Low-Complexity Multiuser Channel Estimation With Aperiodic Spreading Codes

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***Abstract*—Signal processing techniques for CDMA systems employing aperiodic spreading sequences have gained significant interest recently. Due to the time-varying nature of users' unknown signatures in a multipath communication environment, direct design of blind multiuser detectors is intractable. We focus on estimating the unknown multipath parameters for each active user in the system. The problem is solved in the correlation matching context based on correlations of both the directly received data and the outputs of a bank of matched filters. Three typical scenarios are discussed such as quasisynchronous uplink CDMA system with AWGN, with unknown interference, and downlink CDMA system with AWGN, leading to different solutions. We model the aperiodic spreading codes as random variables. For any *priori*-known distribution of the spreading codes, their statistics up to the fourth order can be evaluated, resulting in extremely low computational complexity of the methods. The identifiability of the channel parameters only depends on the nonsingularity of a deterministic matrix determined by known system parameters. In the case of unknown code statistics, the methods can be modified to be still applicable by estimating those code statistics from given spreading codes. However, in such a case, more computations are needed. Comparisons with other existing methods show that the proposed computationally efficient approaches can provide satisfactory results while requiring significantly less computations.**

***Index Terms*—Aperiodic spreading, channel estimation, correlation matching, low complexity.**

I. Introduction

DIRECT sequence (DS) code-division multiple-access (CDMA) techniques have been adopted as standards in the third-generation wireless communication networks [6], [24]. In DS-CDMA systems, the bandwidth of the input signal is spread by a sequence with a much higher rate in order to effectively suppress the interference. Two kinds of spreading codes exist. The periodic spreading sequence (short codes) repeats from symbol to symbol, whereas the aperiodic spreading sequence (long codes) has a much longer period compared with the symbol duration.

In the CDMA literature, many efforts focus on the CDMA systems with periodic codes. The exploitation of such short codes implies a time-invariant structure for the interference and facilitates multiuser detection. Due to their analytical tractability, short-code CDMA systems have been extensively studied. Various algorithms to detect a desired user have been developed and theoretical results have been provided [16]. In wireless communications, the communication channel is unknown and, thus, needs to be estimated by using either training sequences or the channel's output only. Based on the channel estimates, the detector can be easily build for a short-code CDMA system [1], [12], [17]. However, the current standards for DS-CDMA systems employ aperiodic spreading codes. The time-varying nature of signatures renders previous channel estimation and multiuser detection methods not directly applicable.

For those reasons, signal processing techniques for CDMA systems with aperiodic spreading codes have gained interest recently. A number of studies for such systems have appeared. Based on the finite alphabet property of the input, an iterative method to estimate the FIR channels and the transmitted symbols is presented in [13]. In [11] and [18], subspace concepts are adopted to identify the multipath channel. The design of blind receivers to suppress the interference from other cells is discussed in [4]. The blind uplink channel estimation method using correlation matching techniques is proposed in [20] based only on the correlation of the channel's output conditioned on the long spreading codes. The algorithm for downlink channel estimation with low computational complexity has been reported in [23] and for uplink channel estimation in [22]. Those low-complexity algorithms typically deal with the unconditional correlations of the output of a bank of matched filters. For the quasi-synchronous uplink CDMA system discussed in [22], many users experience different unknown communication channels. For the interest of only a group of desired users, the algorithm to estimate the channel has been developed. However, the systematic description of the algorithm and proof of some theoretical results have not been performed.

In this paper, we propose novel low complexity algorithms to estimate channel parameters for desired users in different scenarios:

1) uplink CDMA system with AWGN;
2) uplink CDMA system with unknown interference and noise;
3) downlink CDMA system with AWGN.

These cases can be described by *one* input/output relationship. However, the characteristic of the interference induces different solutions. The first case occurs when all users in the system are known, and the background noise is modeled as AWGN. The second case appears when there are unknown interfers, and the structure of the interference is not available. Different from [22] and [23], we formulate the problem by constructing and minimizing a correlation matching cost function. Correlation matching (or correlation fitting) only uses the second-order statistics (SOS) of the data vector and, thus, offers some superi-

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ority in complexity over higher order statistics (HOS) methods. Moment matching has been applied to various problems such as fractionally spaced channel estimation [5], and time-varying system identification [15]. Estimation of downlink and uplink short-code CDMA channels has been reported in [2] and [21], respectively.

For the CDMA system with aperiodic spreading codes, it is shown in [20] that the channel parameters for all active users or a single user can be estimated from the conditional autocorrelation matrix of the channel's output conditioned on the spreading codes. When all users are considered, the computational complexity is significant since inversion of a large matrix has to be performed many times. In this work, complexity will be significantly reduced. With *a priori*-known up to fourth-order statistics of the spreading codes, some code-related matrices can be precomputed instead of being estimated online. The identifiability of the channel parameters only depends on the nonsingularity of a deterministic matrix in each case. It is derived in a closed form and determined by known system parameters. In the case of unknown code statistics, the methods can be modified to be still applicable by estimating those code statistics from given spreading codes. However, in such a case, more computations are involved. Simulation results are provided for typical scenarios. Comparisons with other existing methods show that the proposed computationally efficient methods can provide satisfactory results while requiring significantly fewer computations.

The rest of the paper has the following structure. The problem is first formulated for three different scenarios in Section II. Section III describes in detail our low-complexity multiuser channel estimation methods corresponding to those three cases. An identifiability issue is studied, and expressions of some deterministic quantities are derived in Section IV. Simulation results and comparisons with other methods are shown in Section V. Finally, some conclusions are drawn in Section VI.

II. Problem Formulation

First, consider a quasi-synchronous uplink CDMA system [9] that employs random spreading codes. Assume it is known that J mobile stations (or J users) communicate with a base station. User j ($j = 1, \ldots, J$) has the information bit stream $w_j(n)$ to transmit through a multipath channel $g_j(m)$. All channels are assumed to have maximum order q. During the nth bit period, user j is assigned a random spreading code $c_{j,n}(k)$ for the kth chip ($k = 0, \ldots, P-1$) with spreading factor P. The signal from user j arrives at the base station with delay δ_j ($0 \le \delta_j \ll P$) in chip period. Then, the received discrete-time signal can be written as (see Fig. 1 and [20]).

$$y(n) = \sum_{j=1}^{J} \sum_{m=0}^{q} g_j(m) s_j(n-m-\delta_j) + v(n) \tag{1}$$

where

$$s_j(n) = \sum_{k=-\infty}^{\infty} w_j(k) c_{j,k}(n-kP) \tag{2}$$

$w_j(n)$ has power $\sigma_{w_j}^2 = E\{|w_j(n)|^2\}$, and $v(n)$ is modeled as zero-mean AWGN with variance $\sigma_v^2 = E\{|v(n)|^2\}$ if all

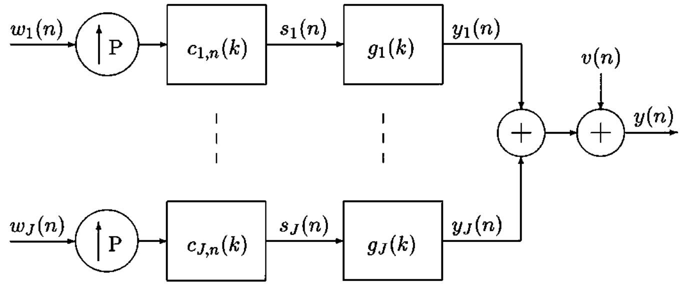

Fig. 1. CDMA system with aperiodic spreading.

users in the system are known. In the case of the presence of unidentified interfers, it is reasonable to model it as a zero-mean random variable with unknown second-order statistics.

With quasisynchronization, it is possible to eliminate the intersymbol interference in the received data. We collect only $L = P - \mu$ samples around the nth bit interval in a vector $\boldsymbol{y}(n) = [y(nP+\mu), \ldots, y(nP+P-1)]^T$ with $\mu = \max(q+\delta_j)$ ($j = 1, \ldots, J$). Similarly, let the noise vector be $\boldsymbol{v}(n) = [v(nP+\mu), \ldots, v(nP+P-1)]^T$ and the channel vector for user j be $\boldsymbol{g}_j = [g_j(0), \ldots, g_j(q)]^T$. Then, according to (1), a simple matrix representation follows (see also [20]):

$$\boldsymbol{y}(n) = \sum_{j=1}^{J} \boldsymbol{C}_j(n) \boldsymbol{g}_j w_j(n) + \boldsymbol{v}(n) \tag{3}$$

where the $L \times (q+1)$ code matrix $\boldsymbol{C}_j(n)$ is a truncated version of the following filtering matrix from the $(\mu+1)$th row to the Pth row

$$\tilde{\boldsymbol{C}}_j(n) = \begin{bmatrix} c_{j,n}(0) & & \mathbf{0} \\ \vdots & \ddots & c_{j,n}(0) \\ c_{j,n}(P-1) & & \vdots \\ \mathbf{0} & \ddots & c_{j,n}(P-1) \end{bmatrix} \tag{4}$$

i.e., $\boldsymbol{C}_j(n) = [\tilde{\boldsymbol{C}}_j(n)]_{\mu+1:P,\,1:q+1}$. Their relationship can thus be expressed by using a selection matrix $\boldsymbol{T}$

$$\boldsymbol{C}_j(n) = \boldsymbol{T}\tilde{\boldsymbol{C}}_j(n), \quad \boldsymbol{T} = [\mathbf{0}_{L\times\mu} \boldsymbol{I}_{P-\mu} \mathbf{0}_{L\times q}]. \tag{5}$$

The vector form input/output model (3) can be used to describe different CDMA communication scenarios as discussed before, depending on the modeling of the noise. It can also be used to describe the downlink communication if all $\boldsymbol{g}_j$ are the same. For these cases, we will discuss them respectively. Besides the directly received data vector $\boldsymbol{y}(n)$, we will further exploit the outputs from the kth matched filter, which correlates $\boldsymbol{y}(n)$ with code matrices $\boldsymbol{C}_k^H(n)$

$$\begin{aligned} \boldsymbol{y}_k(n) &\triangleq \boldsymbol{C}_k^H(n) \boldsymbol{y}(n) \\ &= \sum_{j=1}^{J} \boldsymbol{C}_k^H(n) \boldsymbol{C}_j(n) \boldsymbol{g}_j w_j(n) + \boldsymbol{C}_k^H(n) \boldsymbol{v}(n) \end{aligned} \tag{6}$$

for $k = 1, \ldots, J$. Before we derive our solutions, the following common assumptions for these cases are first explicitly made.

1) $w_j(n)$ is i.i.d. random in j and n with zero-mean and variance $\sigma_{w_j}^2$.
2) $c_{j,n}(k)$ is i.i.d. random in j, n and k, with zero-mean, variance $\sigma_c^2 = E\{|c_{j,n}(k)|^2\}$ and fourth-order moment $m_{4c} = E\{|c_{j,n}(k)|^4\}$ for $\forall j$; if $c_{j,n}(k)$ is complex, its real and imaginary parts are also i.i.d.
3) $w_j(n)$, $c_{j,n}(k)$ and $v(n)$ are mutually independent.
4) P, q, and $y(n)$ are known; δ_j and $c_{j,n}(k)$ ($j = 1, \ldots, J$) are also known.

A. Uplink with J Users and Unknown Interference

In this case, there are J users in the system communicating with the base station. Assume that other unknown interfers contribute stationary interference in $\boldsymbol{v}(n)$. Its autocorrelation matrix is denoted by $\boldsymbol{R}_{int}$. The methods to be presented are all based on the SOS of the data. From (3), the autocorrelation matrix of the data vector $\boldsymbol{y}(n)$ is

$$\boldsymbol{R} \triangleq E\{\boldsymbol{y}(n)\boldsymbol{y}^H(n)\} = \sum_{j=1}^{J} E\{\boldsymbol{C}_j(n)\boldsymbol{G}_j\boldsymbol{C}_j^H(n)\} + \boldsymbol{R}_{int} \tag{7}$$

where $\boldsymbol{G}_j = \sigma_{w_j}^2 \boldsymbol{g}_j \boldsymbol{g}_j^H$, and superscript H represents conjugate transpose. Since $\boldsymbol{R}_{int}$ and $\boldsymbol{G}_j$ are unknown, to guarantee identifiability, we will further exploit the outputs of J matched filters. The correlations of their outputs can be computed from (6)

$$\begin{aligned}\boldsymbol{R}_k &\triangleq E\{\boldsymbol{y}_k(n)\boldsymbol{y}_k^H(n)\}\\ &= \sum_{j=1}^{J} E\{\boldsymbol{C}_k^H(n)\boldsymbol{C}_j(n)\boldsymbol{G}_j\boldsymbol{C}_j^H(n)\boldsymbol{C}_k(n)\}\\ &\quad + E\{\boldsymbol{C}_k^H(n)\boldsymbol{R}_{int}\boldsymbol{C}_k(n)\}.\end{aligned} \tag{8}$$

Notice that channel information is embedded in those correlations in (7) and (8) that are linearly parameterized by $\boldsymbol{G}_j$. If $\boldsymbol{G}_j$ is estimated, then the estimate for $\boldsymbol{g}_j$ is obtained by performing SVD on this rank one matrix within a complex scalar ambiguity. The approach can be pursued after those correlations are easily estimated from the received data. Thus, $\boldsymbol{G}_j$ can be estimated based on the correlation matching idea.

B. Uplink with J Users and AWGN

Assume there are no other users except J active users in the system. The unknown interference is only from background noise. Thus, $\boldsymbol{v}(n)$ can be modeled as AWGN with power $E\{|v(n)|^2\} = \sigma_v^2$. Then, $\boldsymbol{R}_{int}$ becomes $\sigma_v^2\boldsymbol{I}$. The number of unknowns significantly decreases. Instead of employing $\boldsymbol{R}$, we use the power of the received signal

$$\eta \triangleq E\{\boldsymbol{y}^H(n)\boldsymbol{y}(n)\} = \sum_{j=1}^{J} E\{\mathrm{tr}[\boldsymbol{C}_j(n)\boldsymbol{G}_j\boldsymbol{C}_j^H(n)]\} + \sigma_v^2 L \tag{9}$$

where "tr" represents the trace of a matrix. Corresponding to (8), we obtain

$$\begin{aligned}\boldsymbol{R}_k = &\sum_{j=1}^{J} E\{\boldsymbol{C}_k^H(n)\boldsymbol{C}_j(n)\boldsymbol{G}_j\boldsymbol{C}_j^H(n)\boldsymbol{C}_k(n)\}\\ &+\sigma_v^2 E\{\boldsymbol{C}_k^H(n)\boldsymbol{C}_k(n)\}.\end{aligned} \tag{10}$$

In such a case, unknowns become $\boldsymbol{G}_j$ and σ_v^2.

C. Downlink with J Users and AWGN

In the downlink, assume all J users have equal transmitted power $\sigma_{w_1}^2 \cdots = \sigma_{w_J}^2 \triangleq \sigma_w^2$. They also experience the same channel $\boldsymbol{g}_1 = \cdots = \boldsymbol{g}_J \triangleq \boldsymbol{g}$. The background noise is assumed to be AWGN with zero mean and variance σ_v^2. Then, we obtain the output power and the autocorrelation matrix of the received data

$$\eta = \sum_{j=1}^{J} E\{\mathrm{tr}[\boldsymbol{C}_j(n)\boldsymbol{G}\boldsymbol{C}_j^H(n)]\} + \sigma_v^2 L \tag{11}$$

$$\begin{aligned}\boldsymbol{R}_k = &\sum_{j=1}^{J} E\{\boldsymbol{C}_k^H(n)\boldsymbol{C}_j(n)\boldsymbol{G}\boldsymbol{C}_j^H(n)\boldsymbol{C}_k(n)\}\\ &+ \sigma_v^2 E\{\boldsymbol{C}_k^H(n)\boldsymbol{C}_k(n)\}\end{aligned} \tag{12}$$

where $\boldsymbol{G} = \sigma_w^2 \boldsymbol{g}\boldsymbol{g}^H$. Therefore, there are only two unknown arguments $\boldsymbol{G}$ and σ_v^2. The method will be possibly simplified.

III. Channel Estimation with Low Complexity

As explained earlier, our unknowns are embedded in the SOS of $\boldsymbol{y}(n)$. These SOS can be matched with their estimate from the data, and the resulting error can be minimized. Corresponding to three cases in Section II-A–C, we will derive our solutions, respectively. In particular, we will discuss the first case in more detail, whereas the other two can be similarly treated.

A. Uplink with J Users and Unknown Interference

Assume all SOS of $\boldsymbol{y}(n)$ are known or estimated. To solve $\boldsymbol{G}_j$ and $\boldsymbol{R}_{int}$, we start from (7) and (8). In order to obtain a closed-form solution, we introduce the vec operation that stacks all columns of a matrix into a vector [10]. It has the following property:

$$vec(\boldsymbol{X}_1\boldsymbol{X}_2\boldsymbol{X}_3) = (\boldsymbol{X}_3 \otimes \boldsymbol{X}_1)vec(\boldsymbol{X}_2).$$

This property shows that $\boldsymbol{X}_2$ can be extracted from the middle of the matrix product to its outside. If we define $\boldsymbol{r}_{int} \triangleq vec(\boldsymbol{R}_{int})$ and $\boldsymbol{d}_j \triangleq vec(\boldsymbol{G}_j)$, then after vec operation on (7) and (8), we can obtain

$$\boldsymbol{r} \triangleq vec(\boldsymbol{R}) = \sum_{j=1}^{J} E\{\boldsymbol{Q}_j(n)\}\,\boldsymbol{d}_j + \boldsymbol{r}_{int} \tag{13}$$

$$\begin{aligned}\boldsymbol{r}_k &\triangleq vec(\boldsymbol{R}_k)\\ &= E\{\boldsymbol{Q}_k^H(n)\sum_{j=1}^{J}\boldsymbol{Q}_j(n)\boldsymbol{d}_j\} + E\{\boldsymbol{Q}_k^H(n)\}\boldsymbol{r}_{int}\end{aligned} \tag{14}$$

where

$$\boldsymbol{Q}_k(n) \triangleq \boldsymbol{C}_k^*(n) \otimes \boldsymbol{C}_k(n) \tag{15}$$

where superscript $*$ denotes complex conjugate, and "$\otimes$" denotes the Kronecker product. Considering J possible values for k, the number of equations (13) and (14) is sufficient to solve our unknowns $\boldsymbol{d}_j$ and $\boldsymbol{r}_{int}$. To see it clearly, first stacking (14) for $k = 1, \ldots, J$ together, we have

$$\boldsymbol{u} = E\{\boldsymbol{Q}^H(n)\boldsymbol{Q}(n)\}\boldsymbol{d} + E\{\boldsymbol{Q}^H(n)\}\boldsymbol{r}_{int} \tag{16}$$

where

$$\boldsymbol{u} \triangleq [\boldsymbol{r}_1^T, \ldots, \boldsymbol{r}_J^T]^T, \quad \boldsymbol{d} \triangleq [\boldsymbol{d}_1^T, \ldots, \boldsymbol{d}_J^T]^T$$
$$\boldsymbol{Q} \triangleq [\boldsymbol{Q}_1, \ldots, \boldsymbol{Q}_J]. \tag{17}$$

Under the previous definitions, (13) can also be rewritten as

$$\boldsymbol{r} = E\{\boldsymbol{Q}(n)\}\boldsymbol{d} + \boldsymbol{r}_{int}. \tag{18}$$

In the current estimation problem, $\boldsymbol{r}_{int}$ is a nuisance parameter. From (16) and (18), $\boldsymbol{r}_{int}$ is eliminated, to obtain an equation of $\boldsymbol{d}$

$$\boldsymbol{S}_1\boldsymbol{d} = \boldsymbol{z}_1 \tag{19}$$

where[1]

$$\boldsymbol{S}_1 \triangleq E\{\boldsymbol{Q}^H(n)\boldsymbol{Q}(n)\} - E\{\boldsymbol{Q}^H(n)\}E\{\boldsymbol{Q}(n)\},$$
$$\boldsymbol{z}_1 \triangleq \boldsymbol{u} - E\{\boldsymbol{Q}^H(n)\}\boldsymbol{r}. \tag{20}$$

Equation (19) is fundamental to our correlation-based channel estimation method. Assume that the estimate for $\boldsymbol{z}_1$ from N data vectors is $\hat{\boldsymbol{z}}_1$. As a common practice, it is obtained by its sample average based on the sample averages for $\boldsymbol{u}$ and $\boldsymbol{r}$

$$\hat{\boldsymbol{z}}_1 = \frac{1}{N}\sum_{n=1}^{N} \boldsymbol{z}_1(n),$$
$$\boldsymbol{z}_1(n) = [\boldsymbol{Q}^H(n) - E\{\boldsymbol{Q}^H(n)\}]vec[\boldsymbol{y}(n)\boldsymbol{y}^H(n)] \tag{21}$$

where $\boldsymbol{Q}^H(n)vec[\boldsymbol{y}(n)\boldsymbol{y}^H(n)]$ is the instantaneous estimate for $\boldsymbol{u}$, which is obtained by using the fact that $vec[\boldsymbol{y}_j(n)\boldsymbol{y}_j^H(n)] = \boldsymbol{Q}_j^H(n)vec[\boldsymbol{y}(n)\boldsymbol{y}^H(n)]$. In the expressions of $\boldsymbol{S}_1$ and $\boldsymbol{z}_1(n)$, the code-related matrices $E\{\boldsymbol{Q}^H(n)\boldsymbol{Q}(n)\}$ and $E\{\boldsymbol{Q}(n)\}$ are data irrelevant and can be precomputed. This will be discussed later. Motivated by (19), $\boldsymbol{d}$ can be solved by minimizing the following matching error $\|\boldsymbol{S}_1\boldsymbol{d} - \hat{\boldsymbol{z}}_1\|^2$. The solution to this problem is easily found to be

$$\hat{\boldsymbol{d}} = (\boldsymbol{S}_1^H\boldsymbol{S}_1)^{-1}\boldsymbol{S}_1^H\hat{\boldsymbol{z}}_1. \tag{22}$$

In obtaining (22), we have assumed that $\boldsymbol{S}_1$ has full column rank. This topic will be discussed in detail in Section IV. Some preliminary observation can be made about this deterministic matrix. It depends on the statistics of code matrices and, thus, can be theoretically precomputed if the statistics are derived from the distribution of the spreading codes $c_{j,k}(n)$. Therefore, the complexity to perform (22) is significantly reduced. However, in the case of unknown code statistics, the expected value in (20) should be estimated from its sample average. Thus, the inversion of the corresponding matrix has to be either performed or updated, based on the matrix inversion lemma to estimate $(\boldsymbol{S}_1^H\boldsymbol{S}_1)^{-1}\boldsymbol{S}_1^H$ for different number of available data vectors. Adaptive implementation is very straightforward (e.g., [23]) and is thus omitted.

We will proceed to discuss two other cases mentioned before in a much similar way. Similar arguments will be applied to two more deterministic matrices $\boldsymbol{S}_2$ and $\boldsymbol{S}_3$, which will appear in the next two subsections.

[1]Here, subscript $(\cdot)_1$ means the first (current) case. Subscripts $(\cdot)_2$ and $(\cdot)_3$ will be similarly used in the corresponding equations for other two cases in the subsequent subsections.

B. Uplink with J Users and AWGN

Similar steps can be taken on (9) and (10) to obtain $\boldsymbol{d}$. First, (9) can be expressed explicitly by unknowns based on the property of "tr"

$$\eta = E\{vec^H[\boldsymbol{H}(n)]\}\,\boldsymbol{d} + \sigma_v^2 L \tag{23}$$

where

$$\boldsymbol{H}(n) \triangleq [\boldsymbol{H}_1(n), \ldots, \boldsymbol{H}_J(n)], \boldsymbol{H}_j(n) \triangleq \boldsymbol{C}_j^H(n)\boldsymbol{C}_j(n). \tag{24}$$

After the vec operation on both sides of (10) and stacking J equations, we have

$$\boldsymbol{u} = E\{\boldsymbol{Q}^H(n)\boldsymbol{Q}(n)\}\,\boldsymbol{d} + \sigma_v^2 E\{vec[\boldsymbol{H}(n)]\}. \tag{25}$$

Eliminating nuisance parameter σ_v^2 from (23) and (25), we obtain

$$\boldsymbol{S}_2\,\boldsymbol{d} = \boldsymbol{z}_2 \tag{26}$$

where

$$\boldsymbol{S}_2 \triangleq LE\{\boldsymbol{Q}^H(n)\boldsymbol{Q}(n)\} - E\{vec[\boldsymbol{H}(n)]\}E\{vec^H[\boldsymbol{H}(n)]\}$$
$$\boldsymbol{z}_2 \triangleq L\boldsymbol{u} - \eta E\{vec[\boldsymbol{H}(n)]\}. \tag{27}$$

Similarly, $E\{\boldsymbol{Q}^H(n)\boldsymbol{Q}(n)\}$ and $E\{vec[\boldsymbol{H}(n)]\}$ can be precomputed. Assume that $\hat{\boldsymbol{z}}_2$ is an estimate for $\boldsymbol{z}_2$

$$\hat{\boldsymbol{z}}_2 = \frac{1}{N}\sum_{n=1}^{N} \boldsymbol{z}_2(n),$$
$$\boldsymbol{z}_2(n) = L\boldsymbol{Q}^H(n)vec[\boldsymbol{y}(n)\boldsymbol{y}^H(n)] - \boldsymbol{y}^H(n)\boldsymbol{y}(n)E\{vec[\boldsymbol{H}(n)]\} \tag{28}$$

where $\boldsymbol{Q}^H(n)vec[\boldsymbol{y}(n)\boldsymbol{y}^H(n)]$ is the instantaneous estimate for $\boldsymbol{u}$, as discussed before. According to (26), we minimize $\|\boldsymbol{S}_2\,\boldsymbol{d} - \hat{\boldsymbol{z}}_2\|^2$ and obtain

$$\hat{\boldsymbol{d}} = (\boldsymbol{S}_2^H\boldsymbol{S}_2)^{-1}\boldsymbol{S}_2^H\hat{\boldsymbol{z}}_2. \tag{29}$$

C. Downlink with J Users and AWGN

Let us define $\boldsymbol{x} \triangleq vec(\boldsymbol{G})$. In the downlink, it is reasonable to focus only on the kth mobile user. Rewriting (11) and taking the vec on (12), we get

$$\eta = \sum_{j=1}^{J} E\{vec^H[\boldsymbol{H}_j(n)]\}\boldsymbol{x} + \sigma_v^2 L \tag{30}$$

$$\boldsymbol{r}_k = \sum_{j=1}^{J} E\{\boldsymbol{Q}_k^H(n)\boldsymbol{Q}_j(n)\}\boldsymbol{x} + \sigma_v^2 E\{vec[\boldsymbol{H}_k(n)]\}. \tag{31}$$

By eliminating σ_v^2 from these two equations, we obtain

$$\boldsymbol{S}_3\boldsymbol{x} = \boldsymbol{z}_3 \tag{32}$$

where

$$\boldsymbol{S}_3 \triangleq L\sum_{j=1}^{J} E\{\boldsymbol{Q}_k^H(n)\boldsymbol{Q}_j(n)\} - E\{vec[\boldsymbol{H}_k(n)]\}\sum_{j=1}^{J} E\{vec^H[\boldsymbol{H}_j(n)]\} \tag{33}$$

$$z_3 \triangleq L\boldsymbol{r}_k - \eta E\{vec[\boldsymbol{H}_k(n)]\}. \tag{34}$$

Code-related quantities are precomputed from given statistics of the codes. $\boldsymbol{z}_3$ is estimated from data as

$$\hat{\boldsymbol{z}}_3 = \frac{1}{N}\sum_{n=1}^{N} \boldsymbol{z}_3(n)$$
$$\boldsymbol{z}_3(n) = L\boldsymbol{Q}_k^H(n)vec[\boldsymbol{y}(n)\boldsymbol{y}^H(n)] - \boldsymbol{y}^H(n)\boldsymbol{y}(n)E\{vec[\boldsymbol{H}_k(n)]\}. \tag{35}$$

By minimizing $||\boldsymbol{S}_3\boldsymbol{x} - \hat{\boldsymbol{z}}_3||^2$, $\boldsymbol{x}$ can be estimated as

$$\hat{\boldsymbol{x}} = (\boldsymbol{S}_3^H\boldsymbol{S}_3)^{-1}\boldsymbol{S}_3^H\hat{\boldsymbol{z}}_3. \tag{36}$$

D. Brief Summary

In all these three cases, the low computational complexity of the proposed method is achieved by precomputing some deterministic matrices involved in the equations. In the first two cases, the complexity is about $O(J^2(q+1)^4)$, and in the third case, it is only about $O((q+1)^4)$. However, if those code-related matrices $\boldsymbol{S}_1$, $\boldsymbol{S}_2$, and $\boldsymbol{S}_3$ are estimated from given codes by sample averages (e.g., [20]), the involved matrix inversion will require extra complexity about $O(J^3(q+1)^6)$ and $O((q+1)^6)$, respectively. Therefore, the proposed precomputation approach requires much less computation. In order to achieve low complexity, the next section will be devoted to the evaluation of such quantities from the *priori* knowledge about the statistics of the spreading codes.

IV. Evaluation of Deterministic Quantities

Our estimator exists only if $\boldsymbol{S}_1$ (or $\boldsymbol{S}_2$ or $\boldsymbol{S}_3$) has full column rank. These are deterministic matrices. Under our assumptions in Section II, it can be observed that they are only determined by system parameters, irrespective of channels. Before we present the results, let us define some matrices for notational convenience in the following. $\boldsymbol{X}$ is a $(P+q)\times(P+q)$ Jordan matrix whose first subdiagonal entries below the main diagonal are unity while all remaining entries are zeros:

$$\boldsymbol{X} \triangleq \begin{bmatrix} 0 & & & \\ 1 & \ddots & & \\ & \ddots & \ddots & \\ 0 & & 1 & 0 \end{bmatrix}.$$

Matrix $\boldsymbol{M}$ has dimension $(P+q)\times(q+1)$ with an identity matrix as a subblock

$$\boldsymbol{M} \triangleq [\boldsymbol{I}_{q+1}\boldsymbol{0}]^T.$$

Transposes of $\boldsymbol{X}$, $\boldsymbol{T}$ [which are defined in (5)], and $\boldsymbol{M}$ are denoted by

$$\tilde{\boldsymbol{X}} \triangleq \boldsymbol{X}^T, \quad \tilde{\boldsymbol{T}} \triangleq \boldsymbol{T}^T, \quad \tilde{\boldsymbol{M}} \triangleq \boldsymbol{M}^T.$$

The 0th power of $\boldsymbol{X}$ and $\tilde{\boldsymbol{X}}$ are defined as $\boldsymbol{X}^0 \triangleq \tilde{\boldsymbol{X}}^0 \triangleq \boldsymbol{I}$.

Lemma 1: Under our assumptions in Section II, $\boldsymbol{S}_1$, $\boldsymbol{S}_2$, and $\boldsymbol{S}_3$ have the following expressions:

$$\boldsymbol{S}_1 = \boldsymbol{I}_J \otimes \left(\boldsymbol{B} - \sigma_c^4 \sum_{l_1,l_2=0}^{P-1} \boldsymbol{U}_{l_1,l_2,l_1,l_2}\right) \tag{37}$$

$$\boldsymbol{S}_2 = \begin{bmatrix} \boldsymbol{B}_1 & \boldsymbol{B}_2 & \cdots & \boldsymbol{B}_2 \\ \boldsymbol{B}_2 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \boldsymbol{B}_2 \\ \boldsymbol{B}_2 & \cdots & \boldsymbol{B}_2 & \boldsymbol{B}_1 \end{bmatrix} \tag{38}$$

$$\boldsymbol{S}_3 = L\boldsymbol{B} + \sigma_c^4(J-1)L\sum_{l_1,l_2=0}^{P-1}\boldsymbol{U}_{l_1,l_2,l_1,l_2} - \sigma_c^4 JL^2 vec(\boldsymbol{I}_{q+1})vec^T(\boldsymbol{I}_{q+1}) \tag{39}$$

where $\boldsymbol{B}_1$ and $\boldsymbol{B}_2$ are constant matrices

$$\boldsymbol{B}_1 \triangleq L\boldsymbol{B} - \sigma_c^4 L^2 vec(\boldsymbol{I}_{q+1})vec^T(\boldsymbol{I}_{q+1}) \tag{40}$$

$$\boldsymbol{B}_2 \triangleq \sigma_c^4 L \sum_{l_1,l_2=0}^{P-1}\boldsymbol{U}_{l_1,l_2,l_1,l_2} - \sigma_c^4 L^2 vec(\boldsymbol{I}_{q+1})vec^T(\boldsymbol{I}_{q+1}) \tag{41}$$

and

$$\boldsymbol{U}_{l_1,l_2,l_3,l_4} \triangleq \left(\tilde{\boldsymbol{M}}\tilde{\boldsymbol{X}}^{l_1}\tilde{\boldsymbol{T}}\boldsymbol{T}\boldsymbol{X}^{l_2}\boldsymbol{M}\right) \otimes \left(\tilde{\boldsymbol{M}}\tilde{\boldsymbol{X}}^{l_3}\tilde{\boldsymbol{T}}\boldsymbol{T}\boldsymbol{X}^{l_4}\boldsymbol{M}\right). \tag{42}$$

$\boldsymbol{B}$ depends on whether the spreading codes are real or complex. With real spreading codes, $\boldsymbol{B}$ has the following expression:

$$\boldsymbol{B} = \sigma_c^4 \sum_{l_1,l_2=0}^{P-1}(\boldsymbol{U}_{l_1,l_1,l_2,l_2} + \boldsymbol{U}_{l_1,l_2,l_1,l_2} + \boldsymbol{U}_{l_1,l_2,l_2,l_1}) + (m_{4c} - 3\sigma_c^4)\sum_{l=0}^{P-1}\boldsymbol{U}_{l,l,l,l}. \tag{43}$$

With complex spreading codes, $\boldsymbol{B}$ becomes

$$\boldsymbol{B} = \sigma_c^4 \sum_{l_1,l_2=0}^{P-1}(\boldsymbol{U}_{l_1,l_1,l_2,l_2} + \boldsymbol{U}_{l_1,l_2,l_1,l_2}) + (m_{4c} - 2\sigma_c^4)\sum_{l=0}^{P-1}\boldsymbol{U}_{l,l,l,l}. \tag{44}$$

Proof: See Appendix A. □

In order to achieve low complexity, in addition to matrices $\boldsymbol{S}_1$, $\boldsymbol{S}_2$, and $\boldsymbol{S}_3$, other matrices also need to be precomputed in $\boldsymbol{z}_1(n)$ [in (21)], $\boldsymbol{z}_2(n)$ [in (28)], and $\boldsymbol{z}_3(n)$ [in (35)], respectively. These matrices are $E\{\boldsymbol{Q}(n)\}$, $E\{\boldsymbol{H}(n)\}$ and $E\{\boldsymbol{H}_k(n)\}$. From (A.1), (A.4), and (A.5) in the Appendix, we can directly obtain them. For clarity, we present these results in the following lemma.

Lemma 2: Under our assumptions in Section II, $E\{\boldsymbol{Q}(n)\}$, $E\{\boldsymbol{H}(n)\}$ and $E\{\boldsymbol{H}_k(n)\}$ have the following expressions:

$$E\{\boldsymbol{Q}_k(n)\} = \sigma_c^2 \sum_{l=0}^{P-1} (\boldsymbol{T}\boldsymbol{X}^l\boldsymbol{M}) \otimes (\boldsymbol{T}\boldsymbol{X}^l\boldsymbol{M})$$

$$E\{\boldsymbol{Q}(n)\} = [E\{\boldsymbol{Q}_k(n)\}, \ldots, E\{\boldsymbol{Q}_k(n)\}] \tag{45}$$

$$E\{\boldsymbol{H}_k(n)\} = \sigma_c^2 L\boldsymbol{I}_{q+1}$$

$$E\{\boldsymbol{H}(n)\} = \sigma_c^2 L[\boldsymbol{I}_{q+1}, \ldots, \boldsymbol{I}_{q+1}]. \tag{46}$$

According to these lemmas, the rank of $\boldsymbol{S}_1, \boldsymbol{S}_2$, or $\boldsymbol{S}_3$ depends on the system parameters such as σ_c^2, m_{4c}, P, q, J, and the delay relevant parameter μ. However, they are independent of the channel coefficients. For a large range of given parameters, it is observed from our numerical test that these matrices have full rank. The rank of $\boldsymbol{S}_1$, $\boldsymbol{S}_2$, or $\boldsymbol{S}_3$ can always be *a priori* checked. However, it is intractable for a general proof [20] since several parameters are involved. In addition to the rank condition of $\boldsymbol{S}_i$ ($i = 1, 2, 3$), our solutions also depend on $\hat{\boldsymbol{z}}_i$. Since it can be easily found that $\hat{\boldsymbol{z}}_i$ is a consistent estimate of $\boldsymbol{z}_i$, as $N \to \infty$, we have

$$\lim_{N\to\infty} \hat{\boldsymbol{z}}_i = \boldsymbol{z}_i. \tag{47}$$

Therefore, we can obtain

$$\lim_{N\to\infty} \hat{\boldsymbol{d}} = \boldsymbol{d}, \quad \lim_{N\to\infty} \hat{\boldsymbol{x}} = \boldsymbol{x}. \tag{48}$$

This shows that our estimators asymptotically converge to their true values respectively.

V. Simulations

We simulate a CDMA system with random spreading codes and multipath effect. The mean square error (MSE) of the channel estimate is adopted as the performance measure that is $E\{\|\hat{\boldsymbol{g}} - (\boldsymbol{g}/\|\boldsymbol{g}\|)\|^2\}$, where $\hat{\boldsymbol{g}}$ is an estimate for the normalized channel vector $\boldsymbol{g}/\|\boldsymbol{g}\|$. For the current proposed blind methods, $\boldsymbol{g}$ can only be estimated as $e^{j\phi}\boldsymbol{g}$. The phase ambiguity ϕ cannot be avoided. It is assumed to have been resolved for the adopted measure to be meaningful.[2] Three typical cases discussed in the text are tested

1) uplink with unknown interference;
2) uplink with AWGN;
3) downlink with AWGN.

In each case, the result from the proposed method with precomputation is compared with that of the method without precomputation (instead, it is compared by on-line estimation of related quantities). Each user transmits binary bits $\{\pm 1\}$ with equal power. The spreading factor is set to be 16. The spreading codes for all users are randomly generated and take values $\{\pm 1 \pm j\}$ with equal probability. Therefore, $m_{4c} = 4$, and $\sigma_c^2 = 2$. Channel vectors for all users are complex and have four coefficients ($q = 3$). Both the real and imaginary parts of each coefficient are randomly selected from an interval $(-1, +1)$.

[2]For example, one may constrain the first coefficient in $\boldsymbol{g}$ to be real and positive.

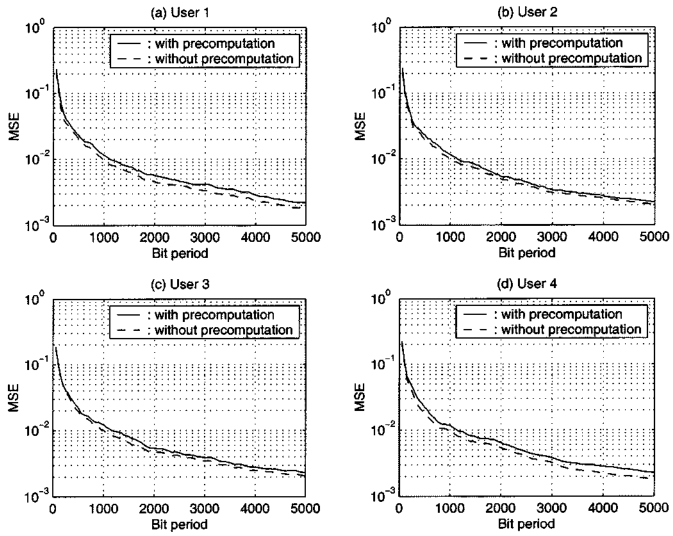

Fig. 2. Uplink channel estimation errors for four out of five known users with ten additional unknown interfers and AWGN.

Then, the vector is normalized to have unit norm. A 15-dB white Gaussian noise is added to the system. In total, 50 independent realizations are performed to obtain the average results.

In the first case, there are totally 15 active synchronous users (delays are assumed to be equal) and 15 dB AWGN. Users' channel coefficients are independently generated. Among all users, only five users are known by the receiver. Another ten users act as unknown interfers. With these parameters, the matrices $\boldsymbol{S}_1$ and $E\{\boldsymbol{Q}(n)\}$ can be precomputed based on the lemmas, resulting in low complexity of the method. They can also be estimated on-line (without precomputation) from given spreading codes known to the receiver. The effect of data length on the channel MSE is plotted in Fig. 2(a)–(d) for four out of five users. Solid lines represent the results with precomputation, whereas dashed lines represent without precomputation. It is observed that both methods provide satisfactory channel estimate, as indicated by the small errors in the figure after 5000 bit periods (about 2×10^{-3}). However, a little difference exists between them. When $\boldsymbol{S}_1$ and $E\{\boldsymbol{Q}(n)\}$ are estimated from the spreading codes, the channel estimation error is smaller at the expense of much higher computational complexity. The difference may vary from user to user, but in a very small amount, because of power control. The superiority of the method without precomputation is due to the fact that with estimated matrices, the method captures the variation of the codes, whereas with precomputed matrices, it only employs statistical property of the codes.

We also test two other cases. In the second case, there are only five active users and 15 dB AWGN. The receiver has exact knowledge about all users' spreading codes. $\boldsymbol{S}_2$ and $E\{\boldsymbol{H}(n)\}$ are precomputed. The result is presented in Fig. 3. Similar conclusion can be made. However, the convergence level of MSE is much lower than that in Fig. 2 (about 4×10^{-4}) since the receiver has the code information of all users in the system, and the problem is not overmodeled. In the third case, all five users experience a common multipath channel with 15 dB AWGN. The MSE is shown in Fig. 4. The MSE is comparable with those in

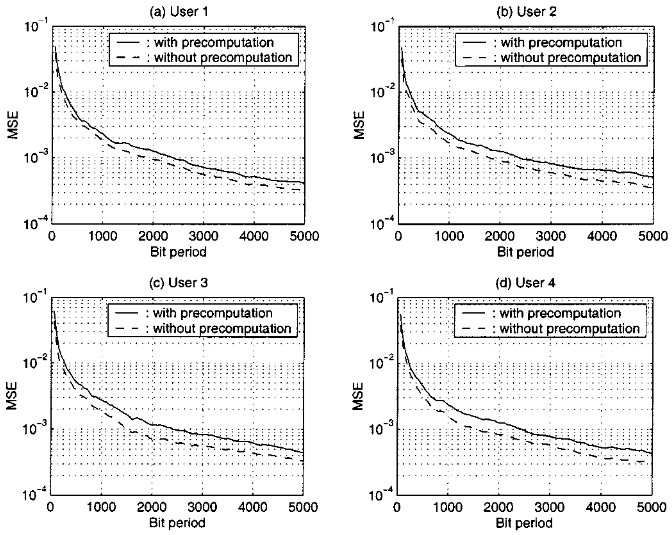

Fig. 3. Uplink channel estimation errors for four out of five known users only with AWGN.

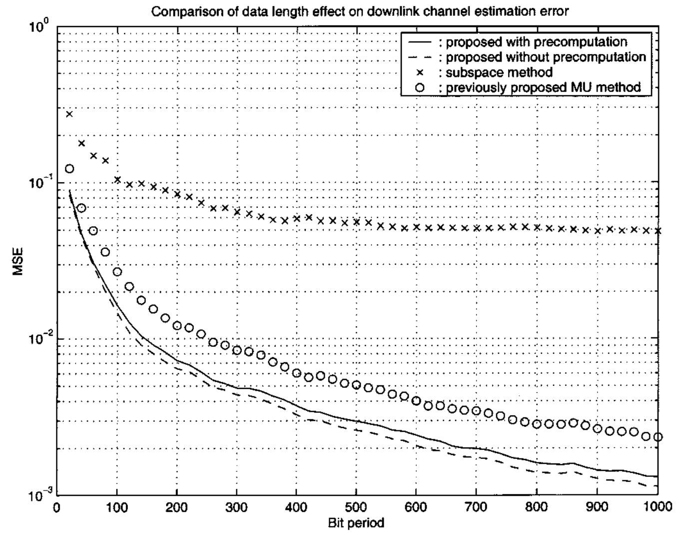

Fig. 5. Effect of data length on downlink channel estimation errors for different methods.

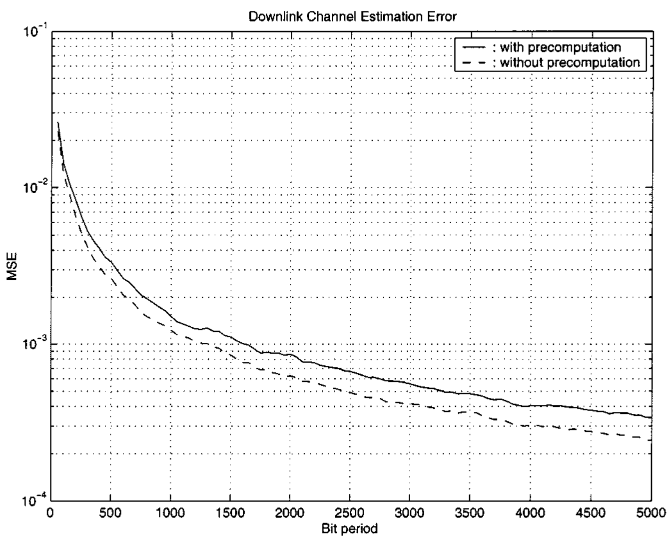

Fig. 4. Downlink channel estimation errors with AWGN.

Fig. 3 (about 3×10^{-4}) based on the accurate modeling. However, the complexity is much lower than that required to obtain Fig. 3.

To gain an insight of how well the proposed methods perform, we compare them with the previously proposed multiuser (MU) method [20] and the subspace method [18] for a downlink CDMA system. We experimented with different effects such as data length, system load (in terms of the number of users), and background noise. The channel vector is chosen to be $[1\ 0.25+0.125j\ 0.1111+0.0370j]^T$, which is same as in [18]. First, the data length effect on the channel MSE is studied for a ten-user CDMA system. The spreading factor is still set to be 16. In order to apply [18], ten users are divided into two groups, with the first group of four users based on the given parameters. The channel estimation errors are shown in Fig. 5. The solid line and dashed line are for the proposed methods with precomputation and without precomputation, respectively. The circles are for the MU method [20], and the crosses are for the subspace method [18]. These notations will be consistent in all the following figures. Among all methods, the proposed method with precomputation requires significantly low complexity. It can be seen that the proposed methods are better than other two in terms of MSE. With on-line estimation of deterministic quantities, the code variation can be captured, and thus, the method without precomputation gives smaller errors in channel estimation. The MU method assumes that different users have individual channels. Therefore, ten virtual channel vectors are estimated at the same time. However, the channel is common in the downlink. This overparameterization induces some performance loss (about 10^{-3}). In such a system, the subspace method converges to a much higher error level after 1000 bit periods (5×10^{-2}). The reason is explained in [20] since the severe interference exists with many users in the system.

Motivated by the previous figure, the effect of the system load is further investigated. We plot the MSE after 1000 bit periods with a large range of number of users in the system in Fig. 6. Group 1 always has single user in testing the subspace method. If we focus on this method, it is found that the MSE changes dramatically with the number of users. It increases from 0.3×10^{-4} to 0.8×10^{-1} when J increases from 1 to 16. This shows that the method is sensitive to the actual load of the system. The currently and previously proposed methods are less sensitive to the load based on this figure. When there are 16 users, the MSEs of the proposed methods can still achieve a level below 0.5×10^{-2}. With many users, the discrepancy of the proposed method with precomputation from the proposed method without precomputation is negligible, as indicated by closeness of the solid line and the dashed line. However, the MU method deviates from currently proposed methods with more number of users. Taking into account the computational cost and the performance together, the proposed low complexity method is more desired.

The background noise also affects the channel estimation errors of different estimators. All parameters in the experiment are set as those to obtain Fig. 5, except that we obtain the results for different noise levels (SNR) after 1000 bit periods. The results are presented in Fig. 7. For a large range of SNRs

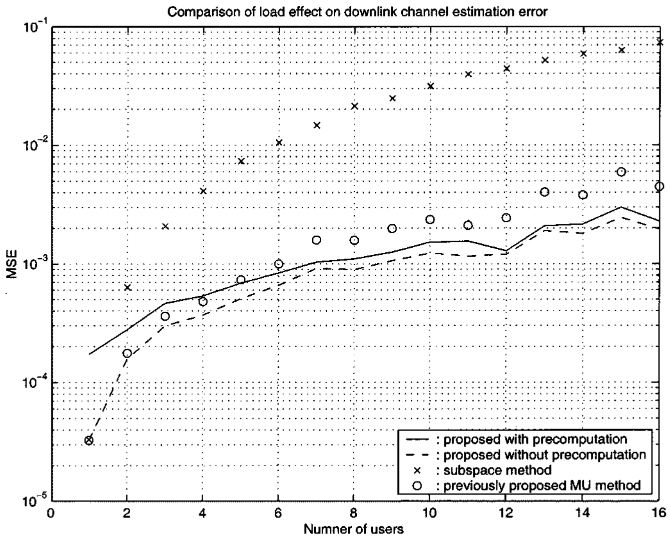

Fig. 6. Effect of system load on downlink channel estimation errors for different methods.

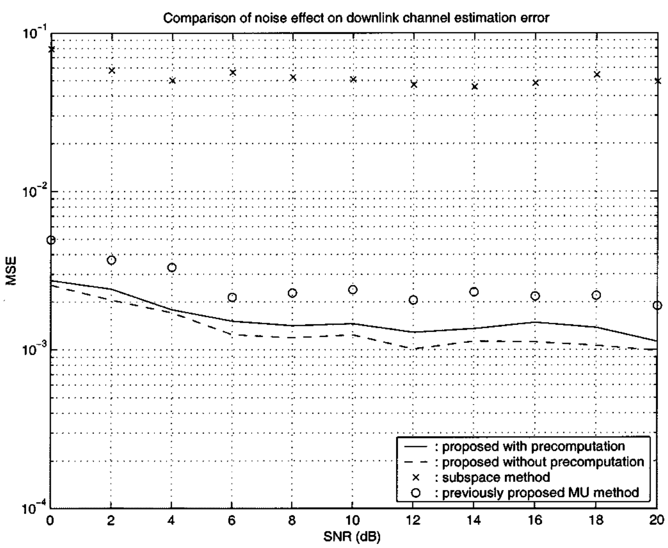

Fig. 7. Effect of background noise on downlink channel estimation errors for different methods.

(from 0 dB to 20 dB), the MSE based on the subspace method is again observed to be much higher. The proposed methods with precomputation and without precomputation show similar performance. Since the MSEs of all these methods only change slightly with SNRs according to this figure, we can conclude that background noise is not a dominant factor in determining the performance of all these methods for this simulation scenario.

We also test the effect of channel order mismatch on the performance of the proposed low complexity channel estimator. CDMA downlink with five users is simulated. The true channel order is set to be 3. Channel coefficients are similarly generated as before. Four different cases are tested when channel order is assumed to be 2 to 5 (underestimate, exactly estimate, overestimate by 1, and overestimate by 2). Correspondingly, the average MSEs from 50 realizations are plotted in Fig. 8 by "x," solid

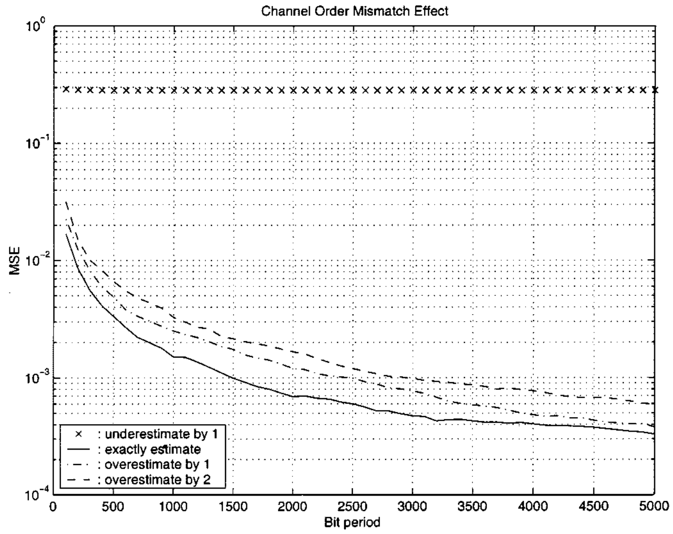

Fig. 8. Effect of channel order mismatch on downlink channel estimation error.

line, dashed-dotted line, and dashed line, respectively. It can be seen that when the order is overestimated, the performance loss is negligible. However, if the order is underestimated, the estimator yields unsatisfactory results. This suggests that overestimated channel order is preferred when exact channel order is not available.

VI. CONCLUSIONS

In this paper, low complexity in multiuser channel estimation is achieved by precomputing some deterministic code-related quantities. The multipath parameters are then estimated based on the correlation matching idea. Compared with the previously proposed method based on on-line estimation of those quantities, the currently proposed channel estimators show negligible performance loss but significantly reduce the computational cost. Their performance is also comparable with the subspace method when the system is lightly loaded. However, fewer computations are required. Moreover, the proposed methods outperform the subspace method for a heavy load system.

APPENDIX
PROOF OF LEMMA 1

By examining definitions of $\boldsymbol{S}_1$, $\boldsymbol{S}_2$, and $\boldsymbol{S}_3$, typically, $E\{\boldsymbol{Q}_k^H(n)\boldsymbol{Q}_k(n)\}$, $E\{\boldsymbol{Q}_k(n)\}$, and $E\{\boldsymbol{H}_k(n)\}$ are required to evaluate these matrices. For notational convenience, we denote

$$E\{\boldsymbol{Q}_k(n)\} \triangleq \boldsymbol{A}, \quad E\{\boldsymbol{Q}_k^H(n)\boldsymbol{Q}_k(n)\} \triangleq \boldsymbol{B}$$
$$E\{\boldsymbol{H}_k(n)\} \triangleq \boldsymbol{D} \tag{A.1}$$

where $\boldsymbol{Q}_k(n)$ and $\boldsymbol{H}_k(n)$ are determined by code matrix $\boldsymbol{C}_k(n)$ in (5), which is related to $\tilde{\boldsymbol{C}}_k(n)$ in (4). Based on its Toeplitz structure, $\tilde{\boldsymbol{C}}_k(n)$ can be expressed by a series of shift operations as

$$\tilde{\boldsymbol{C}}_k = \sum_{l=0}^{P-1} c_{k,n}(l)\boldsymbol{X}^l\boldsymbol{M}. \tag{A.2}$$

From (5) and (15), we obtain

$$\boldsymbol{Q}_k(n) = \sum_{l_1,l_2=0}^{P-1} c_{k,n}^*(l_1)c_{k,n}(l_2)(\boldsymbol{TX}^{l_1}\boldsymbol{M}) \otimes (\boldsymbol{TX}^{l_2}\boldsymbol{M}). \tag{A.3}$$

According to our assumptions, $E\{c_{k,n}^*(l_1)c_{k,n}(l_2)\} = \delta(l_1 - l_2)$, where $\delta(\cdot)$ is the Kronecker delta function. Then

$$\boldsymbol{A} = \sigma_c^2 \sum_{l=0}^{P-1} (\boldsymbol{TX}^l\boldsymbol{M}) \otimes (\boldsymbol{TX}^l\boldsymbol{M}). \tag{A.4}$$

Matrix $\boldsymbol{D}$ can be similarly evaluated. However, there is a simpler way to achieve it. If we express $\boldsymbol{C}_1(n)$ explicitly by its $q+1$ columns $\boldsymbol{c}_1, \ldots, \boldsymbol{c}_{q+1}$

$$\boldsymbol{C}_1(n) = [\boldsymbol{c}_1, \ldots, \boldsymbol{c}_{q+1}]$$

where $\boldsymbol{c}_i$ is its ith column vector of length L. Then, based on our assumptions on the spreading codes, it can be verified that $E\{\boldsymbol{c}_i^H\boldsymbol{c}_j\} = \sigma_c^2 L\delta(i-j)$. Therefore

$$\boldsymbol{D} = \sigma_c^2 L\boldsymbol{I}_{q+1}. \tag{A.5}$$

To derive an expression for $\boldsymbol{B}$, we use (A.3) and obtain

$$\boldsymbol{B} = \sum_{l_1,l_2,l_3,l_4=0}^{P-1} E\{c_{k,n}(l_1)c_{k,n}^*(l_3)c_{k,n}^*(l_2)c_{k,n}(l_4)\} \cdot \boldsymbol{U}_{l_1,l_2,l_3,l_4}. \tag{A.6}$$

Next, we evaluate $E\{c_{k,n}(l_1)c_{k,n}^*(l_3)c_{k,n}^*(l_2)c_{k,n}(l_4)\}$ based on the statistics of the spreading codes. Two cases need to be differentiated: 1) Spreading codes are real, and 2) spreading codes are complex.

Case 1—Real Spreading Codes: If codes are real, then

$$\begin{aligned} &E\{c_{k,n}(l_1)c_{k,n}^*(l_3)c_{k,n}^*(l_2)c_{k,n}(l_4)\} \\ &\quad = E\{c_{k,n}(l_1)c_{k,n}(l_3)c_{k,n}(l_2)c_{k,n}(l_4)\}. \end{aligned}$$

This term survives if $\{l_1 = l_2,\, l_3 = l_4\}$, $\{l_1 = l_3,\, l_2 = l_4\}$, or $\{l_1 = l_4,\, l_2 = l_3\}$. Since

$$\begin{aligned} &E\{c_{k,n}(l_1)c_{k,n}(l_1)c_{k,n}(l_2)c_{k,n}(l_2)\} \\ &\quad = \begin{cases} m_{4c}, & \text{when } l_1 = l_2 \\ \sigma_c^4, & \text{when } l_1 \neq l_2. \end{cases} \end{aligned}$$

Hence, from (A.6), (43) follows.

Case 2—Complex Spreading Codes: Similarly, when spreading codes are complex, $E\{c_{k,n}(l_1)c_{k,n}^*(l_3) c_{k,n}^*(l_2)c_{k,n}(l_4)\}$ survives if $\{l_1 = l_2, l_3 = l_4\}$, or $\{l_1 = l_3,\, l_2 = l_4\}$. Therefore, (44) is proven.

Now, with these results for $\boldsymbol{A}$, $\boldsymbol{B}$, and $\boldsymbol{D}$, we are able to evaluate $\boldsymbol{S}_1$, $\boldsymbol{S}_2$, and $\boldsymbol{S}_3$. First, $\boldsymbol{S}_1$ is defined in (20). If we expand it and use the independent assumption on spreading codes for different users, then it can be found that $\boldsymbol{S}_1$ becomes a block diagonal matrix with the same subblock $\boldsymbol{B} - \boldsymbol{A}^H\boldsymbol{A}$ in the diagonal position. By using the following property of the Kronecker product [10]

$$(\boldsymbol{X}_1 \otimes \boldsymbol{X}_2)^T = \boldsymbol{X}_1^T \otimes \boldsymbol{X}_2^T$$

and using (A.4), we can express $\boldsymbol{A}^H\boldsymbol{A}$ in a compact form

$$\boldsymbol{A}^H\boldsymbol{A} = \sigma_c^4 \sum_{l_1,l_2=0}^{P-1} \boldsymbol{U}_{l_1,l_2,l_1,l_2}. \tag{A.7}$$

Therefore, $\boldsymbol{S}_1$ in (37) is proven.

$\boldsymbol{S}_2$ is given by (27). It is a block Toeplitz matrix. Its diagonal subblocks can be easily found to be equal to $\boldsymbol{B}_1$ in (40). The off-diagonal subblocks are equal to $L\boldsymbol{A}^H\boldsymbol{A} - \sigma_c^4 L^2\, vec(\boldsymbol{I}_{q+1})vec^T(\boldsymbol{I}_{q+1})$, which is $\boldsymbol{B}_2$ in (41) after (A.7) is applied.

$\boldsymbol{S}_3$ is given by (33). Under definitions in (A.1), this matrix is easily observed to satisfy

$$\boldsymbol{S}_3 = L\boldsymbol{B} + L(J-1)\boldsymbol{A}^H\boldsymbol{A} - J vec(\boldsymbol{D})vec^H(\boldsymbol{D}).$$

From (A.4), (A.5), and (A.7), (39) is much straightforward to obtain. □

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