Physical limits on the maximum integration density of conventional digital electronics has motivated extensive investigations of alternative principles suitable for operation at the few-nanometer scale. Single-electron tunneling (for review see, e.g., Ref. 1) seems to be one of the most promising candidates. Two basic types of ultradense single-electron memory and logic have been suggested. The first way is to use single-electron transistors instead of field-effect transistors in digital circuits resembling conventional ones. Another approach is to code information by the presence of an extra electron on a particular conducting island.

Considerable achievements have been made in both directions in recent experiments. However, both approaches require wires for the power supply and for coupling between elements, and that seems to be inconvenient at the few-nanometer scale.

Another suggestion based on single-electron tunneling which does not require wires has been made. It uses the bistable polarization of the basic element (five quantum dots occupied by two electrons). The polarization of one element affects the polarization of the neighbors leading to the information flow. In this approach there is no power supply, and the only driving force is the fixed polarization of “edge” elements. This is the so-called “ground state computing” based on the belief that the whole system will eventually occupy the state of minimal energy. Somewhat different versions of this idea were discussed in Refs. 15 and 16.

The difficulties of “ground state computing” originate from the fact that even for a large number of elements in a circuit the total energy gain is only of the order of the number of elements-element interaction energy. If the circuit operates in the mode of deterministic sequential switching of elements, then the energy gain should be distributed among all switching events. This requires complicated design, which leads to very low parameter margins (inversely proportional to the number of elements even for a simple propagation line), and allows only small circuits. The other mode of operation assumes that the “path” between initial state and final (ground) state contains parts with increased potential energy. It means that the macroscopic quantum processes (“simultaneous” switching of many elements) are necessary to overcome potential barriers which can have “width” comparable to the number of elements in the device (see, for example, the circuit for fan-out in Ref. 14). In this case the transition time will be practically infinite because of the exponential dependence on the number of elements.

Obviously, the simplest way providing robust logical operation is to use the traditional principle of sequential switching of neighboring elements with the total energy dissipation proportional to the number of switching events. The dissipation in this case should be larger than both $k_B T$ ($T$ is the temperature) and the typical energy of quantum fluctuations. (The energy dissipation per logical operation can be less than $k_B T$; however, this usually requires a more complicated system.)

The purpose of the present letter is to demonstrate the possibility of traditional dissipative way of logical computation using single-electron tunneling between small conducting islands. Similar to systems suggested in Refs. 13–16 our “device” does not have wires. The power necessary for computation is supplied by the changes of external electric field leading to a nonequilibrium state.

A chain of closely located conducting islands (Fig. 1a) serves as the basic element of the device. Small “puddles” of 2D electron gas, small metallic droplets on an insulating substrate, or conducting clusters in a dielectric matrix are possible implementations of the islands. For analysis we will use the “orthodox” theory of single-electron tunneling assuming that the tunnel resistance $R$ between neighboring islands is much larger than $R_0 = \pi \hbar / 2 e^2$. For simplicity we consider a uniform chain (equally spaced islands of equal diameter),

$$\text{FIG. 1. (a) The chain of } N=8 \text{ conducting islands } \langle r/L = 0.5/\sqrt{3}, \alpha = \pi/4 \rangle \text{ being the basic element of the logic, and (b) hysteretic diagram of its polarization number } n \text{ as a function of external field } E. \text{ Fractional numbers denote the states with additional electron-hole pairs inside the chain.}$$
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The application of an external in-plane electric field $E$ creates a potential difference between islands. At zero temperature, tunneling between neighboring islands $i$ and $j$ becomes possible when the potential difference $\Delta \phi$ due to the external field exceeds the Coulomb blockade threshold

$$\Delta \phi_i = C^{-1}_{ii} \frac{e}{2} (C^{-1})_{ii} - C_{jj} (C^{-1})_{jj},$$

where $C^{-1}$ is the inverse capacitance matrix. For spherical islands with radius $r$ much less than the spacing $L$ (see Fig. 1a) one can use the approximation

$$\Delta \phi = EL \cos \alpha, \quad (C^{-1})_{ii} = \frac{1}{4 \pi \varepsilon \varepsilon_0 r^2}, \quad (C^{-1})_{jj} = \frac{1}{4 \pi \varepsilon \varepsilon_0 L},$$

which gives the threshold field $E_i = E_0$, $E_0 = (e/4 \pi \varepsilon \varepsilon_0 L \cos \alpha)(1/r - 1/L)$. Exact electrostatic calculations show a decrease of the ratio $E_i/E_0$ with an increase of $r/L$. ($E_i = 0.963 E_0$ in Fig. 1b).

The external field drags the electron-hole pair toward the edges of the chain, creating the polarized state of the chain. The field of the pair prevents the creation of the next pairs if $E$ is sufficiently close to $E_i$. Further increase of external field can lead to an increase of the number of electrons and holes on the edge islands (Fig. 1b). If the field decreases, the polarization number $n$ also decreases; however, the threshold for the pair annihilation is lower than the threshold for the pair creation. This leads to multistability which is similar to multistability in voltage-biased arrays of tunnel junctions.\textsuperscript{1,5–7,10,11} We will use states $n = 0$ and $n = 1$ to represent logical zero and unity.

The polarization change can propagate along a line of closely located chains (Fig. 2a). Suppose that all chains are not polarized initially, and $E$ is slightly less than $E_i$. This is a metastable state. If one chain becomes polarized, the field of extra electron (hole) on the edge island increases the potential difference between neighboring islands of the next chain (Fig. 2a). This makes tunneling energetically favorable and leads to polarization of the next chain (electrons do not tunnel between chains because of the large distance). This in turn polarizes the next chain and so on. The energy is dissipated during the switching of each chain leading to simple deterministic dynamics of the circuit.

This dynamics has been simulated using a standard Monte Carlo approach based on the “orthodox” theory of single-electron tunneling. In contrast to usual single-electron circuits the electric field is no longer concentrated within tunnel junctions. Hence, the long-range interaction between islands is important, and this requires determination of the full capacitance matrix as a function of the island arrangement. The capacitance matrix for spherical islands was calculated numerically using the method of multiple electrostatic images.

The external field margins for information propagation depend on the arrangement of islands. For a line of chains consisting of $N = 8$ islands with the geometry shown in Fig. 2a ($r/L = 0.5/\sqrt{2}$, $\alpha = \pi/4$) the operation range is between $0.895 E_0$ and $0.961 E_0$ corresponding to a margin of 7.2%. The latter number gives also a crude estimate of the margins for other parameters (radius, spacing, etc.). It increases with the decrease of the distance between islands (for example, the margin is 8.6% for $r/L = 0.6/\sqrt{2}$).

The line of chains shown in Fig. 2a allows propagation perpendicular to external field from left to right. The unidirectional propagation is a consequence of the asymmetry of the circuit; a mirror image of this line would allow propagation from right to left. Propagation with a velocity component along the field or opposite to it can be achieved even more easily by the use of one long chain.

The natural fan-out of the signal into two lines can be realized if both edge islands of a chain are used to trigger the next chains (Fig. 2a). The circuit shown in Fig. 2b operates correctly for external field within the range from $0.895 E_0$ to $0.944 E_0$ (5.3% margin). Actually, the circuits with even larger number of chains in input and output lines than shown in the figures have been simulated, to ensure the account of contribution from long-range interactions.

A “bi-controlled” chain (fifth from the right in Fig. 3a) which can be triggered by the polarization of either of two neighboring input chains can be used as the basic part of the logical gate OR. The simulation of the circuit shown in Fig. 3a gives the operation range from $0.896 E_0$ to $0.943 E_0$ (5.2% margin).

The logical gate AND can be designed similar to the OR gate, but with slightly larger distance between the “bi-controlled” chain and neighboring input chains, in order to decrease their influence. Another possibility is to make the islands of “bi-controlled” chain slightly smaller in order to increase the Coulomb blockade energy. The simulation of the circuit shown in Fig. 3a with the reduced radius of the islands of the “bi-controlled” chain $r' = 0.95 r$ shows that the circuit operates as an AND gate within range from $0.896 E_0$ to $0.937 E_0$ (4.6% margin).

An important drawback of the present version of the logic is the difficult design of the inverter (logical NOT gate), that is a consequence of the asymmetry between logical zero and unity. This problem can be solved with the use of temporal dynamics. The circuit shown in Fig. 3b imple-
ments the logical function \( \text{NOT A} \cdot \text{AND} \cdot \text{B} \) if the signal from input A comes before the signal B. The signal B will propagate to the output as in the usual propagation line, unless the chains of input A are polarized. The simulation of the circuit gives the operation range for external field from \( 0.895 E_0 \) to \( 0.943 E_0 \) (5.2% margin).

This circuit can be used as NOT A, if logical unity always comes from input B and it always comes later than signal A. The relative delays of signals can be adjusted using propagation lines of the proper length or controlling the tunnel resistance \( R \).

The logical gates considered, together with propagation lines and fan-out circuits, are sufficient for computing. In the simplest mode of operation, all chains inside a device have zero polarization and external field is zero in the initial state. Then external field increases up to a value for which all gates operate correctly, and elements start to switch in accordance with the input information flowing from the edges of the device. The result of the computation is the final polarization of output elements which can be read out, for example, by single-electron transistors.

This simplest mode of operation can obviously be improved by the use of periodic changes of the external field (“clock cycles”). Properly chosen levels of the field can reset some elements but preserve the information in other elements. Performance can be improved by the use of elements with different Coulomb blockade thresholds and use of two in-plane components of external field.

An estimate of the external electric field shows that it is relatively large but still can be much smaller than the breakdown limit of the substrate. For example, for circuits shown in Figs. 2 and 3 made of metallic islands with \( r = 2 \) nm on a quartz substrate the typical field \( E_0 = 4 \times 10^5 \) V/cm (breakdown field is about \( 5 \times 10^6 \) V/cm).

The large Coulomb blockade energy \( W = e^2/4 \pi \varepsilon \varepsilon_0 r \) \((W = 3 \times 10^3) \) K for the parameters mentioned above makes the operation at liquid nitrogen temperature feasible. The use of checking algorithms (control sums, etc.) can allow reliable computation in the presence of rare erroneous switching due to finite temperature and cotunneling.

The energy consumption for parameters above can be estimated as \( U = eEL \cos \alpha = 3 \times 10^{-20} \) J per island per switching. Assuming density \( 10^{12} \) islands/cm\(^2\) and clock frequency \( 10^9 \) Hz one finds a power about \( 30 \) W/cm\(^2\).

The use of nonzero background charges can reduce the necessary external field and power consumption. It can also remove the asymmetry between logical zero and unity and, hence, increase the parameter margins and allow the simpler design of the inverter.

In conclusion, we have demonstrated the possibility of single-electron logic using tunneling between small conducting islands in the presence of an external electric field. The logical functions are realized by specific arrangement of islands without need of wires.

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