Performance Analysis of Minimum Variance CDMA Receivers

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***Abstract*— Constrained optimization of the receiver's output variance has recently been proposed as a relatively simple method for designing blind multiuser detectors for DS-CDMA systems. A single constraint is sufficient for the AWGN case, whereas multiple constraints should be used in a multipath environment. It is shown in this paper that the choice of the constraint parameters in the multipath case can have a significant effect on the system performance. A max/min approach for optimizing the constraint is proposed, resulting in blind solutions with improved performance. It is shown that the performance of the proposed method tends to be close to that of the MMSE receiver at high SNR, whereas the constraint parameters converge to the multipath channel parameters. The proposed method does not require knowledge of the interfering users' codes and timing. Simulation results support those performance claims.**

I. Introduction

CODE division multiple access (CDMA) systems have received considerable attention recently as potentially better alternatives to classical TDMA/FDMA systems for wireless cellular systems [3]. In the direct sequence (DS) CDMA framework, all users transmit at the same time and frequency but use distinct signature sequences to allow signal separation at the receiver. Current CDMA systems typically employ single-user receivers (RAKE receivers, e.g., [12]), which treat the other users as noise and do not attempt to suppress multiuser interference (MUI). However, there is evidence that significant performance gains may be achieved if a multiuser detection approach is employed at the receiver (e.g. [23]). This promise has sparked significant activity in the area resulting in optimal maximum likelihood (ML) structures [22], as well as suboptimal linear solutions [8], [9]. Despite the superiority of the ML receiver [22], recent efforts have focused on simple linear designs [10], [17] in order to reduce the computational complexity.

If training data are available, then the linear MMSE multiuser receiver can be obtained directly by solving a Wiener estimation problem as shown in [10]. In the absence of training data, however, the derivation of a linear receiver with performance that is equal or close to that of the MMSE solution presents a significant challenge. A solution to the blind design problem may be obtained through a two-step approach, that is, estimating the system parameters first using a subspace method [1], [15], [18], and then deriving the MMSE receiver from the system parameters.

Recent developments, however, indicate that simpler methods are possible that directly obtain the receiver parameters through a constrained optimization approach, [4], [16], [19]. Honig *et al.* [4] proposed to optimize the receiver's output energy while constraining the response of the user of interest to a constant. In an additive white Gaussian (AWGN) environment with no multipath, this approach provides a blind solution with MMSE performance. Unfortunately, however, the imposed constraint is very sensitive to signal mismatch and interchip interference (ICI) (see the discussion in [4]), making it unsuitable for systems with multipath dispersions.

An extension of the constraint optimization approach to the multipath case was provided in [16], where the signal mismatch problem was treated by forcing the receiver response to all delayed copies of the signal of interest to zero. With these additional constraints, the method of [4] was made applicable to multipath environments. It should be stressed, however, that the solution in [16] uses an *ad hoc* constraint and does not possess any optimality. In fact, the price paid for allowing multipath effects is that the receiver does not exploit the signal energy, which is contained in the delayed copies of the signal of interest. This observation was made in [19], where an improved constraint was proposed by integrating ideas from the area of array processing and, in particular, from the design of minimum variance beamformers [21]. However, no performance analysis was made in [19], nor were any comparisons made with other linear methods and the MMSE solution. Finally, a different extension of [16] was developed in [7].

It is clear from the above developments that a general framework for these constrained optimization solutions needs to be developed and their performance compared with each other as well as the MMSE receiver.

The current paper attempts to fill this gap by providing a comprehensive analysis of the different variations of constrained minimum variance receivers. We treat the general multipath case and are not restricted to the AWGN channel. In order to provide a unifying formulation for the various constraints on the desired user's response, we let the response to each delayed copy be constrained to a parameter value. In this way, the effects of the choice of these parameters on the system's performance may be studied, and the question of optimizing the constraint may be addressed.

In particular, we focus on optimal constraints that maximize the signal energy (after the interference has been suppressed),

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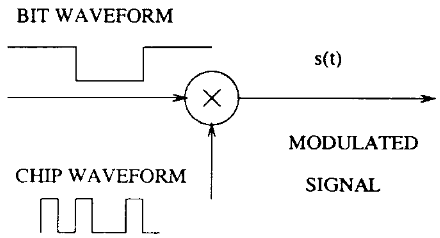

Fig. 1. Spectral spreading: Continuous time model.

utilizing all delayed copies of the signal of interest. It turns out that at high SNR, the performance of this method approaches the performance of the MMSE receiver, making it an attractive candidate among blind solutions. This asymptotic result constitutes the main contribution of this paper.

The rest of the paper is organized as follows: In Sections II and III, a CDMA discrete-time model is presented, and minimum variance receivers are reviewed. In Section IV, the optimal choice of constraints is discussed, and its performance is analyzed. Asymptotic results are developed in Section V as well as comparisons with the MMSE receiver. Finally, some simulation experiments are presented in Section VI and some conclusions in Section VII.

II. Problem Statement

Most treatments of DS-CDMA systems describe the transmitter's spectral spreading operation as a modulation of the narrowband bit rate signal $w_c(t)$ by a higher bandwidth chip waveform $c_c(t)$[1] (see Fig. 1)

$$s_c(t) = w_c(t)c_c(t). \tag{1}$$

This relation provides a conceptually simple understanding of the spectral spreading process but does not provide for the abstract, baseband, discrete-time model needed in the development of multiuser receiver algorithms. Here, we prefer a discrete-time model for the spreading operation, which isolates the effects of the modulation/transmission details of the spread-spectrum signal [17].

We consider a DS-CDMA system with J users and a spreading factor of P chips/information symbol. Let user j $j = 1, \ldots, J$ use a distinct spreading code of length P, $\mathbf{c}_j = [c_j(0), \ldots, c_j(P-1)]^T$. Then, the jth user's discrete-time transmitted signal at the chip rate is given by the multirate convolution (see also [17])

$$s_j(n) = \sum_{k=-\infty}^{\infty} w_j(k)c_j(n-kP) \tag{2}$$

where $w_j(k)$ is a zero-mean, i.i.d. information bearing sequence with variance $\sigma_w^2 = E\{\|w_j(k)\|^2\}$. Let $s_j(n)$ be transmitted through a linear channel with a baseband impulse response $g_{c,j}(t)$ (including the transmitter and receiver filters), and let the receiver collect K samples per chip. Then, the received discrete-time signal $y_j(n)$ due to user j is (e.g. [14])

$$y_j(n) = \sum_{l=-\infty}^{\infty} s_j(l)g_j(n-d_jK-lK) \tag{3}$$

[1]We use subscript c to denote continuous-time signals.

where $g_j(n) = g_{c,j}(t)\mid_{t=\frac{nT_c}{K}}$, T_c is the chip period, and $0 \le d_j < P$ is the delay of user j in chip periods. From (2) and (3), we obtain the input/output relation

$$y_j(n) = \sum_{l=-\infty}^{\infty} w_j(l)h_j(n-d_jK-lKP) \tag{4}$$

$$h_j(n) = \sum_{m=-\infty}^{\infty} c_j(m)g_j(n-mK) \tag{5}$$

(see also Fig. 2); $h_j(n)$ represents the signature pulse of user j, which is a distorted version of the code $c_j(n)$ due to the multipath parameters $g_j(m)$ [cf., (5)]. In the special case where $K = 1$ (one sample per chip), (5) becomes a single rate convolution. Finally, the received signal $y(n)$ is a superposition of the signals from all users plus additive zero-mean, white, Gaussian noise $v(n)$ with variance $\sigma_v^2 = E\{\|v(n)\|^2\}$

$$y(n) = \sum_{j=1}^{J} y_j(n) + v(n). \tag{6}$$

Equations (4)–(6) summarize the DS-CDMA signal model.

We will find it useful in the sequel to use a single rate, multichannel description of (4)–(6). We will also assume, following common practice in communications, that the multipath channels $g_j(n)$ are FIR of maximum order q. Finally, we will present the $K = 1$ case (one sample per chip) to make the exposition clearer; the extension to $K > 1$ is straightforward based on (5). More details on the CDMA signal model can be found in [17].

Let $\mathbf{h}_j = [h_j(0), \ldots, h_j(P+q-1)]^T$ be the signature vector, and let us collect $L = P + q$ measurements of $y_j(n)$ in a vector $\mathbf{y}_j = [y_j(nP), \ldots, y_j(nP+L-1)]^T$. Then, if the receiver is synchronized to user j, ($d_j = 0$), we can see from (4) and (5) that

$$\mathbf{y}_j(n) = \mathbf{h}_j w_j(n) + \tilde{\mathbf{h}}_j w_j(n-1) + \tilde{\tilde{\mathbf{h}}}_j w_j(n+1) \tag{7}$$

where $\tilde{\mathbf{h}}_j = [h_j(P), \ldots, h_j(P+q-1), 0, \ldots, 0]^T$ and $\tilde{\tilde{\mathbf{h}}}_j = [0, \ldots, 0, h_j(0), \ldots, h_j(q-1)]^T$. The effects of the second and third term in (7) are usually ignored if $q \ll P$, but we do not have to ignore them here.

In the methods we will study in the sequel, the system is assumed to be synchronized to the user of interest. Let us assume without loss of generality that user $j = 1$ is the user of interest (therefore, $d_1 = 0$), and define that all users are synchronous if their chip delays $d_j = 0$; otherwise, they are called asynchronous (if $d_j \neq 0$). Then, for the (asynchronous) interfering users (i.e. for $j \neq 1$ and $d_j > q$), (7) becomes[2]

$$\mathbf{y}_j(n) = \tilde{\mathbf{h}}_{j,a} w_j(n-1) + \tilde{\tilde{\mathbf{h}}}_{j,a} w_j(n) \tag{8}$$

where $\tilde{\mathbf{h}}_{j,a} = [h_j(P-d_j), \ldots, h_j(P+q-1), 0, \ldots, 0]^T$, and $\tilde{\tilde{\mathbf{h}}}_{j,a} = [0, \ldots 0, h_j(0), \ldots, h_j(P+q-1-d_j)]^T$. Collecting

[2]If $d_j < q$, a third term needs to be added in (8) as in (7). For a more detailed treatment using the polyphase decomposition of multirate structures see [17] and [20, ch. 4].

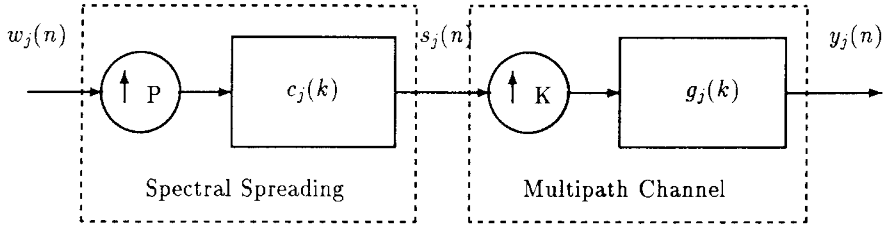

Fig. 2. DS/SS signal in multipath: Discrete-time model.

everything together, we obtain

$$\mathbf{y}(n) = \sum_{j=1}^{J} \mathbf{y}_j(n) = \mathbf{h}_1 w_1(n) + \mathbf{H}\mathbf{w}(n) + \mathbf{v}(n) \tag{9}$$

where $w_1(n)$ is the signal of interest, whereas $\mathbf{H}$ and $\mathbf{w}(n)$ contain all the signatures and interfering bits, respectively, $\mathbf{H} = [\tilde{\mathbf{h}}_1, \tilde{\tilde{\mathbf{h}}}_1, \tilde{\mathbf{h}}_{2,a}, \tilde{\tilde{\mathbf{h}}}_{2,a}, \ldots, \tilde{\mathbf{h}}_{J,a}, \tilde{\tilde{\mathbf{h}}}_{J,a}]$, $\mathbf{w}(n) = [w_1(n-1), w_1(n+1), w_2(n), w_2(n-1), \ldots, w_J(n), w_J(n-1)]^T$.

III. Minimum Variance Receivers

If we focus our attention to linear solutions, then the receiver design problem is equivalent to determining a vector $\mathbf{f}$ such that the receiver output

$$\hat{w}_1(n) = \mathbf{f}^H \mathbf{y}(n)$$

contains less interference and is close to the desired signal $w_1(n)$ in some sense.

The minimum variance approach to selecting the parameter vector uses the output energy as a cost function

$$\mathrm{J}_{\mathrm{MV}}(\mathbf{f}) = E\{\|\hat{w}_1(n)\|^2\} = \mathbf{f}^H \mathbf{R}_y \mathbf{f}, \;\; \mathbf{R}_y = E\{\mathbf{y}(n)\mathbf{y}^H(n)\}. \tag{10}$$

It is clear from (10) that some constraint on $\mathbf{f}$ needs to be imposed when minimizing $J_{\mathrm{MV}}(\mathbf{f})$ in order to avoid the trivial solution $\mathbf{f} = \mathbf{0}$. In particular, it is desirable to constrain the response of the user of interest to a constant, in which case, minimization of the output energy results in minimization of the energy of the interference. The case where no multipath is present was studied in [4]. In that case, $q = 0$, and $\mathbf{h}_j = \alpha_j \mathbf{c}_j$, where α_j is an attenuation factor, and $\mathbf{c}_j = [c_j(0), \ldots, c_j(P-1)]^T$. In that case, the constraint $\mathbf{f}^H \mathbf{h}_1 = 1$ guarantees no signal cancellation, and the minimization of (10) results in a solution with performance identical to that of the MMSE receiver, [4]. Related developments in the area of array processing are known as the "minimum variance distortionless response" (MVDR) beamformer [5].

Unfortunately, constrained optimization methods are known to be very sensitive to signature mismatch due to signal cancellation effects, [5]. Hence, special care needs to be taken when multipath is present [4]. If the multipath signature $\mathbf{h}_1$ were known, the constraint $\mathbf{f}^H \mathbf{h}_1 = 1$ could be used resulting in the solution

$$\mathbf{f}_{\mathrm{MV}} = \left(\mathbf{h}_1^H \mathbf{R}_y^{-1} \mathbf{h}_1\right)^{-1} \mathbf{R}_y^{-1} \mathbf{h}_1 \tag{11}$$

and the minimum output energy (MOE) at the optimal point becomes

$$\mathrm{MOE}(\mathbf{h}_1) = \mathbf{f}_{\mathrm{MV}}^H \mathbf{R}_y \mathbf{f}_{\mathrm{MV}} = \frac{1}{\mathbf{h}_1^H \mathbf{R}_y^{-1} \mathbf{h}_1}. \tag{12}$$

Notice, however, that from (5) (with $K = 1$), we may write

$$\mathbf{h}_1 = \mathbf{C}_1 \mathbf{g}_1 \tag{13}$$

where

$$\mathbf{C}_1 = \begin{bmatrix} c_1(0) & & 0 \\ \vdots & \ddots & c_1(0) \\ c_1(P-1) & & \vdots \\ 0 & \ddots & c_1(P-1) \end{bmatrix}, \quad \mathbf{g}_1 = \begin{bmatrix} g_1(0) \\ \vdots \\ g_1(q) \end{bmatrix}. \tag{14}$$

Hence, $\mathbf{h}_1$ depends on the unknown multipath parameters $\mathbf{g}_1$ and is generally not known.

It was proposed in [19] to avoid the explicit estimation of $\mathbf{g}_1$ by considering a constraint $\mathbf{f}^H \mathbf{h} = 1$, where $\mathbf{h}$ is a parameter vector to be determined. A procedure reminiscent to the Capon estimation method was proposed there for optimizing $\mathbf{h}$. This procedure consists of selecting $\mathbf{h}$ such that MOE ($\mathbf{h}$) in (12) is maximized subject to the constraints $\|\mathbf{h}\| = 1$. If $\mathbf{h}$ is replaced by $\mathbf{h} = \mathbf{C}_1 \mathbf{g}$ for some parameters $\mathbf{g}$, we arrive at the optimization problem

$$\mathbf{g}_{\mathrm{Capon}} = \arg \max_{\mathbf{h} = \mathbf{C}_1 \mathbf{g}} \frac{\mathbf{h}^H \mathbf{h}}{\mathbf{h}^H \mathbf{R}_y^{-1} \mathbf{h}} = \arg \min_{\mathbf{g}} \frac{\mathbf{g}^H \mathbf{C}_1^H \mathbf{R}_y^{-1} \mathbf{C}_1 \mathbf{g}}{\mathbf{g}^H \mathbf{C}_1^H \mathbf{C}_1 \mathbf{g}}. \tag{15}$$

Equation (15) is equivalent to a generalized eigenvalue problem involving the pencil of $(\mathbf{C}_1^H \mathbf{R}_y^{-1} \mathbf{C}_1, \mathbf{C}_1^H \mathbf{C}_1)$. This max/min procedure attempts to maximize the signal component after the interference has been suppressed and was reported to have improved performance [19].

A different approach for handling the multipath case relies on extending the number of constraints. In [16], the receiver vector $\mathbf{f}$ was constrained to $\mathbf{C}_1^H \mathbf{f} = [1, 0, \ldots, 0]^T$. In this way, the response of the signal of interest was constrained to [cf., (13)]

$$\mathbf{f}^H \mathbf{h}_1 = \mathbf{f}^H \mathbf{C}_1 \mathbf{g}_1 = [1, 0, \ldots, 0] \mathbf{g}_1 = g_1(0) = \mathrm{constant}. \tag{16}$$

Since $g_1(0) \neq 0$ (if correct timing is available), this method avoids the signal cancellation problem by forcing the response of delayed copies of the signal of interest to zero. In doing so, however, the method does not exploit all the energy of the received signal and results in suboptimal performance.

It is clear from the above discussion that a host of different methods may be derived by appropriately choosing the constraint in a minimum variance optimization framework. There is a need, therefore, for some performance analysis results to guide us in evaluating the different candidate methods and selecting optimal alternatives. In this paper, we generalize the constraint of (16) by forcing $\mathbf{f}$ to satisfy

$$\mathbf{C}_1^H\mathbf{f}=\mathbf{g} \tag{17}$$

where $\mathbf{g}$ is a general parameter vector, which can be arbitrarily chosen. We study the performance of minimum variance solutions subject to (17) and discuss the effects of the choice of $\mathbf{g}$. These developments also indicate an algorithm for selecting $\mathbf{g}$ to obtain asymptotically (as SNR $\rightarrow\infty$) optimal performance [and relate it with (15)].

IV. SINR Analysis

In the sequel, we derive analytical expressions to compare minimum variance methods with each other and with the MMSE solution. We use the output signal to interference and noise ratio (SINR) as the figure of merit. From (9), the correlation matrix $\mathbf{R}_y$ can be written as

$$\mathbf{R}_y=\mathbf{R}_s+\mathbf{R}_i \tag{18}$$

where

$$\mathbf{R}_s=\sigma_w^2\mathbf{h}_1\mathbf{h}_1^H,\quad \mathbf{R}_i=\sigma_w^2\mathbf{H}\mathbf{H}^H+\sigma_v^2\mathbf{I}. \tag{19}$$

Then, for any linear receiver $\mathbf{f}$, the output SINR is given by

$$\text{SINR}=\frac{\mathbf{f}^H\mathbf{R}_s\mathbf{f}}{\mathbf{f}^H\mathbf{R}_i\mathbf{f}}=\frac{\sigma_w^2\mathbf{f}^H\mathbf{h}_1\mathbf{h}_1^H\mathbf{f}}{\mathbf{f}^H\left(\mathbf{R}_y-\sigma_w^2\mathbf{h}_1\mathbf{h}_1^H\right)\mathbf{f}}. \tag{20}$$

Plugging the MMSE solution $\mathbf{f}_{\text{mse}}=\sigma_w^2\mathbf{R}_y^{-1}\mathbf{h}_1$ into (20), we obtain

$$\text{SINR}_{\text{mse}}=\frac{\sigma_w^2\mathbf{h}_1^H\mathbf{R}_y^{-1}\mathbf{h}_1}{1-\sigma_w^2\mathbf{h}_1^H\mathbf{R}_y^{-1}\mathbf{h}_1}. \tag{21}$$

After some manipulation, (21) may be written as

$$\text{SINR}_{\text{mse}}=\frac{1}{\frac{1}{\sigma_w^2\mathbf{h}_1^H\mathbf{R}_y^{-1}\mathbf{h}_1}-1}. \tag{22}$$

On the other hand, if we consider the minimum variance solution subject to the constraint (17), we obtain the optimal receiver

$$\mathbf{f}_{\text{mv}}=\mathbf{R}_y^{-1}\mathbf{C}_1\left(\mathbf{C}_1^H\mathbf{R}_y^{-1}\mathbf{C}_1\right)^{-1}\mathbf{g} \tag{23}$$

using the method of Lagrange multipliers (e.g., [5]). Moreover, the receiver's output energy can be shown to be

$$\text{MOE}(\mathbf{g})=\mathbf{g}^H\left(\mathbf{C}_1^H\mathbf{R}_y^{-1}\mathbf{C}_1\right)^{-1}\mathbf{g}. \tag{24}$$

Using (23) in the SINR formula (20), we obtain

$$\text{SINR}(\mathbf{g})=\frac{\sigma_w^2\left\|\mathbf{g}^H\left(\mathbf{C}_1^H\mathbf{R}_y^{-1}\mathbf{C}_1\right)^{-1}\mathbf{C}_1^H\mathbf{R}_y^{-1}\mathbf{h}_1\right\|^2}{\mathbf{g}^H\left(\mathbf{C}_1^H\mathbf{R}_y^{-1}\mathbf{C}_1\right)^{-1}\mathbf{g}-\sigma_w^2\left\|\mathbf{g}^H\left(\mathbf{C}_1^H\mathbf{R}_y^{-1}\mathbf{C}_1\right)^{-1}\mathbf{C}_1^H\mathbf{R}_y^{-1}\mathbf{h}_1\right\|^2} \tag{25}$$

which, after some manipulation, and using $\mathbf{h}_1=\mathbf{C}_1\mathbf{g}_1$, reduces to

$$\text{SINR}(\mathbf{g})=\frac{1}{\frac{\mathbf{g}^H(\mathbf{C}_1^H\mathbf{R}_y^{-1}\mathbf{C}_1)^{-1}\mathbf{g}}{\sigma_w^2\|\mathbf{g}^H\mathbf{g}_1\|^2}-1}. \tag{26}$$

Equations (21) and (26) allow the comparison of minimum variance methods with the MMSE solution. It is well known that the MMSE receiver maximizes the SINR [11]; hence, we expect, in general

$$\text{SINR}(\mathbf{g})\leq\text{SINR}_{\text{mse}}. \tag{27}$$

It is clear from (23) that if $\mathbf{g}$ is chosen as

$$\mathbf{g}=\sigma_w^2\left(\mathbf{C}_1^H\mathbf{R}_y^{-1}\mathbf{C}_1\right)\mathbf{g}_1 \tag{28}$$

then equality is achieved in (27) because $\mathbf{f}_{\text{MV}}$ is now identical to the MMSE equalizer

$$\mathbf{f}_{\text{mse}}=\sigma_w^2\mathbf{R}_y^{-1}\mathbf{C}_1\mathbf{g}_1=\sigma_w^2\mathbf{R}_y^{-1}\mathbf{h}_1.$$

This discussion reveals the importance of the choice of $\mathbf{g}$ in the system's performance. Although (28) provides an optimal constraint, it requires knowledge of the channel parameters $\mathbf{g}_1$, which may not be available in a blind setup.

An approach to optimize $\mathbf{g}$ when no channel parameters are *a priori* known might be [along the lines of (15)] to maximize the output signal power after the interference has been suppressed, i.e., to seek $\mathbf{g}$ such that (24) is maximized subject to $\|\mathbf{g}\|=1$. This max/min problem is equivalent to

$$\max_{\mathbf{g}}\frac{\text{MOE}(\mathbf{g})}{\|\mathbf{g}\|^2}=\max_{\mathbf{g}}\frac{\mathbf{g}^H\left(\mathbf{C}_1^H\mathbf{R}_y^{-1}\mathbf{C}_1\right)^{-1}\mathbf{g}}{\mathbf{g}^H\mathbf{g}} \tag{29}$$

The cost function in (29) is a Rayleigh quotient, and hence, the solution to this optimization problem is the eigenvector of $(\mathbf{C}_1^H\mathbf{R}_y^{-1}\mathbf{C}_1)^{-1}$ corresponding to the maximum eigenvalue (or, equivalently, the minimum eigenvector of $(\mathbf{C}_1^H\mathbf{R}_y^{-1}\mathbf{C}_1)$). It would be interesting, therefore, in order to assess the performance of this approach to evaluate (26) when $\mathbf{g}$ is the minimum eigenvector of $\mathbf{C}_1^H\mathbf{R}_y^{-1}\mathbf{C}_1$. Let $\mu_0\leq\mu_1\leq\cdots\leq\mu_q$ and $\mathbf{v}_0,\ldots,\mathbf{v}_q$ denote the eigenvalues and corresponding eigenvectors of $\mathbf{C}_1^H\mathbf{R}_y^{-1}\mathbf{C}_1$, respectively, and let $\mathbf{g}=\mathbf{v}_0$. Then, (26) becomes

$$\text{SINR}(\mathbf{v}_0)=\frac{1}{\frac{1}{\mu_0\sigma_w^2\|\mathbf{v}_0^H\mathbf{g}_1\|^2}-1}. \tag{30}$$

Notice that the SINR in (30) is a monotone increasing function of $\text{T}_0=\mu_0\|\mathbf{v}_0^H\mathbf{g}_1\|^2$, whereas SINR_{mse} in (22) is a monotone increasing function of

$$\text{T}_{\text{mse}}=\mathbf{h}_1^H\mathbf{R}_y^{-1}\mathbf{h}_1=\mathbf{g}_1^H\mathbf{C}_1^H\mathbf{R}_y^{-1}\mathbf{C}_1\mathbf{g}_1=\sum_{i=0}^{q}\mu_i\left\|\mathbf{v}_i^H\mathbf{g}_1\right\|^2. \tag{31}$$

Equation (31) demonstrates the suboptimal nature of the proposed method since

$$\text{T}_{\text{mse}}=\text{T}_0+\sum_{i=1}^{q}\mu_i\left\|\mathbf{v}_i^H\mathbf{g}_1\right\|^2\geq\text{T}_0. \tag{32}$$

This result may seem discouraging. However, T_0 will be very close to T_{mse} as long as the extra error in (32) is very small. This could be true if all spreading codes of other users are nearly orthogonal to $\mathbf{C}_1$. As a result, the performance level tends to be close to that of the MMSE receiver at high SNR with a small penalty, as confirmed by our simulations. The issue addressed here is theorectically explored next.

V. Asymptotic Analysis (SNR $\to \infty$)

In this section, we study the performance of optimal minimum variance solutions in the high SNR region. In particular, we show the following two facts as SNR $\to \infty$.

- The parameter vector $\mathbf{g}$ obtained from (29) converges to $\frac{\mathbf{g}_1}{\|\mathbf{g}_1\|}$.
- The ratio $\frac{\mathrm{SINR}_{\mathrm{mse}}}{\mathrm{SINR}(\mathbf{g})}$ [for $\mathbf{g}$ obtained from (29)] converges to $1+\delta$.

We also derive explicit expressions for the excess penalty δ and the error in estimating $\frac{\mathbf{g}_1}{\|\mathbf{g}_1\|}$ based on $\mathbf{g}$.

The major difficulty in this analysis comes from the fact that the matrix $\mathbf{R}_y^{-1}$ is not well defined if $\sigma_v^2 = 0$. For this reason, we first proceed by expressing the matrix $\mathbf{C}_1^H\mathbf{R}_y^{-1}\mathbf{C}_1$ explicitly in terms of σ_v^2. To this end, we use the following eigendecomposition of $\mathbf{R}_y$:

$$\mathbf{R}_y = [\mathbf{V}_s \quad \mathbf{V}_n]\begin{bmatrix}\mathbf{\Lambda}_s & \mathbf{0}\\ \mathbf{0} & \mathbf{0}\end{bmatrix}\begin{bmatrix}\mathbf{V}_s^H\\ \mathbf{V}_n^H\end{bmatrix} + \sigma_v^2\mathbf{I} \tag{33}$$

where $\mathbf{\Lambda}_s = \mathrm{diag}\{\lambda_1,\ldots,\lambda_\xi\}$,[3] and $\mathbf{V}_s, \mathbf{V}_n$ represents the signal and noise subspaces, respectively. Then, $\mathbf{C}_1^H\mathbf{R}_y^{-1}\mathbf{C}_1$ can be expressed as follows.

Lemma 1: It holds that

$$\sigma_v^2\mathbf{C}_1^H\mathbf{R}_y^{-1}\mathbf{C}_1 = \mathbf{C}_1^H\big[\mathbf{I} - \mathbf{V}_s\mathbf{D}\mathbf{V}_s^H\big]\mathbf{C}_1 \tag{34}$$
$$= \mathbf{A}_0 + \sigma_v^2\mathbf{A}_1 + \sigma_v^4\mathbf{A}_2 + \mathbf{O}\big(\sigma_v^6\big) \tag{35}$$

where

$$\mathbf{D} = \mathrm{diag}\left\{\frac{\lambda_1}{\sigma_v^2+\lambda_1},\ldots,\frac{\lambda_\xi}{\sigma_v^2+\lambda_\xi}\right\},\quad \mathbf{A}_0 = \mathbf{C}_1^H\mathbf{V}_n\mathbf{V}_n^H\mathbf{C}_1 \tag{36}$$

$$\mathbf{A}_1 = \mathbf{C}_1^H\mathbf{V}_s\mathbf{\Lambda}_s^{-1}\mathbf{V}_s^H\mathbf{C}_1,\quad \mathbf{A}_2 = -\mathbf{C}_1^H\mathbf{V}_s\mathbf{\Lambda}_s^{-2}\mathbf{V}_s^H\mathbf{C}_1. \tag{37}$$

Proof: The proof of (34) can be easily achieved. According to (33) and using the fact that $\mathbf{I} = \mathbf{V}_s\mathbf{V}_s^H + \mathbf{V}_n\mathbf{V}_n^H$, we obtain

$$\mathbf{R}_y^{-1} = \frac{1}{\sigma_v^2}\big[\mathbf{I} - \mathbf{V}_s\mathbf{D}\mathbf{V}_s^H\big]. \tag{38}$$

Pre and postmultiplying (38) by $\mathbf{C}_1^H$ and $\mathbf{C}_1$, we arrive at (34).

If we further exploit the fact that

$$\frac{1}{\lambda_i + \sigma_v^2} = \frac{1}{\lambda_i} - \frac{1}{\lambda_i^2}\sigma_v^2 + O\big(\sigma_v^4\big) \tag{39}$$

in (34), we obtain (35). □

Based on this lemma, we can see that as $\sigma_v^2 \to 0$, $\mathbf{D} \to \mathbf{I}$ [cf., (36)], and hence, (34) implies that

$$\sigma_v^2\mathbf{C}_1^H\mathbf{R}_y^{-1}\mathbf{C}_1 \to \mathbf{C}_1^H\big[\mathbf{I} - \mathbf{V}_s\mathbf{V}_s^H\big]\mathbf{C}_1. \tag{40}$$

Therefore, the eigenvector corresponding to the minimum eigenvalue of $\mathbf{C}_1^H\mathbf{R}_y^{-1}\mathbf{C}_1$ converges to the minimizer

$$\min_{\mathbf{g}} \frac{\mathbf{g}^H\mathbf{C}_1^H\big[\mathbf{I} - \mathbf{V}_s\mathbf{V}_s^H\big]\mathbf{C}_1\mathbf{g}}{\mathbf{g}^H\mathbf{g}}. \tag{41}$$

[3] The value of ξ depends on if all users are synchronous or not. For example, if they are asynchronous, then $\xi = 2J+1$ because user 1 contributes three linearly independent signatures [cf., (7)], whereas each of the other $J-1$ users contributes two linearly independent signatures [cf., (8)].

Notice that if a $\mathbf{g}$ is found such that $\mathbf{h} = \mathbf{C}_1\mathbf{g} \in \mathbf{V}_s$, then $\mathbf{g}^H\mathbf{C}_1^H[\mathbf{I} - \mathbf{V}_s\mathbf{V}_s^H]\mathbf{C}_1\mathbf{g} = 0$, and $\mathbf{g}$ is the minimum eigenvector in (41). The choice $\mathbf{g} = \frac{\mathbf{g}_1}{\|\mathbf{g}_1\|}$ clearly qualifies since $\mathbf{h} = \frac{\mathbf{C}_1\mathbf{g}_1}{\|\mathbf{g}_1\|} = \frac{\mathbf{h}_1}{\|\mathbf{g}_1\|} \in \mathbf{V}_s$. The question that remains to be answered is whether $\mathbf{g} = \frac{\mathbf{g}_1}{\|\mathbf{g}_1\|}$ is a unique solution to (41), or if the zero eigenvalue has multiplicity greater that one.

The following proposition presents a sufficient condition that guarantees unique identifiability.

Proposition 1 (Identifiability): Under the assumption

$$\mathbf{AS1}\text{): } [\mathbf{C}_1\mathbf{H}] \text{ has full column rank}$$

there does not exist $\mathbf{g}'$ linearly independent from $\mathbf{g}_1$ such that $\mathbf{h}' \in \mathbf{V}_s$, where $\mathbf{h}' = \mathbf{C}_1\mathbf{g}'$.

Proof: The proof proceeds by contradition. Let both $\mathbf{C}_1\mathbf{g}_1 \in \mathbf{V}_s$ and $\mathbf{C}_1\mathbf{g}' \in \mathbf{V}_s$ for $\mathbf{g}' \neq \mathbf{g}_1$. Since $\mathrm{span}\{\mathbf{V}_s\} = \mathrm{span}\{[\mathbf{h}_1\mathbf{H}]\}$ [cf., (9)], there exist parameters θ' and θ_1' such that

$$\mathbf{C}_1\mathbf{g}' = \mathbf{H}\theta' + \theta_1'\mathbf{h}_1. \tag{42}$$

Using $\mathbf{h}_1 = \mathbf{C}_1\mathbf{g}_1$ in (42), we obtain

$$\mathbf{C}_1\tilde{\mathbf{g}} = \mathbf{H}\theta' \tag{43}$$

where $\tilde{\mathbf{g}} = (\mathbf{g}' - \theta_1'\mathbf{g}_1)$. If $\mathbf{g}$ and $\mathbf{g}'$ are linearly independent, then $\tilde{\mathbf{g}} \neq 0$, and (43) contradicts **AS1)**. □

Combining Proposition 1 with our earlier discussion [cf., (41)], we arrive at our first asymptotic result.

Proposition 2: If $\mathbf{g}$ is obtained by (29) and under the identifiability condition $\mathbf{AS1}$), it holds that

$$\mathbf{g} \to \frac{\mathbf{g}_1}{\|\mathbf{g}_1\|} \text{ as } \sigma_v^2 \to 0. \tag{44}$$

Some remarks are now in order:

1) Proposition 2 shows that at high SNR, (29) may be used as a channel estimation procedure, too. However, (29) combined with (23) provide a complete receiver design approach.
2) The identifiability condition of Proposition 1 is certainly useful in the analysis of the current minimum variance approach. Its applicability, however, is more general, extending to any subspace-based CDMA channel identification scheme, e.g., [1], [15].
3) The direct implementation of (29) may be computationally demanding as it involves the inversion of $\mathbf{R}_y$. Adaptive methods should be investigated that optimize (29) in conjunction with (23) and avoid the inversion of $\mathbf{R}_y$. These developments, however, are outside the scope of this paper and will be reported elsewhere.

We next wish to further quantify Proposition 2 and provide an expression for the error in estimating $\frac{\mathbf{g}_1}{\|\mathbf{g}_1\|}$ when (29) is used. Let us write $\mathbf{g}$ in (29) as

$$\mathbf{g} = \frac{\mathbf{g}_1}{\|\mathbf{g}_1\|} + \Delta\mathbf{g} \tag{45}$$

where $\Delta\mathbf{g}$ denotes the estimation bias and, in general, depends on SNR. In order to express $\Delta\mathbf{g}$ as a function of σ_v^2, we first establish the following lemma by using perturbation theory.

Lemma 2: If the matrix $\mathbf{C}_1^H\mathbf{R}_y^{-1}\mathbf{C}_1$ is expressed as a function of σ_v^2 as in (35), then

$$\sigma_w^2\mathbf{h}_1^H\mathbf{R}_y^{-1}\mathbf{h}_1 = 1 + \sigma_v^2\sigma_w^2\mathbf{g}_1^H\mathbf{A}_2\mathbf{g}_1 + O(\sigma_v^4) \tag{46}$$

$$\sigma_w^2\mathbf{g}_1^H\mathbf{A}_1\mathbf{g}_1 = 1 \tag{47}$$

and the minimum eigenvalue/eigenvector $\gamma_{\min}, \mathbf{g}$ of this matrix are given by

$$\mathbf{g} = \frac{\mathbf{g}_1}{\|\mathbf{g}_1\|} - \sigma_v^2\mathbf{A}_0^\dagger\mathbf{A}_1\frac{\mathbf{g}_1}{\|\mathbf{g}_1\|} + \mathbf{O}(\sigma_v^4) \tag{48}$$

$$\gamma_{\min} = \frac{1}{\sigma_w^2\|\mathbf{g}_1\|^2} + \sigma_v^2\frac{\mathbf{g}_1^H}{\|\mathbf{g}_1\|}\mathbf{A}_2\frac{\mathbf{g}_1}{\|\mathbf{g}_1\|} - \sigma_v^2\frac{\mathbf{g}_1^H}{\|\mathbf{g}_1\|}\mathbf{A}_1\mathbf{A}_0^\dagger\mathbf{A}_1\frac{\mathbf{g}_1}{\|\mathbf{g}_1\|} + O(\sigma_v^4) \tag{49}$$

where † denotes the pseudoinverse.

Proof: See Appendix A. □

By using Lemma 2, we are ready to arrive at the first order estimation error for $\Delta\mathbf{g}$ as follows.

Proposition 3: For small σ_v^2, and if $\mathbf{g}$ is given by (29), then

$$\Delta\mathbf{g} \simeq -\sigma_v^2\left[\mathbf{C}_1^H\mathbf{V}_n\mathbf{V}_n^H\mathbf{C}_1\right]^\dagger\mathbf{C}_1^H\mathbf{B}\mathbf{C}_1\frac{\mathbf{g}_1}{\|\mathbf{g}_1\|}$$

where † denotes pseudoinverse, and

$$\mathbf{B} = \mathbf{V}_s\mathrm{diag}\left\{\frac{1}{\sigma_v^2+\lambda_1}, \ldots, \frac{1}{\sigma_v^2+\lambda_\xi}\right\}\mathbf{V}_s^H.$$

Proof: Substituting (36) and (37) into the second term of the right-hand-side of (48), we obtain the result. □

Combining Propositions 1–3 and Lemmas 1 and 2, we may show that the performance of the proposed method tends to be close to that of the MMSE receiver as $\sigma_v^2 \to 0$. We, therefore, close this section with the following proposition, which establishes the asymptotic property of the minimum variance method.

Proposition 4: If $\mathbf{g}$, $\mathbf{f}_{\mathrm{mv}}$ are obtained from (29) and (23) under the condition **AS1)**, then

$$\frac{\mathrm{SINR}_{\mathrm{mse}}}{\mathrm{SINR}(\mathbf{g})} \to 1 + \delta \text{ as } \sigma_v^2 \to 0$$

where

$$\delta = \frac{\mathbf{g}_1^H\mathbf{A}_1\mathbf{A}_0^\dagger\mathbf{A}_1\mathbf{g}_1}{\mathbf{g}_1^H\mathbf{C}_1^H\mathbf{V}_s\mathbf{\Lambda}_s^{-2}\mathbf{V}_s^H\mathbf{C}_1\mathbf{g}_1}.$$

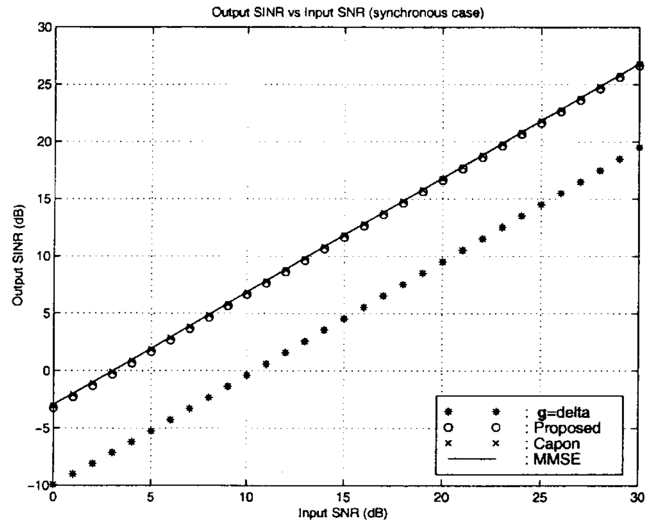

Fig. 3 Performance of the proposed algorithm for synchronous system.

Proof: See Appendix B. □

We should stress here that the previous performance claims are also applicable to the Capon-like method of (15). Indeed, Propositions 2 and 4 are applicable here too since the numerator of (15) is identical to that of (29). Our simulation results indicated similar performance for the two minimum variance alternatives, as explained in the next section. This can be further explained by the fact that in most CDMA systems, the spreading codes are chosen so that they are nearly orthogonal to their time shifts. Hence, we expect $\mathbf{C}_1^H\mathbf{C}_1 \simeq P\mathbf{I}$, which makes (15) approximately equivalent to (29).

VI. Simulations

We tested the proposed methods in a system with ten users and spreading factor $P = 31$. Gold codes of length 31 were used as spreading codes, and each user's signal was transmitted through a (different) multipath channel of length equal to four chips. Both the synchronous and asynchronous cases were tested. In both cases, we simulated a severe near-far case, where the user of interest was a weak one with power 10 dB less than each of the interfering users.

Fig. 3 compares the performance of various minimum variance methods with that of the MMSE receiver for a synchronous system. It depicts the output SINR for a wide range of input SNR. Notice that both proposed methods of (29) and (15) (circles and 'x's, respectively) perform close to the MMSE solution (solid line), whereas if the constraint is not optimized ($\mathbf{g} = [1, 0, \ldots, 0]^T$ depicted in stars), a severe performance loss is experienced. Similar behavior was observed when the method was applied to an asynchronous system as shown in Fig. 4. These results are in accordance with the performance analysis developments of the paper. The next two figures further validate the predictions of Propositions 4 and 3. Fig. 5 shows the ratio $\frac{\mathrm{SINR}(\mathbf{g})}{\mathrm{SINR}_{mmse}}$ versus SNR for both synchronous [Fig. 5(a)] and asynchronous [Fig. 5(b)] systems. The ratio can be observed to tend to be a constant (dashed line), which is close to 1 as SNR $\to \infty$ for both cases, as expected from Proposition 4. Fig. 6 shows the bias in estimating the

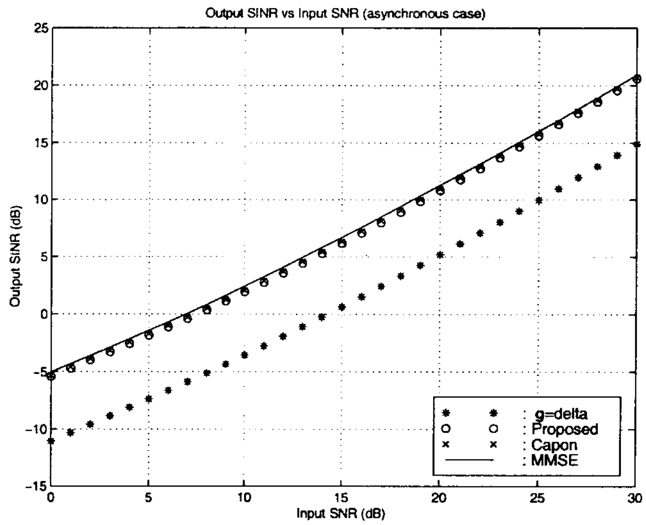

Fig. 4. Performance of the proposed algorithm for asynchronous system.

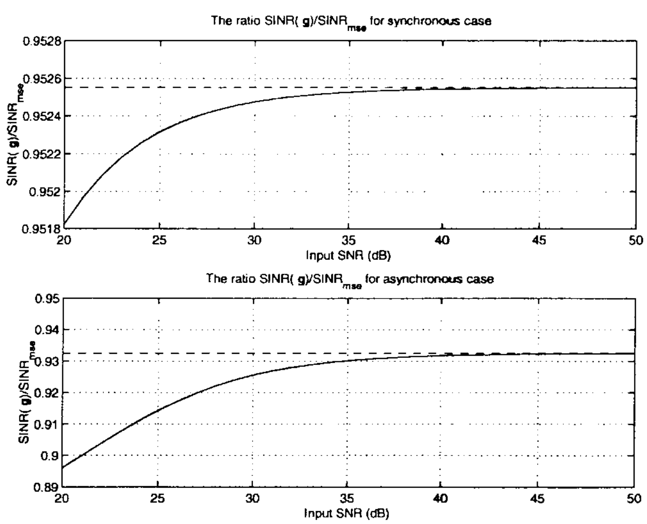

Fig. 5. Ratio SINR/SINR$_{mmse}$ versus SNR.

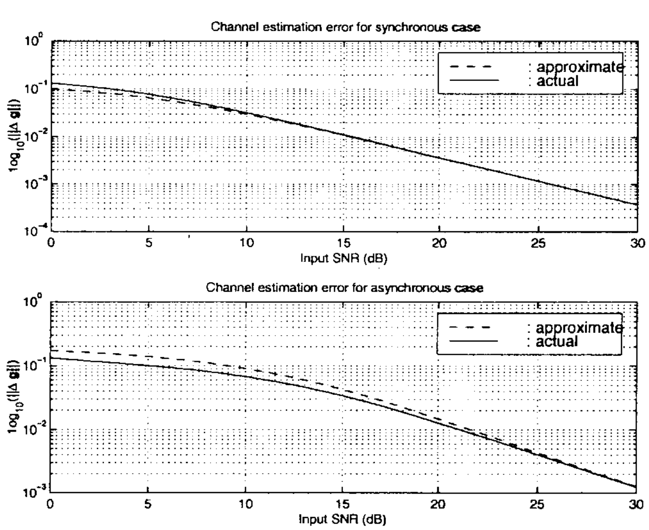

Fig. 6. Actual and approximate estimation error of channel parameters.

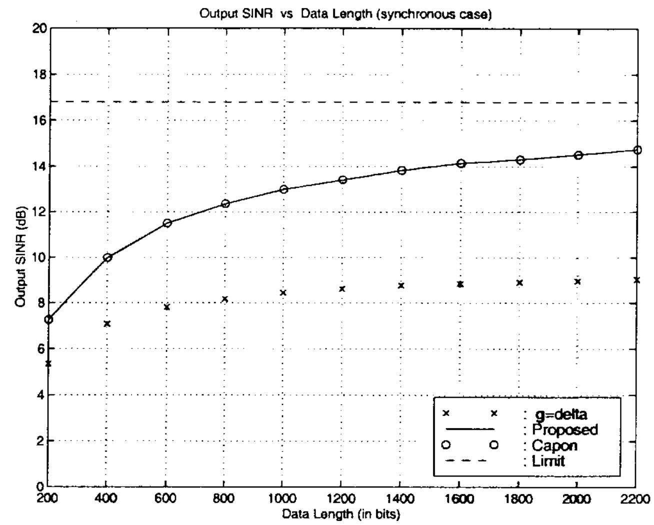

Fig. 7. Output SINR comparison for different data lengths (synchronous).

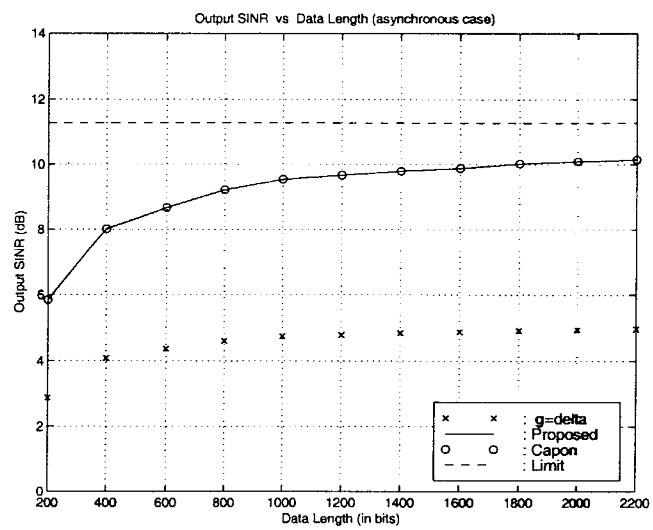

Fig. 8. Output SINR comparison for different data lengths (asynchronous).

channel parameters (in logarithmic scale) when (29) is used (solid line), as a function of SNR. The approximate formula for evaluating the bias (see Proposition 3) was also used (dashed line). The two lines can be seen to converge as SNR $\rightarrow \infty$.

The last simulation experiment investigates the effects of the data length N on the system's performance. The comparisons presented in the previous figures are all based on ideal correlations ($N = \infty$). In Figs. 7 and 8, Monte Carlo results are presented for a synchronous and asynchronous system, respectively, when the correlation matrix is estimated through sample averaging from the available data. The average output SINR is plotted versus data length for SNR $= 20$ dB (50 Monte Carlo iterations per data length point). The proposed methods again exhibit performance similar to that of the Capon method, whereas a severe performance loss is observed if the constraint is not optimized. The limiting case SNR $= \infty$ is also shown as a in dashed line. Finally, Fig. 9 shows the deviation of the estimated channel parameters from their mean values as a function of the data length. The experimental average

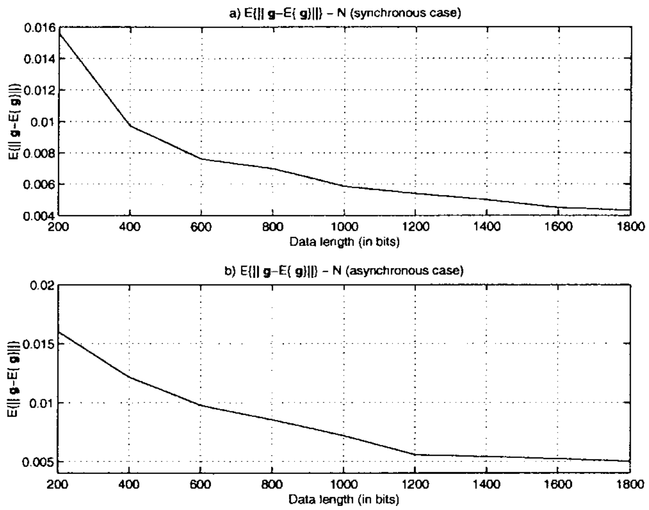

Fig. 9. Average estimation error of channel parameters versus data length (SNR= 20 dB).

deviation of 50 Monte Carlo iterations per point is plotted for the synchronous [Fig. 9(a)] and asynchronous [Fig. 9(b)] case. The deviation is decreasing with N as expected.

VII. Conclusion

Our performance analysis studies indicate that the choice of the constraint vector $\mathbf{g}$ may have a significant effect on the performance of constrained minimum variance CDMA receivers. It appears that a max/min approach, where the minimum output variance is maximized with respect to $\mathbf{g}$, yields a blind method with significant performance benefits. It is shown that its performance tends to be close to that of the MMSE receiver at high SNR, whereas the obtained constraint vector converges to the multipath channel parameters. Further research is required to develop computationally efficient adaptive versions of this method and study their performance.

Appendix A
Proof of Lemma 2

We first use the expression SINR_{mse} in (21) to prove (46) and (47). It is well known that $\text{SINR}_{\text{mse}} \to \infty$ as $\sigma_v^2 \to 0$ (as the MMSE solution converges to the zero forcing one). Hence, from (21), we must have $\sigma_w^2 \mathbf{h}_1^H \mathbf{R}_y^{-1} \mathbf{h}_1 \to 1$ as $\sigma_v^2 \to 0$. By pre and postmultiplying (35) by $\mathbf{g}_1^H$ and $\mathbf{g}_1$ and recalling that $\mathbf{h}_1 = \mathbf{C}_1 \mathbf{g}_1$ and that $\mathbf{V}_n^H \mathbf{h}_1 = 0$, we obtain

$$\sigma_w^2 \mathbf{h}_1^H \mathbf{R}_y^{-1} \mathbf{h}_1 = \sigma_w^2 \mathbf{g}_1^H \mathbf{A}_1 \mathbf{g}_1 + \sigma_v^2 \sigma_w^2 \mathbf{g}_1^H \mathbf{A}_2 \mathbf{g}_1 + O(\sigma_v^4). \quad (50)$$

Taking the limit to both sides of (50) as $\sigma_v^2 \to 0$, we then obtain (47). By substituting (47) in (50), (46) is achieved.

According to perturbation theory [13], if a matrix $\mathbf{A}$ can be expressed as

$$\mathbf{A} = \mathbf{A}_0 + \sigma_v^2 \mathbf{A}_1 + \sigma_v^4 \mathbf{A}_2 + \cdots \quad (51)$$

and $(\gamma_0, \mathbf{v}_0), (\gamma, \mathbf{v})$ are eigenvalue/eigenvector pairs of $\mathbf{A}_0$ and $\mathbf{A}$, respectively, then there exist $\gamma_1, \gamma_2, \ldots$ and $\mathbf{v}_1, \mathbf{v}_2, \ldots$ such that

$$\gamma = \gamma_0 + \sigma_v^2 \gamma_1 + \sigma_v^4 \gamma_2 + \cdots \quad (52)$$

$$\mathbf{v} = \mathbf{v}_0 + \sigma_v^2 \mathbf{v}_1 + \sigma_v^4 \mathbf{v}_2 + \cdots \quad (53)$$

$\forall \sigma_v^2$ in some neighborhood of $\sigma_v^2 = 0$. By substituting (52) and (53) in the eigenvalue problem $\mathbf{A}\mathbf{v} = \gamma \mathbf{v}$ and equating equal powers of σ_v^2, we obtain the equations (cf., [2])

$$\mathbf{A}_0 \mathbf{v}_0 = \gamma_0 \mathbf{v}_0 \quad (54)$$

$$\mathbf{A}_0 \mathbf{v}_1 + \mathbf{A}_1 \mathbf{v}_0 = \gamma_0 \mathbf{v}_1 + \gamma_1 \mathbf{v}_0 \quad (55)$$

$$\mathbf{A}_0 \mathbf{v}_2 + \mathbf{A}_1 \mathbf{v}_1 + \mathbf{A}_2 \mathbf{v}_0 = \gamma_0 \mathbf{v}_2 + \gamma_1 \mathbf{v}_1 + \gamma_2 \mathbf{v}_0. \quad (56)$$

Equation (54) offers no new information. The other two, however, may be premultiplied by $\mathbf{v}_0^H$ to yield

$$\gamma_1 = \mathbf{v}_0^H \mathbf{A}_1 \mathbf{v}_0 \quad (57)$$

$$\gamma_2 = \mathbf{v}_0^H \mathbf{A}_1 \mathbf{v}_1 + \mathbf{v}_0^H \mathbf{A}_2 \mathbf{v}_0 - \gamma_1 \mathbf{v}_0^H \mathbf{v}_1. \quad (58)$$

In our case, consider $\mathbf{A} = \sigma_v^2 \mathbf{C}_1^H \mathbf{R}_y^{-1} \mathbf{C}_1$; then, $\mathbf{A}_0, \mathbf{A}_1, \mathbf{A}_2$ are given by (36) and (37), and we are interested in the perturbation of the null eigenvector $\mathbf{v}_0 = \frac{\mathbf{g}_1}{\|\mathbf{g}_1\|}$ corresponding to the eigenvalue $\gamma_0 = 0$. According to (47), (57) yields $\gamma_1 = \frac{1}{\sigma_w^2 \|\mathbf{g}_1\|^2}$. Further, it can be shown [6] that

$$\mathbf{v}_1 = -\mathbf{A}_0^{\dagger} \mathbf{A}_1 \mathbf{v}_0. \quad (59)$$

Equation (59) shows that $\mathbf{v}_1^H \mathbf{v}_0 = 0$ since $\mathbf{v}_0^H \mathbf{A}_0 = \mathbf{0}$, and hence, $\mathbf{v}_0^H \mathbf{A}_0^{\dagger} = \mathbf{0}$. Thus, the last term of (58) can be ignored

$$\gamma_2 = -\mathbf{v}_0^H \mathbf{A}_1 \mathbf{A}_0^{\dagger} \mathbf{A}_1 \mathbf{v}_0 + \mathbf{v}_0^H \mathbf{A}_2 \mathbf{v}_0. \quad (60)$$

Substituting (60) and (59) into (52) and (53) and dividing by σ_v^2 ($\frac{1}{\sigma_v^2}\mathbf{A}$ has eigenvalue $\gamma_{\min} = c\frac{\gamma}{\sigma_v^2}$), we obtain the desired result.

□

Appendix B
Proof of Proposition 4

We will express SINR for the MMSE receiver in (21) and the proposed method in (26) as the explicit functions of σ_v^2 by using (35) and Lemma 2. Then, the ratio of $\frac{\text{SINR}_{\text{mse}}}{\text{SINR}(\mathbf{g})}$ will be evaluated.

First, we start with SINR_{mse} in (21). Substituting (46) in Lemma 2 into (21), we obtain

$$\text{SINR}_{\text{mse}} = \frac{1 + O(\sigma_v^2)}{\sigma_v^2 \sigma_w^2 \mathbf{g}_1^H \mathbf{A}_2 \mathbf{g}_1 + O(\sigma_v^4)}. \quad (61)$$

Next, we turn our attention to $\text{SINR}(\mathbf{g})$ in (26). We have assumed that the minimum eigenvalue/eigenvector of $\mathbf{C}_1^H \mathbf{R}_y^{-1} \mathbf{C}_1$ is $\gamma_{\min}, \mathbf{g}$, which are given by (48) and (49) in Lemma 2. Then, (26) can be written as

$$\text{SINR}(\mathbf{g}) = \frac{\sigma_w^2 \gamma_{\min} \|\mathbf{g}^H \mathbf{g}_1\|^2}{1 - \sigma_w^2 \gamma_{\min} \|\mathbf{g}^H \mathbf{g}_1\|^2}. \quad (62)$$

Noting that $\mathbf{g}_1^H \mathbf{A}_0^{\dagger} = \mathbf{0}$ since $\mathbf{g}_1$ is a null eigenvector of $\mathbf{A}_0$ and $\mathbf{A}_0^{\dagger}$, we may write

$$\|\mathbf{g}^H \mathbf{g}_1\|^2 = \|\mathbf{g}_1\|^2 + O(\sigma_v^4). \quad (63)$$

Finally, substituting (63) and (49) in (62), we arrive at

$$\mathrm{SINR}(\mathbf{g}) = \frac{1+O(\sigma_v^2)}{\sigma_v^2\sigma_w^2\mathbf{g}_1^H\mathbf{A}_2\mathbf{g}_1 - \sigma_v^2\sigma_w^2\mathbf{g}_1^H\mathbf{A}_1\mathbf{A}_0^\dagger\mathbf{A}_1\mathbf{g}_1 + O(\sigma_v^4)}. \quad (64)$$

Having managed to express the SINR as a function of σ_v^2, we may now combine (64) with (61) to compute the limit

$$\lim_{\sigma_v^2\to 0}\frac{\mathrm{SINR}_{\mathrm{mse}}}{\mathrm{SINR}(\mathbf{g})} = 1 - \frac{\mathbf{g}_1^H\mathbf{A}_1\mathbf{A}_0^\dagger\mathbf{A}_1\mathbf{g}_1}{\mathbf{g}_1^H\mathbf{A}_2\mathbf{g}_1}. \quad (65)$$

By substituting $\mathbf{A}_2$ in (35) into (65), we obtain the expression for δ in Proposition 4. □

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