

# Code-Constrained Blind Detection of CDMA Signals in Multipath Channels

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**Abstract**—In this letter, a constant-modulus-algorithm-based multiuser detection scheme is proposed for a communication system under multipath propagation. To mitigate channel distortion and multiuser interference, we integrate multiple constraints into the optimization criterion. According to our analysis, the ability of the detector to remove all interference is ensured in the absence of noise when the constraints are properly preselected. However, in the presence of noise, the constraints highly affect the performance of the receiver. In order to optimally combine signals from different paths to achieve performance gains, those constraints can also be treated as variables and jointly optimized with the receiver, as verified by numerical examples.

**Index Terms**—Constant modulus algorithm, constrained optimization, multiuser detection.

## I. INTRODUCTION

THE CONSTANT modulus algorithm (CMA) is a very effective blind approach to combating intersymbol interference (ISI) when a communication channel is frequency-selective [2], [3]. Recently, the method has been applied to detect code-division-multiple-access (CDMA) signals based on a constrained optimization technique. With given spreading codes of the desired user, the CMA-based detector is forced to satisfy one or a set of linear constraints such that signals from the desired user are detected in a flat fading channel [6], [10] or multipath environment [4], [7]. Although those CMA-based approaches have shown better detection performance than some second-order-based methods, they exhibit certain disadvantages. It is clear that [6] and [10] suffer from signature mismatch. Among those capable of multipath mitigation, [4] exhibits local minima and inability to optimally combine signal components from different paths. The approach in [7] is a batch iterative algorithm. Its global convergence has not been established analytically.

In this letter, we adopt the CMA criterion but propose multiple constraints to detect the desired signal in a multipath propagation environment. Based on our analysis, the constraints can be properly preselected to guarantee global convergence of the algorithm. However, in the presence of noise, the constraints highly affect the performance of the receiver. In order to optimally combine signals from different paths to achieve performance gains, those constraints can also be treated as vari-

ables and jointly optimized with the receiver. Although, similar to the conventional CMA, our constrained CMA suffers from slower convergence than some existing linear multiuser detection methods such as in [1] and [8], the proposed receivers can be adaptively implemented with low complexity and yield satisfactory performance.

## II. DATA MODEL

Consider a direct-sequence CDMA (DS-SS) system with  $J$  users. User  $j$  is assigned a periodic spreading sequence  $c_j(k)$  ( $k = 0, \dots, P-1$ ). Let the chip sequence be transmitted through a multipath channel with unknown coefficients  $g_j(n)$ . Then the received chip-rate discrete-time signal  $y(n)$  has the form [9]

$$y(n) = \sum_{j=1}^J \sum_{l=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} w_j(l)c_j(n-m-d_j-lP)g_j(m)+v(n) \quad (1)$$

where  $w_j(n)$  is the  $n$ th information symbol from user  $j$  and assumed to take either  $+1$  or  $-1$  with equal probability;  $d_j$  is the chip delay of user  $j$ ; and  $v(n)$  is zero-mean additive white Gaussian noise (AWGN). All quantities in (1) are assumed to be real in this letter, and the maximum channel order for all users is denoted as  $q$ . Without loss of generality, user 1 is treated as the user of interest. The receiver is assumed to be synchronized to this user. After collecting  $L = \nu P$  measurements in a vector  $\mathbf{y}_n = [y(nP), \dots, y(nP + L - 1)]^T$ , the received data vector becomes [7]

$$\mathbf{y}_n = \mathbf{H}\mathbf{w}(n) + \mathbf{v}_n = \mathbf{h}_1 w_1(n + \nu_0) + \mathbf{H}_{\text{int}} \mathbf{w}_{\text{int}}(n) + \mathbf{v}_n \quad (2)$$

where  $\mathbf{H} = [\mathbf{h}_1, \mathbf{H}_{\text{int}}]$ ,  $\mathbf{h}_1 \triangleq \mathbf{C}_1 \mathbf{g}_1$  is the signature vector of the desired symbol  $w_1(n + \nu_0)$  with a time offset  $0 \leq \nu_0 \leq \nu$ ;  $\mathbf{C}_1$  is the code-filtering matrix whose first  $\nu_0 P$  rows are all zeros;  $\mathbf{g}_1$  is the channel vector

$$\mathbf{C}_1 = \begin{bmatrix} \mathbf{0} & \dots & \mathbf{0} \\ c_1(0) & & 0 \\ \vdots & \ddots & c_1(0) \\ c_1(P-1) & & \vdots \\ 0 & \ddots & c_1(P-1) \\ \mathbf{0} & \dots & \mathbf{0} \end{bmatrix} \quad \mathbf{g}_1 = \begin{bmatrix} g_1(0) \\ \vdots \\ g_1(q) \end{bmatrix};$$

$\mathbf{w}(n) = [w_1(n + \nu_0), \mathbf{w}_{\text{int}}^T(n)]^T$ ;  $\mathbf{w}_{\text{int}}(n)$  is an interference vector including ISI and multiuser interference (MUI);  $\mathbf{H}_{\text{int}}$  is

Manuscript received May 3, 2002; revised August 2, 2002. The associate editor coordinating the review of this manuscript and approving it for publication was Prof. Victor A. N. Barroso.

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Digital Object Identifier 10.1109/LSP.2002.806043

the corresponding signature matrix; and  $\mathbf{v}_n$  is the noise vector. The particular structure of  $\mathbf{h}_1$  will be exploited to derive a blind detector that is capable of combating multipath distortion and suppressing both ISI and MUI.

Throughout this letter, we make the following assumptions: **AS1**) all users' information sequences are temporally independent and identically distributed with unit power and mutually independent; **AS2**) channel noise  $\mathbf{v}_n$  is white Gaussian and independent of input signals; **AS3**) matrix  $[\mathbf{C}_1, \text{span}\{\mathbf{H}_{\text{int}}\}]$  has full column rank, where  $\text{span}\{\mathbf{H}_{\text{int}}\}$  represents any set of bases of the space spanned by all columns of  $\mathbf{H}_{\text{int}}$ . The first two assumptions are common in most multiuser detection approaches. **AS3** can be easily satisfied for the uplink communication due to distinct channel fading of each user. For the downlink communication, even though the full column rank condition on  $\mathbf{H}$  has been shown to be mostly violated [5], **AS3** requires only the linear independence between  $\mathbf{C}_1$  and the range space of the interfering signatures, which is much less restrictive than that in [8] and, thus, enhance the applicability of the approach proposed next.

### III. CONSTRAINED CMA-BASED MULTIUSER DETECTION

Our objective is to design a linear detector  $\mathbf{f}$  to blindly detect  $w_1(n+\nu_0)$ . We adopt Godard's CMA criterion but with multiple constraints arranged in a vector [9]

$$\min_{\mathbf{f}} \mathcal{J} = E \left\{ (z_n^2 - 1)^2 \right\} \text{ subject to } \mathbf{C}_1^T \mathbf{f} = \mathbf{g} \quad (3)$$

where  $z_n = \mathbf{f}^T \mathbf{y}_n$  is the detector's output;  $\mathbf{g}$  is a preselected constraint vector aiming to properly combine the desired signal components from different paths in the presence of multipath distortion. In a special case of flat fading, (3) reduces to the method described in [10]. According to (2),  $z_n$  becomes

$$z_n = \mathbf{f}^T \mathbf{C}_1 \mathbf{g}_1 w_1(n + \nu_0) + \mathbf{f}^T \mathbf{H}_{\text{int}} \mathbf{w}_{\text{int}}(n) + \mathbf{f}^T \mathbf{v}_n \quad (4)$$

where the contribution  $\mathbf{f}^T \mathbf{C}_1 \mathbf{g}_1$  from the desired symbol becomes a constant  $\mathbf{g}^T \mathbf{g}_1$  due to the constraints in (3). Its power  $(\mathbf{g}^T \mathbf{g}_1)^2$  plays a critical role in removing both ISI and MUI, as shown in the following theorem. The theorem will also suggest how to select the constraint vector  $\mathbf{g}$ .

*Theorem:* Under **AS1** and **AS3**, if  $\mathbf{g}$  is chosen to satisfy  $(\mathbf{g}^T \mathbf{g}_1)^2 > (1/3)$ , then  $\mathbf{f}$  obtained from (3) will remove ISI and MUI completely in the absence of noise. Otherwise, residual interference exists.

*Proof:* Under **AS3**,  $\mathbf{f}^T \mathbf{H}_{\text{int}}$  in (4) is linearly independent of  $\mathbf{f}^T \mathbf{C}_1 = \mathbf{g}^T$ . Therefore,  $\mathbf{f}^T \mathbf{H}_{\text{int}}$  can be replaced by a new set of optimization variables independent of constraints. On the other hand,  $\mathbf{H}_{\text{int}}$  is not necessarily of full column rank in a multipath environment [5], and  $\mathbf{f}^T \mathbf{H}_{\text{int}}$  cannot be defined in general as a set of new variables as in [10]. Suppose  $\mathbf{H}_{\text{int}}$  has  $M$  columns but has rank  $I \leq M$ . Arrange  $I$  independent columns of  $\mathbf{H}_{\text{int}}$  in a matrix  $\mathbf{\Psi}$ . Then  $\mathbf{H}_{\text{int}} = \mathbf{\Psi} \mathbf{\Gamma}$  where  $\mathbf{\Gamma} \triangleq [\boldsymbol{\gamma}_1, \dots, \boldsymbol{\gamma}_M]$  has rank  $I$ . Therefore, we can define a set of new independent variables as  $\boldsymbol{\alpha} \triangleq (\mathbf{f}^T \mathbf{\Psi})^T$ . Then  $\mathbf{f}^T \mathbf{H}_{\text{int}} = \boldsymbol{\alpha}^T \mathbf{\Gamma}$ . On the basis of the above analysis, (3) can

be transformed to the following unconstrained function with respect to  $\boldsymbol{\alpha}$  after explicitly evaluating the expectation [3]

$$\begin{aligned} \min_{\boldsymbol{\alpha}} \mathcal{J} = & -2 \left[ (\mathbf{g}^T \mathbf{g}_1)^4 + \sum_{i=1}^M (\boldsymbol{\alpha}^T \boldsymbol{\gamma}_i)^4 \right] \\ & + 3 \left[ (\mathbf{g}^T \mathbf{g}_1)^2 + \sum_{i=1}^M (\boldsymbol{\alpha}^T \boldsymbol{\gamma}_i)^2 \right]^2 \\ & - 2 \left[ (\mathbf{g}^T \mathbf{g}_1)^2 + \sum_{i=1}^M (\boldsymbol{\alpha}^T \boldsymbol{\gamma}_i)^2 \right] + 1. \end{aligned} \quad (5)$$

Next, we obtain the stationary points of (5). Taking the derivative of (5) with respect to  $\boldsymbol{\alpha}$  and forcing it to be zero yields

$$\nabla_{\boldsymbol{\alpha}} \mathcal{J} = (12\phi - 4) \sum_{i=1}^M \boldsymbol{\alpha}^T \boldsymbol{\gamma}_i \boldsymbol{\gamma}_i - 8 \sum_{i=1}^M (\boldsymbol{\alpha}^T \boldsymbol{\gamma}_i)^3 \boldsymbol{\gamma}_i = \mathbf{0} \quad (6)$$

where  $\phi = (\mathbf{g}^T \mathbf{g}_1)^2 + \sum_i (\boldsymbol{\alpha}^T \boldsymbol{\gamma}_i)^2$ . Rearranging all terms in (6), we have

$$\nabla_{\boldsymbol{\alpha}} \mathcal{J} = \left\{ \sum_{i=1}^M \boldsymbol{\gamma}_i [12\phi - 4 - 8(\boldsymbol{\alpha}^T \boldsymbol{\gamma}_i)^2] \boldsymbol{\gamma}_i^T \right\} \boldsymbol{\alpha} = \mathbf{0}$$

which is further expressed in the following matrix form

$$\nabla_{\boldsymbol{\alpha}} \mathcal{J} = \mathbf{\Gamma} \text{diag}\{12\phi - 4 - 8(\boldsymbol{\alpha}^T \boldsymbol{\gamma}_i)^2\} \mathbf{\Gamma}^T \boldsymbol{\alpha} = \mathbf{0}. \quad (7)$$

Equation (7) immediately suggests that  $\bar{\boldsymbol{\alpha}} = \mathbf{0}$  is a stationary point. However, other nonzero stationary points are difficult to obtain in a closed form. We will show that when  $(\mathbf{g}^T \mathbf{g}_1)^2 > (1/3)$ ,  $\bar{\boldsymbol{\alpha}} = \mathbf{0}$  is the unique minimum point, while any nonzero stationary point which satisfies (6) [or (7)] is impossible to be a minimum point.

First, we obtain the Hessian matrix by taking the derivative of (6) with respect to  $\boldsymbol{\alpha}$

$$\begin{aligned} \mathcal{H}(\boldsymbol{\alpha}) = & \sum_i [12\phi - 4 - 24(\boldsymbol{\alpha}^T \boldsymbol{\gamma}_i)^2] \boldsymbol{\gamma}_i \boldsymbol{\gamma}_i^T \\ & + 24 \left( \sum_i \boldsymbol{\alpha}^T \boldsymbol{\gamma}_i \boldsymbol{\gamma}_i \right) \left( \sum_j \boldsymbol{\alpha}^T \boldsymbol{\gamma}_j \boldsymbol{\gamma}_j^T \right). \end{aligned} \quad (8)$$

At the stationary point  $\bar{\boldsymbol{\alpha}} = \mathbf{0}$ , (8) reduces to  $\mathcal{H}(\mathbf{0}) = 4[3(\mathbf{g}^T \mathbf{g}_1)^2 - 1]\mathbf{\Gamma}\mathbf{\Gamma}^T$ . Recalling that  $\mathbf{\Gamma}$  has full rank,  $\mathbf{\Gamma}\mathbf{\Gamma}^T$  is positive definite. It can be easily concluded that  $\bar{\boldsymbol{\alpha}} = \mathbf{0}$  is a minimum point when  $(\mathbf{g}^T \mathbf{g}_1)^2 > (1/3)$ . However, at any nonzero stationary point  $\bar{\boldsymbol{\alpha}}$ , it is not easy to obtain the property of  $\mathcal{H}(\bar{\boldsymbol{\alpha}})$  directly from (8). Thus, we left and right multiply (8) by  $\bar{\boldsymbol{\alpha}}$  and check the resulting quantity. At  $\bar{\boldsymbol{\alpha}}$ , after replacing  $\phi$  by  $\bar{\phi} = \phi(\bar{\boldsymbol{\alpha}})$  and  $\sum_j (\bar{\boldsymbol{\alpha}}^T \boldsymbol{\gamma}_j)^2$  by  $\bar{\phi} - (\mathbf{g}^T \mathbf{g}_1)^2$ , we obtain

$$\begin{aligned} \bar{\boldsymbol{\alpha}}^T \mathcal{H}(\bar{\boldsymbol{\alpha}}) \bar{\boldsymbol{\alpha}} = & 3 \sum_i [12\bar{\phi} - 4 - 8(\bar{\boldsymbol{\alpha}}^T \boldsymbol{\gamma}_i)^2] (\bar{\boldsymbol{\alpha}}^T \boldsymbol{\gamma}_i)^2 \\ & + 8 \sum_i [1 - 3(\mathbf{g}^T \mathbf{g}_1)^2] (\bar{\boldsymbol{\alpha}}^T \boldsymbol{\gamma}_i)^2. \end{aligned} \quad (9)$$

Notice that  $\bar{\boldsymbol{\alpha}}$  satisfies (6). Left multiplying (6) by  $\bar{\boldsymbol{\alpha}}^T$  clearly zeros out the first part in (9). Therefore  $\bar{\boldsymbol{\alpha}}^T \mathcal{H}(\bar{\boldsymbol{\alpha}}) \bar{\boldsymbol{\alpha}} = 8[1 - 3(\mathbf{g}^T \mathbf{g}_1)^2] \sum_i (\bar{\boldsymbol{\alpha}}^T \boldsymbol{\gamma}_i)^2$ . Obviously, if  $(\mathbf{g}^T \mathbf{g}_1)^2 > (1/3)$  then  $\bar{\boldsymbol{\alpha}}^T \mathcal{H}(\bar{\boldsymbol{\alpha}}) \bar{\boldsymbol{\alpha}} < 0$ , which implies that  $\mathcal{H}(\bar{\boldsymbol{\alpha}})$  is nonpositive definite. Therefore, any nonzero stationary point from minimizing

(3) cannot be a minimum point. Hence, we can conclude that if  $(\mathbf{g}^T \mathbf{g}_1)^2 > (1/3)$ , then  $\bar{\boldsymbol{\alpha}} = \mathbf{0}$  is the only minimum point. Since  $\bar{\boldsymbol{\alpha}} = \mathbf{0}$  implies  $\mathbf{f}^T \mathbf{H}_{\text{int}} = \mathbf{0}^T$ , the receiver obtained from (3) will remove both ISI and MUI completely. In the case of  $(\mathbf{g}^T \mathbf{g}_1)^2 < (1/3)$ ,  $\mathcal{H}(\mathbf{0})$  becomes negative definite. Then,  $\bar{\boldsymbol{\alpha}} = \mathbf{0}$  is not a minimum point. Nonzero stationary points cause  $\mathbf{f}^T \mathbf{H}_{\text{int}} = \boldsymbol{\alpha}^T \boldsymbol{\Gamma}$  to be nonzero, since  $\boldsymbol{\Gamma}$  has full column rank. This situation results in some residual interference.  $\square$

It is worth to mention that in a special case when the channel is flat fading and  $\mathbf{H}_{\text{int}}$  has full column rank (i.e.,  $I = M$ ), all previous analyses are applicable and will reduce to those in [6], [10] by taking  $\boldsymbol{\Psi} = \mathbf{H}_{\text{int}}$  and correspondingly  $\boldsymbol{\Gamma}$  to be an identity matrix.

To easily implement the proposed approach, we decompose the receiver in a generalized sidelobe canceler structure as [9]

$$\mathbf{f} = \mathbf{C}_1 (\mathbf{C}_1^T \mathbf{C}_1)^{-1} \mathbf{g} + \boldsymbol{\Pi}_{\mathbf{c}}^\perp \mathbf{u} \quad \boldsymbol{\Pi}_{\mathbf{c}}^\perp \triangleq \mathbf{I} - \mathbf{C}_1 (\mathbf{C}_1^T \mathbf{C}_1)^{-1} \mathbf{C}_1^T. \quad (10)$$

The constrained optimization problem is then transformed into an unconstrained one. Under this decomposition, (3) becomes an unconstrained function of  $\mathbf{u}$ . The derivative of  $\mathcal{J}$  with respect to  $\mathbf{u}$  is easily found to be  $4\boldsymbol{\Pi}_{\mathbf{c}}^\perp E\{(z_n^2 - 1)z_n \mathbf{y}_n\}$ . By using the stochastic gradient descent method and instantaneous approximation to the expected value, we readily obtain the following update of  $\mathbf{u}$  when  $\mathbf{g}$  is preselected according to our theorem

$$\mathbf{u}(n+1) = \mathbf{u}(n) - \mu_u (z_n^2 - 1) z_n \boldsymbol{\Pi}_{\mathbf{c}}^\perp \mathbf{y}_n \quad (11)$$

where a factor of four in the gradient has been absorbed into the step size  $\mu_u$ . After  $\mathbf{u}$  is obtained,  $\mathbf{f}$  can be constructed according to (10).

In order to maximize the power of the desired symbol while maximally suppressing the interference, the constraint vector  $\mathbf{g}$  can be further optimized as discussed next.

#### IV. OPTIMIZATION OF THE CONSTRAINT VECTOR

In Section III, we have shown that if  $(\mathbf{g}^T \mathbf{g}_1)^2 > (1/3)$ , then interference can be completely canceled in the absence of noise. At the unique minimum point  $\bar{\boldsymbol{\alpha}} = \mathbf{0}$ , the residual CMA cost becomes  $\mathcal{J}_{\min}(\mathbf{g}) = [(\mathbf{g}^T \mathbf{g}_1)^2 - 1]^2$  according to (5). It is a function of  $\mathbf{g}^T \mathbf{g}_1$  if  $\mathbf{g}$  is treated as a variable. Meanwhile, the power of the desired symbol under our constraints is observed to be  $(\mathbf{f}^T \mathbf{h}_1)^2 = (\mathbf{g}^T \mathbf{g}_1)^2$ , which also depends on  $\mathbf{g}$ . Therefore, the preselected  $\mathbf{g}$  in our previous discussion highly affects the receiver's performance. In order to achieve the minimum CMA cost and obtain the maximum power for the desired symbol, we propose to further minimize the residual cost with respect to  $\mathbf{g}$  under a norm constraint

$$\min_{\mathbf{g}} \mathcal{J}_{\min}(\mathbf{g}) \text{ subject to } \|\mathbf{g}\| = \rho. \quad (12)$$

A similar idea has been applied in the minimum output energy (MOE)-based approach [9], which maximizes the residual output power of the receiver after constrained minimization in the first step. The norm constraint in (12) aims to force  $\mathbf{g}$  to be in the direction of  $\mathbf{g}_1$  in order for optimal combining of the signal components from different paths. To guarantee zero interference and zero cost, which require  $(\mathbf{g}^T \mathbf{g}_1)^2 > (1/3)$  and  $(\mathbf{g}^T \mathbf{g}_1)^2 = 1$  respectively,  $\rho$  should be chosen to satisfy

$(1/\sqrt{3}\|\mathbf{g}_1\|) < \rho \leq (1/\|\mathbf{g}_1\|)$ . In the case of unknown channel parameters, an empirical choice  $(1/\sqrt{3}) < \rho \leq 1$  is adoptable, since  $\|\mathbf{g}_1\| < 1$  for most wireless communication systems.

With decomposition (10), the CMA cost function becomes a function of both  $\mathbf{g}$  and  $\mathbf{u}$ . After taking the derivative with respect to  $\mathbf{g}$ , which gives  $(\mathbf{C}_1^T \mathbf{C}_1)^{-1} \mathbf{C}_1^T E\{(z_n^2 - 1)z_n \mathbf{y}_n\}$ , an update equation similar to (11) follows

$$\begin{aligned} \mathbf{g}(n+1) &= \mathbf{g}(n) - \mu_g (z_n^2 - 1) z_n (\mathbf{C}_1^T \mathbf{C}_1)^{-1} \mathbf{C}_1^T \mathbf{y}_n \\ \mathbf{g}(n+1) &\leftarrow \frac{\rho}{\|\mathbf{g}(n+1)\|} \mathbf{g}(n+1) \end{aligned} \quad (13)$$

where  $\mu_g$  is a step size. Once  $\mathbf{u}$  and  $\mathbf{g}$  are updated by (11) and (13) respectively, the receiver can be obtained by (10) at different time. The two step sizes  $\mu_u$  and  $\mu_g$  can be independently adjusted in the light of [9] and [11].

According to our previous analysis,  $(\mathbf{g}^T \mathbf{g}_1)^2 > (1/3)$  should be satisfied to guarantee the convergence of the algorithm. Under the norm constraint on  $\mathbf{g}$ , this can be achieved by good initialization of  $\mathbf{g}$  and proper selection of  $\mu_g$ . Satisfactory initialization is achievable by resorting to the MOE method [9]. Since the constraint vector in [9] has been shown to converge at a satisfactory rate to  $\mathbf{g}_1$  within a scalar ambiguity in the absence of AWGN, we may initially run the MOE algorithm and then switch to our algorithm with sufficiently small  $\mu_g$  to guarantee  $(\mathbf{g}^T \mathbf{g}_1)^2 > (1/3)$ . The turning point may be any point after convergence of the MOE algorithm. Typically, a few hundred iterations are sufficient, based on the simulation results in [9]. However, how to properly initialize  $\mathbf{g}$  without assistance of other methods in order to guarantee favorable convergence still constitutes an open topic.

#### V. SIMULATIONS

We consider a CDMA system with spreading factor  $P = 12$  and four equal power users. Each user transmits binary phase-shift-keying signals through individual multipath channels of order three. Each channel coefficient is Gaussian distributed. The first path has unit power, while each of the remaining paths has power of 0.3. Delays of interfering users are uniformly distributed from zero to  $P - 1$ . Each user's spreading sequence, channel, and delay are randomly generated in each of 100 realizations. We set  $\nu = 5$  and  $\nu_0 = 3$ . In the first experiment, we use the theoretical cost function (3) to test the effect of the constraint vector on the performance of the proposed CMA receiver when constraints are fixed (termed as CMA-FC). The total interference power over different  $(\mathbf{g}^T \mathbf{g}_1)^2$  for a noiseless system is plotted in Fig. 1. The critical value of  $(\{1\}/\{3\})$  is experimentally observed, justifying our analysis. Next, we compare our adaptive CMA receiver when constraints are updated (termed as CMA-UC), our adaptive CMA-FC receiver, the adaptive MOE receiver [9], Li and Fan's adaptive receiver (LF) [4], and the ideal minimum-mean-square-error (MMSE) receiver. The constraint vector obtained from the MOE method after 800 iterations is used directly for the CMA-FC receiver and as an initial vector to update the constraint vector required by the CMA-UC receiver. Those 800 received data vectors are also used for estimating the best delay in [4]. Fig. 2 compares bit error rates (BERs) of those receivers over various

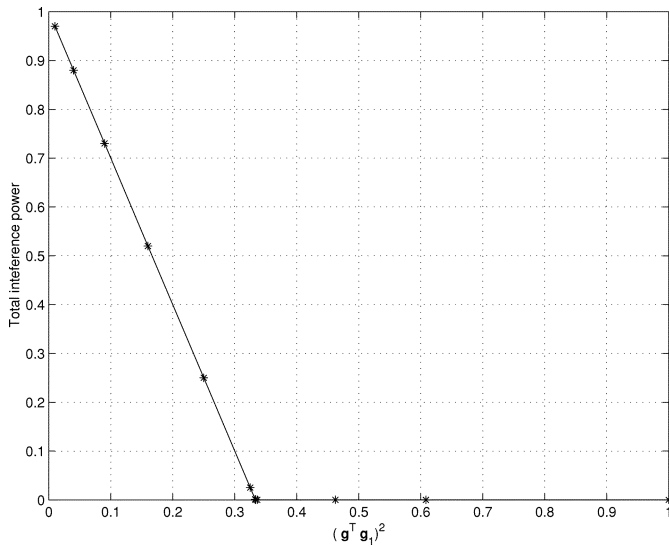


Fig. 1. Total interference power versus  $(\mathbf{g}^T \mathbf{g}_1)^2$  for the proposed CMA-FC receiver.

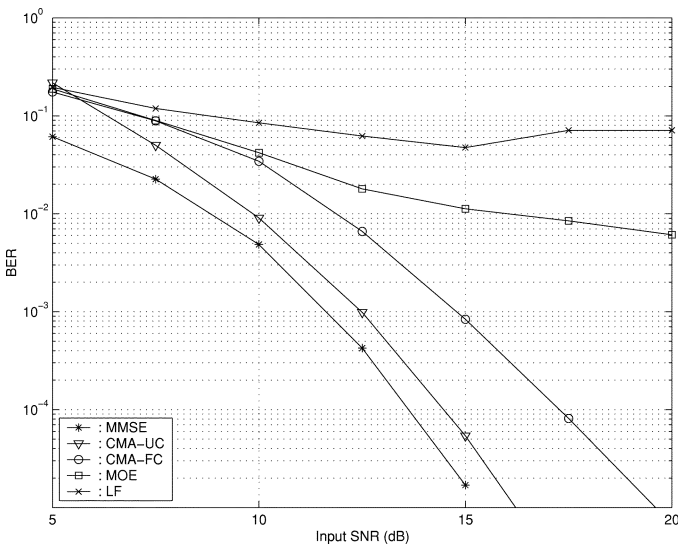


Fig. 2. BER versus input SNR for different receivers.

input SNRs. It can be observed that both proposed receivers have better performance than the MOE and LF receivers. On the other hand, the performance of the CMA-UC receiver is close to that of the ideal MMSE receiver, and better than that of the CMA-FC receiver. Theoretical justification of the superiority of the proposed constrained CMA receivers to the MOE receiver and analytical comparison with the MMSE receiver in the presence of noise will be studied in the future. In the third experiment, we compared our adaptive CMA-UC receiver with the MOE and MMSE receivers under a two-ray Rayleigh fading environment [9] but with equal power. For each user, the channel delay spread was randomly generated according to uniform distribution from one to five chip periods in each realization. Fig. 3 illustrates the BERs of different approaches versus input SNRs. It is observed that our CMA-UC receiver performs closely to the MMSE receiver and significantly outperforms the MOE receiver. However, performance of each receiver degrades due to channel fading when this figure is compared with the previous one.

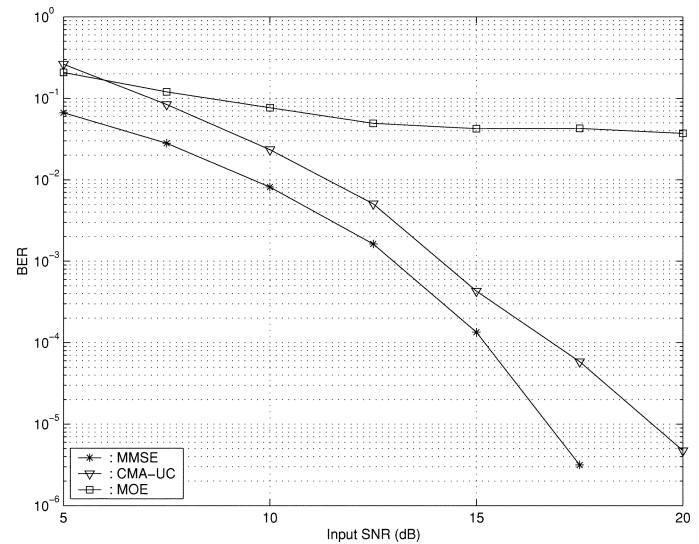


Fig. 3. BER versus input SNR under two-ray Rayleigh fading.

## VI. CONCLUSION

We have proposed a blind multiuser detector to combat multipath distortions and suppress both ISI and MUI. The method is based on the CMA criterion with multiple constraints on the detector. The choice of constraints affects the performance of the detector. It is shown that under certain conditions on the constraints, the algorithm enjoys global convergence in the absence of noise. We also seek optimal constraint parameters by jointly updating them with the detector. Initialization for the proposed algorithm can be performed by the MOE approach to guarantee favorable convergence.

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