Quantum efficiency of binary-outcome solid-state detectors of qubits

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Outline:

- Intro: Quantum efficiency of linear detectors
- Definitions of quantum efficiency for binary-outcome detectors
- Quantum efficiency for several models
  - indirect projective measurements
  - linear detector in binary-output regime
  - detector for phase qubit
  - tunneling into continuum

Support:
Quantum efficiency of linear detectors

Idea of definition (Korotkov-1998):

\[ \Gamma \geq 1/2\tau_m \]  
(informational bound)

ensemble  “measurement time”
decoherence rate (to reach signal-to-noise =1)

⇒ quantum efficiency (ideality)  \[ \eta = 1/2\Gamma \tau_m \]

Ideal detector (η=1) does not decohere qubit (pure → pure)

Slightly different definitions (A.K., 2000):

\[ \Gamma \geq 1/2\tau_m + K^2 S / 4 \]  
S – output noise
K – output-backaction
noise correlation

⇒ efficiency  \[ \tilde{\eta} = (1/2\tau_m + K^2 S / 4) / \Gamma \]

or  \[ \tilde{\eta} = (1/2\tau_m) / (\Gamma - K^2 S / 4) \]

Equivalent to the energy sensitivity limitations known since 1980s:

\[ (\varepsilon_O \varepsilon_B)^{1/2} \geq \hbar / 2, \]  
\[ (\varepsilon_O \varepsilon_B - \varepsilon_{OB})^{1/2} \geq \hbar / 2, \]  
(Clarke, Tesche, Likharev, Caves, etc.)
Now general binary-output detector
(try to use the same idea)

We consider realistic detectors (not the “orthodox” projective measurement!)

Measurement fidelities $F_0$ and $F_1$

$F_0 = \text{probability to get result 0 for a qubit in state } |0\rangle,$

$F_1 = \text{probability to get result 1 for a qubit in state } |1\rangle$

Ideal detector (pure qubit state $\rightarrow$ pure)

Use POVM language: linear measurement operators $M_0$ and $M_1$ (result 0 or 1)

$$\rho \rightarrow \frac{M_i \rho M_i^\dagger}{\text{Tr}(M_i \rho M_i^\dagger)} \quad \text{for result } i, \quad \text{probability } P_i = \text{Tr}(M_i \rho M_i^\dagger)$$

Each operator $M_i : 8 -1(\text{phase}) = 7$ real parameters

$7+7=14$, but completeness $(M_0^+M_0+M_1^+M_1=1)$, so $14 - 4=10$

Ideal binary-output detector of a qubit is described by $\mathbf{10}$ real parameters (including fidelities $F_0$ and $F_1$)
Non-ideal binary-output detectors

Again use POVM language, now arbitrary one-qubit quantum operation (superoperator) for each measurement result
⇒ 16 + 16 – 4 = 28 real parameters for a general (non-ideal) detector

28 (general) – 10 (ideal) = 18 (quantum efficiencies)

Therefore, quantum efficiency (ideality) of a general binary-outcome detector is described by 18 real parameters.

Too many!!! Impractical. What to do?

Consider only “QND” detectors
(qubit does not evolve itself during measurement, $\sigma_z$-coupling)

$|0\rangle \rightarrow |0\rangle$, $|1\rangle \rightarrow |1\rangle$

Try to use the informational bound (as for linear detectors)
Decoherence bound for a QND detector

General description of a QND detector: only 6 parameters
(fidelities $F_0$ and $F_1$, decoherences $D_0$ and $D_1$, and angles $\phi_0$ and $\phi_1$)

result 0:
\[
\begin{pmatrix}
\rho_{00} & \rho_{01} \\
\rho_{10} & \rho_{11}
\end{pmatrix}
\rightarrow
\frac{1}{P_0}
\begin{pmatrix}
F_0\rho_{00} & \sqrt{F_0(1-F_1)} e^{-D_0} e^{i\phi_0} \rho_{01} \\
\text{c.c.} & (1-F_1)\rho_{11}
\end{pmatrix}
\]

result 1:
\[
\begin{pmatrix}
\rho_{00} & \rho_{01} \\
\rho_{10} & \rho_{11}
\end{pmatrix}
\rightarrow
\frac{1}{P_1}
\begin{pmatrix}
(1-F_0)\rho_{00} & \sqrt{(1-F_0)F_1} e^{-D_1} e^{i\phi_1} \rho_{01} \\
\text{c.c.} & F_1 \rho_{11}
\end{pmatrix}
\]

probabilities:
\[P_0 = F_0 \rho_{00} + (1-F_1) \rho_{11}, \quad P_1 = (1-F_0) \rho_{00} + F_1 \rho_{11}\]

Average over results:
\[
\begin{pmatrix}
\rho_{00} & \rho_{01} \\
\rho_{10} & \rho_{11}
\end{pmatrix}
\rightarrow
\begin{pmatrix}
\rho_{00} & e^{-D_{\text{av}}} e^{i\phi_{\text{av}}} \rho_{01} \\
\text{c.c.} & \rho_{11}
\end{pmatrix}
\]

\[e^{-D_{\text{av}}} e^{i\phi_{\text{av}}} = \sqrt{F_0(1-F_1)} e^{-D_0} e^{i\phi_0} + \sqrt{(1-F_0)F_1} e^{-D_1} e^{i\phi_1}\]

$\Rightarrow$ ensemble decoherence bound
\[D_{\text{av}} \geq D_{\text{min}} = -\ln[\sqrt{F_0(1-F_1)} + \sqrt{(1-F_0)F_1}]\]
Definitions of quantum efficiency (actual decoherence vs. informational bound)

Similar to the first definition for linear detectors

\[ \eta = \frac{D_{\text{min}}}{D_{\text{av}}} \]

Taking into account phase correlation:

\[ \tilde{\eta} = \frac{-\ln \left| \sqrt{F_0(1 - F_1)} + \sqrt{(1 - F_0)F_1} e^{i(\phi_1 - \phi_0)} \right|}{D_{\text{av}}} \]

or

\[ \tilde{\eta} = \frac{-\ln \left[ \sqrt{F_0(1 - F_1)} + \sqrt{(1 - F_0)F_1} \right]}{-\ln \left[ \sqrt{F_0(1 - F_1)} e^{-D_0} + \sqrt{(1 - F_0)F_1} e^{-D_1} \right]} \]

Also meaningful to define quantum efficiency for each result of the measurement:

\[ 1 - \eta_0 = \frac{D_0}{D_0 - \ln \sqrt{F_0(1 - F_1)}} , \quad 1 - \eta_1 = \frac{D_1}{D_1 - \ln \sqrt{(1 - F_0)F_1}} \]

(useful for “asymmetric” and “half-destructive” detectors, as for phase qubits)
Quantum efficiency for several detector models

Model 1: Indirect projective measurement

Evolution:

\[(\alpha |0\rangle + \beta |1\rangle) |0_a\rangle \rightarrow \alpha |0\rangle (c_{00} |0_a\rangle + c_{10} |1_a\rangle) + \beta |1\rangle (c_{01} |0_a\rangle + c_{11} |1_a\rangle) \rightarrow \]

\[
\begin{cases}
(\alpha c_{00} |0\rangle + \beta c_{01} |1\rangle) / \text{Norm, if result 0} \\
(\alpha c_{10} |0\rangle + \beta c_{11} |1\rangle) / \text{Norm, if result 1}
\end{cases}
\]

Then

\[F_0 = |c_{00}|^2, \quad F_1 = |c_{11}|^2, \quad \phi_0 = \text{arg}(c_{00}^* c_{01}), \quad \phi_1 = \text{arg}(c_{10}^* c_{11}), \quad D_0 = 0, \quad D_1 = 0\]

And so

\[\eta_0 = \eta_1 = 1, \quad \tilde{\eta} = \tilde{\eta} = 1 \quad \text{(ideal)}\]

but \(\eta \neq 1\), if \(\phi_0 \neq \phi_1\)
Model 2: Linear detector in binary-output regime

Even for an ideal linear detector the threshold detector is significantly non-ideal.

Why? Because we loose information!

\[ F_0 = \frac{1 + \text{erf}(r + s)}{2} \]
\[ F_1 = \frac{1 + \text{erf}(-r + s)}{2} \]

where \( r \) is threshold and
\[ s = \sqrt{t / 2 \tau_m} \] is measurement strength.

Results:
\[ \eta_0 = \frac{-\ln \sqrt{F_0(1 - F_1)}}{-\ln[(1 + \text{erf}(r))/2] + s^2 + \gamma t} \]
\[ \eta_1(r) = \eta_0(-r) \]
\[ \eta = \frac{-\ln[\sqrt{F_0(1 - F_1)} + \sqrt{(1 - F_0)F_1}]}{s^2 + \gamma t} \]
\[ \eta \leq \frac{2}{\pi} \]
Model 3: Partial-collapse measurement of a phase qubit

N. Katz et al., Science-2006 (Martinis’ group)

Result 0 (“null result”), then
\[ \alpha |0\rangle + \beta |1\rangle \rightarrow \frac{\alpha |0\rangle + e^{i\phi} \beta e^{-\Gamma t/2} |1\rangle}{\text{Norm}} \]

Result 1, then qubit destroyed

For this model \( \eta_0 = 1 \), while \( \eta_1 \) and \( \eta \) cannot be defined ("half-destructive" measurement)

If imperfections are taken into account (Pryadko-Korotkov, 2007), then finite quantum efficiency for null-result case: \( \eta_0 < 1 \).
Model 4: Detector based on tunneling into continuum

Probability to tunnel out ($p_0$ or $p_1$) depends on the qubit state:

$$F_0 = 1 - p_0, \quad F_1 = p_1$$

Non-destructive detector for both measurement results.

In simple case (when $p_0 < p_1 << 1$):

Result 0 (no tunneling), then

$$\alpha |0\rangle + \beta |1\rangle \rightarrow \frac{\alpha \sqrt{1 - p_0} |0\rangle + \beta \sqrt{1 - p_1} e^{i\phi_0} |1\rangle}{\text{Norm}}$$

Result 1 (tunneling), then

$$\alpha |0\rangle + \beta |1\rangle \rightarrow \frac{\alpha \sqrt{p_0} |0\rangle + \beta \sqrt{p_1} e^{i\phi_1} |1\rangle}{\text{Norm}}$$

Then ideal detector:

$$\eta_0 = \eta_1 = 1,$$

$$\tilde{\eta} = \tilde{\eta} = 1$$

$$\eta = 1 \text{ if } \phi_0 = \phi_1$$

Can such regime be realized by a real SQUID or by a bifurcation detector?
Conclusions

- Derived a simple informational bound on the qubit ensemble decoherence due to measurement by a binary-outcome detector

- Introduced corresponding definitions for the detector quantum efficiency

- Calculated the quantum efficiency in several models