Extended-Time Demand Bids: A New Bidding Framework to Accommodate Time-Shiftable Loads

Mahdi Kohansal, Student Member, IEEE, Hamed Mohsenian-Rad, Senior Member, IEEE

Abstract—Time-shiftable loads play an important role in creating load flexibility and enhancing demand response and peak-load shaving programs. However, recent studies have suggested that time-shiftable loads may face load synchronization and market instability if they are deployed at high penetrations such that they become price maker. To tackle this problem, in this paper, we propose a new demand bidding framework that recognizes the special characteristics of time-shiftable loads. The bids in this new bidding framework are called extended-time demand bids. They are either extended-time self-schedule bids or extended-time economic bids. The bidding concept, its visualization, and its mathematical representation are presented. Using the bids data in the California energy market, we show that the new bidding structure is beneficial not only to the power system as a whole but also to the consumers that are capable of shifting a portion of their loads. The new bidding structure also increases the market competitiveness due to expanding the competition domain and increasing demand elasticity with temporal dependencies.

Keywords: Time-shiftable loads, extended-time demand bids, price bids, energy bids, day-ahead market, demand response, price competitiveness, load synchronization, social welfare.

NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T$</td>
<td>Number of hours in the hourly market</td>
</tr>
<tr>
<td>$G$</td>
<td>Set of supply bids</td>
</tr>
<tr>
<td>$D$</td>
<td>Set of demand bids</td>
</tr>
<tr>
<td>$e$</td>
<td>Energy component of a bid</td>
</tr>
<tr>
<td>$p$</td>
<td>Price component of a bid</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Start-time component of a bid</td>
</tr>
<tr>
<td>$\beta$</td>
<td>End-time component of a bid</td>
</tr>
<tr>
<td>$i$</td>
<td>Bidder index</td>
</tr>
<tr>
<td>$t$</td>
<td>Hourly time index</td>
</tr>
<tr>
<td>$q$</td>
<td>Cleared energy in the market</td>
</tr>
<tr>
<td>$\pi$</td>
<td>Cleared price in the market</td>
</tr>
<tr>
<td>$RS$</td>
<td>Indicating a regular self-schedule bid</td>
</tr>
<tr>
<td>$RE$</td>
<td>Indicating a regular economic bid</td>
</tr>
<tr>
<td>$ES$</td>
<td>Indicating an extended-time self-schedule bid</td>
</tr>
<tr>
<td>$EE$</td>
<td>Indicating an extended-time economic bid</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Penetration of time-shiftable loads</td>
</tr>
<tr>
<td>$\Delta$</td>
<td>Time-shiftable load flexible time duration</td>
</tr>
<tr>
<td>$\lambda, \nu, \pi$</td>
<td>Lagrange multipliers in economic dispatch</td>
</tr>
</tbody>
</table>

I. INTRODUCTION

A time-shiftable load is a task that requires consuming a certain total energy to finish, but its operation can be scheduled any time within a given time frame, where the end of such time frame is the deadline to finish operation. Some examples of time-shiftable loads are as follows: charging plug-in electric vehicles [1], irrigation pumps [2], batch processes in data centers and computer servers [3]–[5], various home appliances such as dish-washer, washing machine, and dryer [6]–[8], intelligent pools [9], and certain industrial equipment [10], [11]. Time-shiftable loads are also sometimes referred to as deferrable loads with deadlines, c.f. [12], [13].

The operational flexibility in time-shiftable loads makes them a valuable resource in the wholesale electricity market. Accordingly, an optimal flexibility mechanism for time-shiftable loads has recently been proposed in [14]. Both self-scheduling and economic bidding are considered. However, the analysis in [14] is valid only if the time-shiftable load is small and price-taker so that its operation does not affect the price. Given the great interests among utilities to expand their demand response potential, c.f., [15], such price-taker assumption may no longer be accurate in the near future.

The price-maker operation of large time-shiftable loads has also recently been studied, e.g., in [16]–[19]. In [16] and [17], the authors explained how a single large time-shiftable load can submit self-schedule and economic bids to the wholesale market, respectively. The wholesale market interaction among multiple large time-shiftable loads is investigated in [19] using game theory. It is shown that a market with multiple strategic time-shiftable loads may not always have a Nash equilibrium. That is, such market may not always be stable.

In this paper, we consider the case where an arbitrary number of time-shiftable loads participate in the wholesale electricity market. Unlike in [19], where the competition among time-shiftable loads causes market instability and lack of equilibrium, here, we propose a new demand bidding framework that recognizes the special characteristics of time-shiftable loads. The bids in this new bidding framework are called extended-time demand bids. They are either extended-time self-schedule bids or extended-time economic bids.

The contributions in this paper are summarized as follows:

- A new demand bidding framework is proposed to accommodate time-shiftable loads in the market. The bids include the information for the acceptable operation time-frame. Both extended-time self-schedule bids and extended-time economic bids are considered.
- The new bidding structure is beneficial not only to the power system as a whole but also to the consumers that are capable of shifting a portion of their loads. On one hand, it helps the power system by increasing the social welfare across all generators and loads. On the other hand, time-shiftable loads are cleared at cheaper cost.
- The new bidding structure also increases the market competitiveness due to expanding the competition domain and increasing demand elasticity with temporal dependencies.

The authors are with the Department of Electrical Engineering, University of California, Riverside, CA, USA, e-mail: {mkohansal, hamed}@ece.ucr.edu. This work was supported by NSF grants ECCS 1253516, ECCS 1307756, and CNS 1319798. The corresponding author is H. Mohsenian-Rad.
Unlike in [14], the new bidding model incorporates the impact of large and price-maker time-shiftable loads. Also, unlike in [19], the proposed bidding model is not prone to load synchronization and market instability.

II. BIDDING CONCEPT AND ITS VISUALIZATION

Consider an electricity market, where an Independent System Operator (ISO) receives and processes the supply and demand bids from generator and load companies, respectively. Each bid can be either a self-schedule bid or an economic bid. An economic bid specifies an energy quantity \( e \) in MWh and a price quantity \( p \) in \$/MWh. If an economic bid is a demand bid it indicates that the buyer is willing to purchase up to \( e \) MWh energy with a price no higher than \( p \) \$/MWh. In contrast, self-schedule bids do not specify any price quantity. For example, a self-schedule demand bid indicates that the buyer is willing to purchase exactly \( e \) MWh at any cleared market price.

Currently, the demand bids, whether of type self-schedule or economic, are specific to a particular hour. As a result, they cannot directly accommodate time-shiftable loads. In fact, based on the current market structure, if a demand response aggregator with time-shiftable load seeks to participate in the energy market, then it must submit several separate demand bids at each hour, without having the right tools to indicate the inter-temporal dependency across its bids. As we discussed in Section I, lack of recognizing the time-flexibility in time-shiftable loads can cause market instability [19].

To tackle the above challenges, in this paper, we propose the concept of extended-time demand bidding as follows:

- **Extended-time Self-Schedule Demand Bid**: It includes an energy quantity \( e \), a start-time \( \alpha \), and an end-time \( \beta \). It indicates that the buyer is willing to purchase an exact total of \( e \) MWh at any price and between hours \( \alpha \) and \( \beta \).

- **Extended-time Economic Demand Bid**: It includes an energy quantity \( e \), a price quantity \( p \), a start-time \( \alpha \), and an end-time \( \beta \). It indicates that the buyer is willing to purchase up to a total of \( e \) MWh at a price no higher than \( p \) \$/MWh between hours \( \alpha \) and \( \beta \).

Note that, we always have \( \alpha \leq \beta \). For the special case where \( \alpha = \beta \), an extended-time bid reduces to a regular bid.

Fig. 1 shows the impact of time-shiftable self-schedule demand bidding on a two-time-slots market, where \( \alpha = 1 \) and \( \beta = 2 \). Once the ISO receives the bid, it must decide on the value of \( \theta \), i.e., the portion of the total needed energy \( e \) that is going to be procured at hour \( \alpha = 1 \), while the portion \( 1 - \theta \) is going to be procured at hour \( \beta = 2 \). As shown in Figs. 1(a) and 1(b), by increasing \( \theta \), the aggregated demand curve at hour \( \alpha = 1 \) shifts to the right, resulting in a higher price at this hour, while the aggregated demand curve at hour \( \beta = 2 \) shifts to the left, resulting in a lower price at this hour. The price curves versus parameter \( \theta \) are plotted in Fig. 1(c).

The price curves versus parameter \( \theta \) are plotted in Fig. 1(c). Considering the two time slots combined, the changes in the total social welfare in the power system versus parameter \( \theta \) are illustrated in Fig. 1(d). Here, the social welfare is calculated across both generators and loads. Based on this curve, the ISO schedules the operation of the time-shiftable load to consume \( \theta^* e \) MWh at time slot \( \alpha = 1 \) and \((1 - \theta^*) e \) at time slot \( \beta = 2 \). As intended, the total energy consumption adds up to \( e \).

![Fig. 1. Handling an extended-time demand bid in a two-time-slots market: (a) Supply and demand curves at the first hour; (b) Supply and demand curve at the second hour; (c) Price variation with respect to the time-shiftable demand bid at the two hours; (d) Aggregated social welfare across the two hours.](image)

III. MATHEMATICAL REPRESENTATION

In this section, we discuss how the new bidding structure can be incorporated into the economic dispatch problem that is formulated and solved by the ISO. Suppose, the market contains \( T = 24 \) hours. Based on the notations in the nomenclature, the economic dispatch problem in presence of extended-time demand bids can be formulated as

\[
\begin{align*}
\max_q & \quad \sum_{t=1}^{T} \left( \sum_{i \in D_{t}^{RE}} p_i q_{i,t} - \sum_{j \in G_{t}^{RE}} p_j q_{j,t} \right) \\
\text{s.t.} & \quad \sum_{i \in D} q_{i,t} = \sum_{j \in G} q_{j,t}, \quad \forall t \\
& \quad q_{i,t} \leq e_{i}, \quad \forall t, \forall i \in D^{RE} \\
& \quad q_{i,t} = e_{i}, \quad \forall t, \forall i \in D^{RS} \\
& \quad \sum_{t=\alpha}^{\beta} q_{i,t} \leq e_{i}, \quad \forall i \in D^{EE} \\
& \quad \sum_{t=\alpha}^{\beta} q_{i,t} = e_{i}, \quad \forall i \in D^{ES} \\
& \quad q_{i,t} \geq 0, \quad \forall t, \forall i \in D \\
\text{Generator } j \text{ Constraints} & \quad \forall j \in G.
\end{align*}
\]

The objective is to maximize the total social welfare of the power system over the market time horizon. Two changes are made in the economic dispatch problem to incorporate
extended-time demand bids. First, in the objective function, the welfare for each time-shiftable load $i$ that submits extended-time economic demand bid is defined over the entire flexible operation period from $\alpha_i$ to $\beta_i$. Second, in the constraints, the target energy levels for time-shiftable loads are calculated over the entire flexible operation period from $\alpha_i$ to $\beta_i$, whether the extended-time bid is self-schedule or economic.

Next, we derive the Karush-Kuhn-Tucker (KKT) optimality conditions with respect to the extended-time economic demand bid variables. Suppose $\pi_i$ is the Lagrange multiplier for the energy balance constraint at hour $t$, which represents the price at that hour. Let $\lambda_i$ denote the Lagrange multiplier for total demand constraint for time-shiftable load $i$ and $v_{i,t}$ denote the Lagrange multiplier for the constraint that shows the cleared energy cannot be negative. The KKT optimality conditions for load $i \in D^{EE}$ and at hour $t$ are obtained as

$$\frac{dL}{dq_{i,t}} = -p_i + \pi_i + \lambda_i - v_{i,t} = 0,$$

$$\lambda_i(\sum_{t=\alpha_i}^{\beta_i} q_{i,t} - e_i) = 0,$$

$$\lambda_i \geq 0,$$

$$v_{i,t} q_{i,t} = 0,$$

$$v_{i,t} \geq 0.$$  

\begin{equation}
\text{(2)}
\end{equation}

\begin{equation}
\text{(3)}
\end{equation}

\begin{equation}
\text{(4)}
\end{equation}

\begin{equation}
\text{(5)}
\end{equation}

\begin{equation}
\text{(6)}
\end{equation}

We can now show the following results.

**Theorem 1**: If the price bid $p_i$ in an extended-time economic demand bid is greater than $\pi_i$, where $\alpha_i \leq t \leq \beta_i$, then the total energy bid $e_i$ will be cleared, i.e., $\sum_{t=\alpha_i}^{\beta_i} q_{i,t} = e_i$

**Proof**: Without loss of generality, let us assume that $\pi_{t_0}$ is the minimum price between hour $\alpha_i$ and $\beta_i$. If $p_i > \pi_{t_0}$, then from the KKT conditions, the following relations hold:

$$p_i > \pi_{t_0} \Rightarrow p_i - \pi_{t_0} > 0 \Rightarrow \lambda_i - v_{i,t} > 0 \ldots$$

$$\ldots \Rightarrow v_{i,t} > 0 \Rightarrow \lambda_i > 0 \Rightarrow \sum_{t=\alpha_i}^{\beta_i} q_{i,t} = e_i$$

Similarly, we can show that if $p_i$ is less than the minimum price during hours $\alpha_i$ and $\beta_i$, then the bid is not cleared. 

From Theorem 1, the behavior of an extended-time economic bid is similar to a regular economic bid, i.e. the cleared energy and the market price match the bidder’s desire.

**Theorem 2**: Consider an extended-time bid and two hours with two different cleared market prices that are within the time frame $\alpha_i$ and $\beta_i$. The amount of cleared energy of this extended-time bid at the more expensive hour is zero.

**Proof**: First, assume that the extended-time bid is of type economic. Suppose $\pi_{t_1} > \pi_{t_2}$, where $t_1$ and $t_2$ are within the time frame $\alpha_i$ and $\beta_i$. Based on the KKT conditions, the following equalities and inequalities hold:

$$\pi_{t_1} > \pi_{t_2} \Rightarrow p_i - \lambda_i + v_{i,t_1} > p_i - \lambda_i + v_{i,t_2} \Rightarrow \ldots$$

$$\ldots \Rightarrow v_{i,t_1} > v_{i,t_2} \Rightarrow v_{i,t_2} \geq 0 \Rightarrow v_{i,t_1} > 0 \Rightarrow q_{i,t_1} = 0$$

\begin{equation}
\text{(7)}
\end{equation}

\begin{equation}
\text{(8)}
\end{equation}

The case for self-schedule bids can be proved similarly.

From the above theorem, the ISO first clears the extended-time bids at the cheapest hours until all prices become equal. After that, the ISO distributes the time-shiftable loads among different hours so that the hourly prices maintain similar.

Fig. 2 shows the hourly cleared energy and price on January 15, 2014 for different time-shiftable load penetration levels $\gamma$. Here, $\gamma$ % of self-schedule demand bids and $\gamma$ % of economic demand bids are assumed to be replaced by extended-time bids with $\alpha = 1$ and $\beta = 24$. We can see that the amount of cleared load and the cleared market price reduce during peak hours as we increase the penetration of extended-time bids. In fact, the time-shiftable loads have been shifted to off-peak hours, causing the peak-to-average ratio (PAR) reduce, which makes the system more reliable. Moreover, by increasing $\gamma$, the prices at different hours become equal, which is predictable based on the Theorem 2. After that, by increasing the penetration level, there will be no change in the market price. Therefore, after a certain penetration threshold, in this case at about $\gamma = 18\%$, the system reaches a saturation point at which increasing $\gamma$ does not affect the cleared market prices.

Fig. 3(a) shows the social welfare of the power system versus the penetration of the extended-time bids. We can see that increasing $\gamma$ results in increasing the social welfare. Moreover, as mentioned in section III, besides the system benefits, the extended-time bids are beneficial to the time-shiftable loads. This is shown in Fig. 3(b), where the average cleared prices are compared for extended-time and regular demand bids. We can see that the average price of extended-time bids is always less than that of regular bids, especially at lower penetration levels. Based on Theorem 2, in lower amounts of $\gamma$, ISO clears the extended-time bids in off-pick hours which have lower prices. By increasing penetration, the system reaches a saturation point, at which the prices become
almost equal at different hours. Accordingly, the average prices for different demand bid types converge to each other.

One of the key problems in presentation of time-shiftable loads is load synchronization, where all or a large number of time-shiftable loads shift their load to off-peak hours, creating a new peak hour [18], [21]. Next, we show that this problem can be tackled if we use extended-time demand bids. The results are shown in Fig. 4. Here, the peak-to-average (PAR) is plotted versus the time-shiftable load penetration level for two bidding demand scenarios. We can see that PAR is high if zero or only a small percentage of the loads are time-shiftable. As we increase $\gamma$, the PAR reduces almost similarly for the two demand bidding scenarios. However, beyond a certain penetration level when there is a considerable percentage of time-shiftable loads, the PAR starts increasing, instead of decreasing, if the regular demand bids are used for time-shiftable loads. This is due to the load synchronization problem that we mentioned earlier. However, by applying the proposed extended-time bidding framework, we continue benefiting from the time-flexibility in time-shiftable loads and lowering the prices even at higher penetrations of time-shiftable loads without suffering from load synchronization.

Finally, we assess the market outcome for different values of $\Delta = \beta - \alpha + 1$, i.e., the flexible time duration for time-shiftable loads. The results are shown in Fig. 5. We can see that increasing $\Delta$ can potentially help in peak load shaving and lower peak load prices. However, by comparing the results in Figs. 2 and 5, one can conclude that increasing the penetration of time-shiftable loads, i.e., the volume of flexible loads is often more beneficial compared to increasing the time flexibility of a small volume of time-shiftable loads.

V. IMPACT ON MARKET COMPETITIVENESS

In this section, we present an example to study the impact of extended-time demand bidding on price competitiveness. Consider a market over $T = 3$ hours. The true generation and demand bids based on the true marginal costs are shown in Tables I and II, respectively. Since self-schedule bidding is a special case of economic bidding, where the price bid is infinity, all bids are assumed to be economic bids. There are 11 generators and 11 loads in each time slot. Let us assume that generator $j = 3$ at hour $t = 1$, generator $j = 5$ at hour $t = 2$ and generator $j = 4$ at hour $t = 3$ submit their bids strategically. Similar to the previous section, we assumed that a portion $\gamma$ of each load is time-shiftable, where $\alpha = 1$ and $\beta = 3$. To find Nash equilibrium among the three strategically bidding generators, we followed the general method in [22] and used an exhaustive search with resolution 1$/MWh$ for the price bid and 1MWh for energy bid. The results are shown in Table III. We can see that by increasing $\gamma$, the differences between noncompetitive and competitive prices reduces. It means that by applying extended-time bids, there is less potential for the three strategic generators to exercise market power. Furthermore, at $\gamma = 16\%$, one of the Nash equilibria under extended-time bidding is equal to the true market equilibrium and the prices at the other equilibrium point are only 1$ more than the competitive prices. In contrast,
TABLE I

<table>
<thead>
<tr>
<th>Generator Index</th>
<th>(Energy e, Price p)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>t = 1</td>
</tr>
<tr>
<td>1</td>
<td>(10 , 5)</td>
</tr>
<tr>
<td>2</td>
<td>(8 , 7)</td>
</tr>
<tr>
<td>3</td>
<td>(25 , 11)</td>
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<tr>
<td>4</td>
<td>(15 , 17)</td>
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<td>5</td>
<td>(14 , 25)</td>
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<td>(10 , 28)</td>
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<td>7</td>
<td>(12 , 40)</td>
</tr>
<tr>
<td>8</td>
<td>(10 , 48)</td>
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<tr>
<td>9</td>
<td>(10 , 55)</td>
</tr>
<tr>
<td>10</td>
<td>(11 , 60)</td>
</tr>
<tr>
<td>11</td>
<td>(9 , 68)</td>
</tr>
</tbody>
</table>

at \( \gamma = 0 \), which is the representation of the existing market framework, the differences of the cleared and competitive prices are \( 32 - 28 = \$4 \) at hour \( t = 1 \), \( 36 - 30 = \$6 \) at hour \( t = 2 \), and \( 38 - 35 = \$3 \) at hour \( t = 3 \).

VI. CONCLUSION

Given the growing interests in time-shiftable loads and considering the challenges in large-scale integration of such loads in energy markets, in this paper, we propose extended-time demand bids that are tailored around the special characteristics of time-shiftable loads. The proposed new bidding framework is compatible with the existing market structures as it allows both self-schedule and economic bids to become extended-time demand bids. The bidding concept, its visualization, and its mathematical representation are presented. It is shown that the proposed bidding structure can prevent the typical load synchronization problem for time-shiftable loads. Furthermore, it is beneficial to the power system as a whole and the consumers with time-shiftable loads. The new demand bidding structure also has the potential to increase the market competitiveness and contribute to mitigating market power.

REFERENCES


