Two-qubit decoherence mechanisms revealed via quantum process tomography


QPT basics

Quantum operation (completely positive **linear** map)

(i.e. positive Hermitian (d.m.) → positive Hermitian (d.m.); “completely” means even with ancilla)

$$\rho = L[\rho^0], \quad \rho_{ij} = \sum_{k,l=0}^{d-1} L_{ij,kl} \rho_{kl}^0$$

Can be represented with Kraus operators (not uniquely):  

$$L[\rho^0] = \sum_n K_n \rho^0 K_n^\dagger$$

Unique representation:  

$$L[\rho^0] = \sum_{m,n=0}^{d^2-1} \chi_{mn} E_m \rho^0 E_n^\dagger$$

where $E_m$ is a chosen (arbitrary) basis of operators

“Pauli basis”: products of $(I,X,Y,Z)$ for each qubit (jargon: $X=\sigma_X$, etc.)

Sometimes modified Pauli basis: $(I,X,-iY,Z)$
\[ L[\rho^0] = \sum_{m,n=0}^{d^2-1} \chi_{mn} E_m \rho^0 E_n^\dagger \]

- not quite intuitive
- Hermitian, \(d^2 \times d^2\), positive-semidefinite
- \(\text{Tr } \chi = 1 \) (usually)
- for a unitary operation \(U\)
  \[ \chi_{mn} = k_m k_n^*, \quad U = \sum_{n=0}^{d^2-1} k_n E_n \]

**Markovian decoherence (\(\lambda\)-matrix)**

\[
\frac{d}{dt} \rho = M \rho \quad \text{then} \quad L = \exp(Mt) \quad \text{(if treat density matrix as a vector)}
\]

\[
M = M_{\text{coh}} + D \quad \text{Can introduce } \lambda\text{-matrix:} \quad D[\rho] = \sum_{m,n=0}^{d^2-1} \lambda_{mn} E_m \rho E_n^\dagger
\]

\(\lambda\)-matrix is similar to \(\chi\)-matrix, but \(\text{Tr } \lambda = 0\) and different dimension (\([\chi] = [\lambda t])\)

If \(M_{\text{coh}} = 0\), then at weak decoherence
\[ \chi \approx \chi' + \lambda t, \quad \chi'_{mn} = \delta_{m0} \delta_{n0} \]
**Local vs. non-local decoherence**

**Uncoupled** 2-qubit system and local decoherence: \( \chi = \chi^{(1)} \otimes \chi^{(2)} \)

Natural to introduce nonlocality parameter for decoherence:

\[
\epsilon_{NL} = \frac{\text{Tr} | \chi - \tilde{\chi} |}{\text{Tr} | \chi - \chi_{\text{ideal}} |}, \quad \tilde{\chi} = \tilde{\chi}^{(1)} \otimes \tilde{\chi}^{(2)}, \quad \tilde{\chi}^{(1)} = \text{Tr}_2 \chi
\]

\[|A| \equiv \sqrt{A^\dagger A}, \quad \text{Tr}|A| - "trace norm"

Not so easy for **coupled** system. However, in Markovian case can use \(\lambda\)-matrix:

\[
\epsilon'_{NL} = \frac{\text{Tr} | \lambda - \tilde{\lambda} |}{\text{Tr} | \lambda |}, \quad \tilde{\lambda} = \tilde{\lambda}^{(1)} \otimes \lambda^{(2)} + \lambda^{(1)} \otimes \tilde{\lambda}^{(2)}, \quad \tilde{\lambda}^{(1)} = \text{Tr}_2 \lambda
\]

\[\epsilon'_{NL} \approx \epsilon_{NL} \quad \text{for no coherent evolution and weak decoherence} \]
Considered models of decoherence

Energy relaxation

\[ \lambda_{00} = -2 \left( \frac{1}{T_1^{(1)}} + \frac{1}{T_1^{(2)}} \right) \]
\[ \lambda_{i1} = \lambda_{22} = \frac{1}{T_1^{(2)}}, \quad \lambda_{44} = \lambda_{88} = \frac{1}{T_1^{(1)}} \]
\[ \lambda_{03} = \lambda_{30} = \frac{1}{T_1^{(2)}}, \quad \lambda_{0,12} = \lambda_{12,0} = \frac{1}{T_1^{(1)}} \]
\[ \lambda_{21} = -\lambda_{12} = i/T_1^{(2)}, \quad \lambda_{84} = -\lambda_{48} = i/T_1^{(1)} \]

Partially correlated (nonlocal) dephasing

\[ \lambda_{00} = -(\frac{1}{T_\phi^{(1)}} + \frac{1}{T_\phi^{(2)}})/2 \]
\[ \lambda_{33} = \frac{1}{2T_\phi^{(2)}}, \quad \lambda_{12,12} = \frac{1}{2T_\phi^{(1)}} \]
\[ \lambda_{3,12} = \lambda_{12,3} = -\lambda_{0,15} = -\lambda_{15,0} = \kappa / 2 \sqrt{T_\phi^{(1)}T_\phi^{(2)}} \]
\[ \kappa - \text{correlation factor } (\kappa=0 \text{ if local}) \]

Noisy coupling

\[ \lambda_{00} = -\Gamma_s \]
\[ \lambda_{55} = \lambda_{10,10} = \lambda_{5,10} = \lambda_{10,5} = \lambda_{0,15} = \lambda_{15,0} = \Gamma_s / 2 \]

Specific pattern for each model!
(usually different elements)

If no evolution (memory), then easy:
\[ \chi \approx \chi^I + \lambda t, \quad \chi^I_{mn} = \delta_{m0} \delta_{n0} \]
\[ \lambda = \lambda_{\text{enrel}} + \lambda_{\text{corr deph}} + \lambda_{\text{noisycoupl}} \]
If coupled qubits, then should not be easy \[ \chi \neq \chi_{\text{ideal}} + \lambda t \]

However, miraculously, for sqrt{iswap} and for largest extra elements of \( \chi \), it is still a simple addition, that makes distinguishing decoherence models very simple.
QPT for one-qubit uncollapsing

Kyle Keane (started recently)

Nadav Katz et al., PRL-08

Fidelity decreases at large $p$ due to T1 and selection

Theoretically fidelity goes to 0.25 at $p = 1$

Actually, problem with definition of $\chi$-matrix, because probability depends on initial state