bunch of paths, and
\begin{equation}
W_l^{(i)} = \min \{ X^{(i-1)}_{l,1}, X^{(i)}_{l,1}, \ldots, X^{(H(i)-2)}_{l,i} + (i-2) \}.
\end{equation}
If $H(i) = L - i + 1, \forall L$, the normalized expected route lifetime can be shown to be
\begin{equation}
C_{PDMP-SR}(N, L) = \frac{H_N}{L} + \Delta C,
\end{equation}
with $\Delta C := \sum_{j=1}^{N} j^i \frac{1}{x^N} (1 - e^{-\lambda t})^{N-1} dx$.

D. SP-PR

In SP-PR, the relays in each tier can choose to transmit and receive data within a single channel. Symbol synchronization and space-time modulation can be achieved at the physical layer if a single or multiple narrow-band symbol carrier(s) are used. Without an accurate symbol synchronization, space-time codes designed for frequency selective channels should be used [6], which effectively expands a (virtual) symbol interval. The lifetime of the route is $T_{SP-PR} = \min \{ Z_i, Z_{i+1}, \ldots, Z_k \}$, where $Z_k = \max \{ X^{(k)}_i, X^{(k+1)}_i, \ldots, X^{(k+2)}_i \}$. The cdf of the route lifetime is:
\begin{equation}
F_{SP-PR}(t) = 1 - [1 - (1 - e^{-\lambda t})^{L+1}].
\end{equation}
Thus, the normalized expected lifetime of the route is
\begin{equation}
C_{SP-PR}(N, L) = \int_0^1 [1 - (1 - x)^N]^L dx
= \sum_{k=1}^{L} \left( \frac{L}{k} \right) (-1)^{k+1} H_{k,N}.
\end{equation}

E. Asymptotical lifetimes of CDMP-SR, PDMP-SR and SP-PR

Using the following fact of the harmonic number $H_N$ ($\forall N \geq 1$):
\begin{equation}
\ln(N) + \gamma \leq H_N \leq \ln(N) + \gamma + \frac{1}{2N},
\end{equation}
with $\gamma \approx 0.577216$ (Euler-Mascheroni Constant), we can show that
\begin{equation}
\lim_{N \to \infty} C_{CDMP-SR}(N, L) = \frac{1}{L} \ln(N),
\end{equation}
\begin{equation}
\lim_{N \to \infty} C_{PDMP-SR}(N, L) = \frac{1}{L} \ln(N),
\end{equation}
\begin{equation}
\lim_{N \to \infty} C_{SP-PR}(N, L) = \ln(N).
\end{equation}

It is interesting to observe that when $N \to \infty$, the $C$ values of CDMP-SR, PDMP-SR and SP-PR all increase in the order of $\ln(N)$, CDMP-SR and PDMP-SR have the same normalized expected route lifetime, and the lifetime of SP-PR is $L$ times as long as that of CDMP-SR and PDMP-SR.

VI. ROUTING OVERHEADS

The total routing overhead ($O$) of a routing strategy is the sum of the overheads in route discovery ($O_{RD}$) and route maintenance ($O_{RM}$). There are also two stages in route discovery, i.e., route request ($O_{R}$) and route reply ($O_{A}$). We assume that each routing packet has the same length (in bits), and thus the total number of routing packets transmitted in the network represents the routing overhead.
A. Overhead in Route Request

The route request stage for all routing strategies considered in this paper is the same. So, a common overhead of route request is shared by all of SP-SR, CDMP-SR, PDMP-SR and SP-PR.

Assume that the lifetime counter of a RREQ packet is set at the source to be $T_f$. Then, a RREQ packet can eventually reach all nodes that are within $T_f$ hops away from the source. Each node reached by a RREQ packet within $T_f - 1$ hops rebroadcasts the packet once. According to Assumption 1, the expected number of nodes within $T_f - 1$ hops is

$$E(\text{nodes within } (T_f - 1) \text{ hops}) \approx \rho \pi (T_f - 1)^2 r_0^2.$$  \hspace{1cm} (20)

So, the expected flooding overhead $E(O_F)$ is approximately $\rho \pi (T_f - 1)^2 r_0^2$. It will be seen that this overhead dominates all other overheads in routing.

B. Overhead in Route Reply

When the destination receives a RREQ packet, it sends a RREP packet back to the source through the relay nodes. The overhead in this stage (i.e., the number of transmissions of a RREP packet) depends on the choice of the routing strategies. We can show that

1) For SP-SR: $O_R = L + 1.

2) For CDMP-SR: $O_R = \sum_{i=2}^{N} H(i) + (L + 1) \geq N(L + 1).

3) For PDMP-SR:

$$O_R = \sum_{i=1}^{L} \sum_{k=2}^{N} H(i)^{(k)} + \sum_{k=2}^{N} H(k) + L + 1 \geq (L + 1)[(N - 1)L/2] + N].$$ \hspace{1cm} (21)

4) For SP-RP:

$$E(O_R) = L(\rho \pi r_0^2) + 3L + 1 \hspace{1cm} (22)$$

where the first term is because of the one-hop flooding from each Head Relay along the primary path.

C. Overhead in Route Maintenance

The overhead in route maintenance is the number of RERR packets transmitted when a breakage happens.

1) SP-SR: As there is only one path in SP-SR, any node failure in this path means a breakage occurs. When the $i$th node in the path fails, the $(i - 1)$th node can detect it, then generate and send a RERR packet back to the source along the reverse path. The expected number of RERR packets (or transmissions of such RERR packets) is $E(O_{RM}) = \sum_{i=1}^{N} H(i) = N(L - 1)/2.$  

2) CDMP-SR: As there are $N$ disjoint paths from the source to the destination and any failure in the active path(s) will be reported to the source by RERR packets, the total number of RERR packets transmitted by the time all paths fail is

$$E(O_{RM}) = \sum_{i=1}^{N} H(i) = \sum_{i=1}^{N} H(i)^{(i)} = \sum_{i=1}^{N} H(i) + (L + 1) \geq (N - 1)L/2 + N].$$ \hspace{1cm} (23)

3) PDMP-SR: In this case, each relay on the primary path is equipped with $N - 1$ alternative paths to the destination. We assume that the data initially are forwarded along the primary path. If the $i$th relay (1 $\leq i \leq L$) in the primary path fails, the $(i - 1)$th relay detects it and uses one or more of its alternative paths to forward data [7]. We can show that the expected number of the RERR packets transmitted during the lifetime of PDMP-SR is

$$E(O_{RM}) \geq \sum_{i=1}^{L} \frac{1}{T} \sum_{j=1}^{L} G_i, \hspace{1cm} (24)$$

where the equality holds if and only if $H(i)^{(i)} = L - i + 1$ for all $1 \leq i \leq L$ and $2 \leq k \leq N$, and $G_i$ is the expected number of RERR packets transmitted if the failure starts from the $i$th relay in the primary path (1 $\leq i \leq L$). For $i = 1$, $G_1 = (N - 1)(L - 1)/2$. For $2 \leq i \leq L$, with $p_m = \frac{1}{L}(1 - p)^{L - 1} - (1 - p)^{L - 1}$, $G_i = (L - i + 1)\sum_{k=1}^{L-1} p_m + (1 - \sum_{j=1}^{N-1} p_j) \sum_{k=1}^{N-1} j + G_i$.

4) SP-PR: With the description in section 2, SP-PR has the same overhead as SP-SR in route maintenance, i.e., $E(O_{RM}) = L(L - 1)/2.$

D. Total Normalized Routing Overhead

The total routing overhead is $O = O_F + O_R + O_{RM}$. But as illustrated later, the last two terms are negligible to the first. So, the absolute routing overhead is about the same for all of SP-SR, CDMP-SR, PDMP-SR and SP-PR. But since different route structures have different lifetimes, a more meaningful measure is the normalized routing overhead [15]:

$$E(O)/E(T) = \lambda R, \hspace{1cm} (25)$$

where $T$ is the route lifetime.

VII. NUMERICAL RESULTS

Fig. 1 shows a numerical comparison of the normalized expected route lifetimes (i.e., $C$’s) of the four strategies discussed where $N = 2 \sim 200$ and $L = 3$. Clearly, SP-PR has the best reliability among all strategies, and SP-PR benefits the most by increasing $N$. Fig. 2 shows a comparison of $C$’s for $L = 2 \sim 20$ and $N = 2$. As expected, all values of $C$ decreases with $L$. In general, for $L \geq 2$ and $N \geq 2$, we have

$$C_{SP-SR} < C_{CDMP-SR} < C_{PDMP-SR} < C_{SP-PR}. \hspace{1cm} (26)$$

Fig. 3 shows the expected absolute overheads of the four routing strategies discussed. We assume $\rho \pi r_0^2 = 10$. The difference between the curves is very small. Fig. 4 shows a comparison of the normalized routing overheads (i.e., $R$’s) where $N = 2$ and $L = 2 \sim 20$. We see that SP-PR has the smallest $R$ values among all strategies.
have a higher total-and-effective uniform throughput (i.e., throughput between any pair of nodes) than a network that does not use parallel relays. Such a throughput takes into account several major issues across multiple layers of implementations. Further research into this direction should lead to even more exciting discoveries.

REFERENCES


