Improved Constraint for Multipath Mitigation in Constrained MOE Multiuser Detection

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Abstract: **Constrained optimization method has been shown to be applicable for multiuser detection in the presence of multipath distortion. The performance of the constrained minimum output energy (MOE) detector is close to the minimum mean-square-error (MMSE) detector. However, due to additive noise, optimal constraints are biased estimates for channel parameters, resulting in some performance loss in multiuser detection. This loss becomes more significant when the communication channel has larger background noise (low SNRs). It is revealed in this paper that constrained optimization is closely related to subspace method which provides good estimate for the channel. Motivated by this result, the constrained cost function is modified to improve the channel estimate and thus better detection performance than the previously proposed constrained method. Detailed performance analysis and simulation results are also presented.**

Index Terms: **Multiuser detection, constrained optimization, multipath distortion.**

I. INTRODUCTION

Direct-sequence (DS) code division multiple access (CDMA) has received considerable attention as a multiple access scheme applicable to the third generation wireless networks [1], [2]. In the DS-CDMA framework, all users transmit at the same time and frequency but use distinct signature sequences to allow signal separation at the receiver. However, multiuser interference (MUI) is one of the key factors which degrade the system performance.

In order to achieve better detection performance, MUI has to be suppressed. Various efficient multiuser detection methods have been proposed [3]. The constrained optimization idea from array signal processing [4]–[8] has also been successfully applied to derive a blind minimum output energy (MOE) multiuser detector [9], by minimizing the output energy of the detector subject to a constraint. For a CDMA system with multipath propagation, multiuser detection can be similarly performed by employing a constrained MOE technique with multiple constraints [10]–[12]. Those constraints are priori-chosen such as decorrelating constraints. Constraints can be optimized to mitigate multipath distortion [13]. Different adaptive solutions have been derived in order to achieve low complexity [14]. Additionally, other important detection techniques have also been developed, for example, based on subspace-based channel estimates [15]–[17] or constrained constant modulus ideas[18]–[20].

Compared with the minimum mean-square-error (MMSE) detector, the constrained MOE detector [13] exhibits some performance loss because the optimal constraint vector is a perturbed channel vector. The perturbation depends on the background noise. It can not be neglected especially when the channel has a low signal to noise ratio (SNR). This problem can be mitigated by developing a new constrained method to improve the channel estimate [21].

As is known, subspace method (e.g., [15], [22]) provides a good estimate for the unknown channel. With this estimate, a MMSE detector can be built [17]. However, canonical subspace approach requires batch estimation of partial/whole signal subspace or noise subspace. Usually they are obtained through batch eigenvalue decomposition (EVD) or batch singular value decomposition (SVD) on the data correlation matrix which are computationally expensive. To significantly reduce the complexity, advanced subspace tracking algorithms have been developed [23]–[24].

In this paper, we focus on improvement of the constrained multiuser detector [13]. It is revealed that the constrained optimization method is closely related to the subspace method. Based on subspace decomposition of the autocorrelation matrix, the subspace cost function is found to be upper-bounded by a quadratic function. More surprisingly, this function is different from the constrained cost function by only an additional negative term. Therefore, it is always less than the constrained cost function for any possible choice of the constraint vector. Since the constrained cost function is the reciprocal of the maximum output energy, if this function (upper bound) is adopted as a new cost function, then the corresponding maximum output energy will become larger. Motivated by this result, we modify the constrained cost function and obtain a new one which lies in the neighborhood of the subspace cost function. Therefore, by minimizing the modified cost function, an improved estimate for the channel can be obtained. Then the performance of the corresponding detector will be improved.

Different from the previous cost function, the new cost function involves a weighting factor. It depends on the maximum eigenvalue of the data correlation matrix and thus can be obtained via EVD. Built upon the new constrained vector, the associated detector is studied in detail in the paper in terms of its asymptotic performance. The channel estimation error and the penalty in the output signal to interference plus noise ratio (SINR) (compared with the MMSE detector) are derived in closed forms. Simulation results show that for a large range of noise level, better performance of the proposed detector is obtained compared with previously proposed constrained detector

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[13]. Significant improvement is observed especially when SNR is low.

In order to reduce the computational complexity, adaptive algorithm is also investigated. Since the solution for the constraint vector is found to be an eigenvector of a matrix, it can be easily updated by power method [25], [26], similar to our weighting factor. Therefore, it suffices to update only a few extreme (maximum/minimum) eigen-pairs of some matrices. However, such implementation is different from [14] which describes adaptive realizations of [13].

This paper has the following structure. A multiuser CDMA system model with multipath distortions is described in Section II. In Section III, the relation between the subspace method and constrained optimization method is investigated, and an improved constrained method is developed. Asymptotic performance of the proposed detector is analyzed in Section IV. Adaptive implementation is then provided in Section V, while some numerical examples are presented in Section VI. Finally some conclusions are drawn in the last section.

II. PROBLEM STATEMENT

Consider a DS-CDMA system with J users. User j has input bit stream $w_j(n)$ and is assigned a periodic spreading sequence of length $P, \boldsymbol{c}_j = [c_j(0), \cdots, c_j(P-1)]^T$. Then, the jth user's discrete-time transmitted signal at the chip rate is given by (e.g., [13])

$$s_j(n) = \sum_{k=-\infty}^{\infty} w_j(k) c_j(n - kP), \tag{1}$$

where $w_j(n)$ is assumed to be a zero-mean, i.i.d. information bearing sequence with variance $\sigma_{w_j}^2 = \mathrm{E}\{\|w_j(n)\|^2\}$. Let $s_j(n)$ be transmitted through a linear multipath channel with a discrete-time impulse response $g_j(n)$ (including the transmitter and receiver filters). Then the received discrete-time signal $y_j(n)$ due to user j is

$$y_j(n) = \sum_{l=-\infty}^{\infty} s_j(l) g_j(n - d_j - l), \tag{2}$$

where $0 \leq d_j < P$ is the delay of user j in chip periods. From (1) and (2), the received signal from user j is related to its input by the following

$$y_j(n) = \sum_{l=-\infty}^{\infty} w_j(l) h_j(n - d_j - lP), \tag{3}$$

$$h_j(n) = \sum_{m=-\infty}^{\infty} c_j(m) g_j(n - m). \tag{4}$$

Finally, the received signal $y(n)$ is a superposition of the signals from all users plus zero-mean, additive white Gaussian noise (AWGN) $v(n)$ with variance $\sigma_v^2 = \mathrm{E}\{\|v(n)\|^2\}$

$$y(n) = \sum_{j=1}^{J} y_j(n) + v(n). \tag{5}$$

In this paper, we focus on detection of a particular user in the presence of multipath distortion and MUI. Without loss of generality, we assume user 1 is the desired user with given spreading codes, and the receiver is synchronized to this user. Also assume its multipath channel has maximum order q. To explicitly show the input/output relationship, we adopt vector/matrix representation. First we collect unknown channel coefficients in a vector $\boldsymbol{g}_1 = [g_1(0), \cdots, g_1(q)]^T$. We also collect $P+q$ chip rate samples of $y(n)$ from $y(nP)$ to $y(nP+P+q-1)$ in a vector $\boldsymbol{y}(n)$, then from (3), (4), and (5), we can obtain

$$\boldsymbol{y}(n) = \boldsymbol{C}_1 \boldsymbol{g}_1 w_1(n) + \boldsymbol{H}\boldsymbol{w}(n) + \boldsymbol{v}(n), \tag{6}$$

where $\boldsymbol{C}_1$ is a Toeplitz matrix constructed from code vector $\boldsymbol{c}_1$

$$\boldsymbol{C}_1 = \begin{bmatrix} c_1(0) & & \mathbf{0} \\ \vdots & \ddots & c_1(0) \\ c_1(P-1) & & \vdots \\ \mathbf{0} & \ddots & c_1(P-1) \end{bmatrix}, \tag{7}$$

$w_1(n)$ is the signal of interest, while $\boldsymbol{w}(n)$ includes intersymbol interference from its previous bit and the next bit, and MUI from other $J-1$ interfering users. Each column in $\boldsymbol{H}$ is the signature of a corresponding entry in $\boldsymbol{w}(n)$. $\boldsymbol{v}(n)$ is the AWGN vector. Under input/output model (6), a linear detector is proposed in [13] by minimizing the output energy of the detector while constraining the response from the desired user to be a constant. After this consideration, the approach is equivalent to minimizing the total energy of interference plus noise. It has been shown that the performance of the detector is close to the MMSE detector. However, the optimal constraint vector is a biased channel vector. The performance loss is derived to monotonically increase with the noise power. Therefore, the situation becomes severe especially when the channel is much noisy (SNR is very low). We will show that performance improvement can be made by modifying the cost function used in [13].

As is known, subspace method provides very accurate channel estimate. As a matter of fact, the constrained method is closely related to it. We will first expose the relationship between these two methods. Then we will derive our improved cost function which is in the neighborhood of the subspace cost function and gives better performance than the previous constrained method.

III. IMPROVED CONSTRAINED OPTIMIZATION METHOD

A. Connection of Constrained Method with Subspace Method

We start by analyzing the constrained cost function $\boldsymbol{b}^H(\boldsymbol{C}_1^H \boldsymbol{R}^{-1} \boldsymbol{C}_1)\boldsymbol{b}$ from which the optimal constraint vector $\boldsymbol{b}_{opt}$ is obtained [13], where $\boldsymbol{R} = E\{\boldsymbol{y}(n)\boldsymbol{y}^H(n)\}$ is the data correlation matrix. $\boldsymbol{b}_{opt}$ is an eigenvector of objective matrix $\boldsymbol{C}_1^H \boldsymbol{R}^{-1} \boldsymbol{C}_1$ corresponding to its minimum eigenvalue. In the canonical subspace method, either signal subspace or noise subspace is employed. Thus to seek the connection between two methods, we perform EVD on $\boldsymbol{R}$

$$\boldsymbol{R} = \boldsymbol{U}\boldsymbol{\Lambda}\boldsymbol{U}^H = \boldsymbol{U}_s(\boldsymbol{\Lambda}_s + \sigma_v^2\boldsymbol{I})\boldsymbol{U}_s^H + \boldsymbol{U}_n\boldsymbol{\Lambda}_n\boldsymbol{U}_n^H, \quad (8)$$

where

$$\boldsymbol{U} = [\boldsymbol{U}_s\ \boldsymbol{U}_n],\ \ \boldsymbol{\Lambda} = diag(\boldsymbol{\Lambda}_s + \sigma_v^2\boldsymbol{I}, \boldsymbol{\Lambda}_n),\ \ \boldsymbol{\Lambda}_n = \sigma_v^2\boldsymbol{I}.$$

It is well known that $\boldsymbol{U}_s$ spans the signal subspace which is also a range space of $[\boldsymbol{C}_1\boldsymbol{g}_1, \boldsymbol{H}]$ and $\boldsymbol{U}_n$ spans the noise subspace. From (8), $\boldsymbol{R}^{-1}$ is expressed as

$$\boldsymbol{R}^{-1} = \boldsymbol{U}_s diag(\frac{1}{\lambda_i^2 + \sigma_v^2})\boldsymbol{U}_s^H + \frac{1}{\sigma_v^2}\boldsymbol{U}_n\boldsymbol{U}_n^H. \quad (9)$$

Replacing $\boldsymbol{R}^{-1}$ by (9), the constrained cost function is thus decomposed and related to $\boldsymbol{U}_n$ by

$$\begin{aligned}\boldsymbol{b}^H(\boldsymbol{C}_1^H\boldsymbol{R}^{-1}\boldsymbol{C}_1)\boldsymbol{b} =& \boldsymbol{b}^H\boldsymbol{C}_1^H\boldsymbol{U}_s diag(\frac{1}{\lambda_i^2 + \sigma_v^2})\boldsymbol{U}_s^H\boldsymbol{C}_1\boldsymbol{b} \\ &+ \frac{1}{\sigma_v^2}\boldsymbol{b}^H\boldsymbol{C}_1^H\boldsymbol{U}_n\boldsymbol{U}_n^H\boldsymbol{C}_1\boldsymbol{b}.\end{aligned}$$

Since the desired signature $\boldsymbol{C}_1\boldsymbol{g}_1$ is orthogonal to the noise subspace, the subspace method seeks channel estimate by minimizing the cost function $\boldsymbol{b}^H\boldsymbol{C}_1^H\boldsymbol{U}_n\boldsymbol{U}_n^H\boldsymbol{C}_1\boldsymbol{b}$. Thus from (10) it becomes clear that certain connection exists between two methods. If we denote the constrained cost function and subspace cost function by ξ_c and ξ_s, respectively

$$\xi_c = \boldsymbol{b}^H(\boldsymbol{C}_1^H\boldsymbol{R}^{-1}\boldsymbol{C}_1)\boldsymbol{b},\ \ \xi_s = \boldsymbol{b}^H\boldsymbol{C}_1^H\boldsymbol{U}_n\boldsymbol{U}_n^H\boldsymbol{C}_1\boldsymbol{b},$$

then from (10), they are related to each other by

$$\sigma_v^2\xi_c = \xi_s + \gamma(\boldsymbol{b}), \quad (10)$$

where $\gamma(\boldsymbol{b}) \geq 0$ and depends on σ_v^2 by

$$\gamma(\boldsymbol{b}) = \boldsymbol{b}^H\boldsymbol{C}_1^H\boldsymbol{U}_s diag(\frac{\sigma_v^2}{\lambda_i^2 + \sigma_v^2})\boldsymbol{U}_s^H\boldsymbol{C}_1\boldsymbol{b}.$$

Based on the fact that scaling ξ_c by σ_v^2 does not affect our solution for $\boldsymbol{b}_{opt}$ which is of our interest, both $\sigma_v^2\xi_c$ and ξ_c will be regarded as the constrained cost function later. Eq. (10) shows that as $\sigma_v^2 \to 0$, $\gamma(\boldsymbol{b}) \to 0$. Then the constrained cost function tends to be the subspace cost function. It is known that minimization of ξ_s will give the exact estimate for $\boldsymbol{g}_1$. Therefore as $\sigma_v^2 \to 0$, the optimal constraint vector provides a unique and exact channel estimate. However, in the presence of noise σ_v^2 (no matter how small it is), an additional term $\gamma(\boldsymbol{b})$ exists in the constrained cost function. Since $\boldsymbol{C}_1\boldsymbol{g}_1 \in \boldsymbol{U}_s$, then $\gamma(\boldsymbol{g}_1) > 0$. Therefore an error in estimating channel vector $\boldsymbol{g}_1$ is inevitable by the constrained method. Some performance loss is observed in [13] due to this perturbation in the cost function. The loss is significant especially when the channel is very noisy. To reduce this loss while still maintaining comparative complexity, a new cost function should be designed. We will seek that function to be in the neighborhood of the subspace cost function, but is always small than the constrained cost function. In such a way the cost will be reduced and correspondingly the output power of the desired user will be increased.

B. Improved Method

We will focus on the additional term $\gamma(\boldsymbol{b})$ from which the performance loss results. It is determined by the eigen structure of $\boldsymbol{R}$. In order to avoid the dependence of the method on the complete knowledge about subspaces and thus reduce the computational cost, we will employ $\lambda_{max}^2 + \sigma_v^2$–the maximum eigenvalue of $\boldsymbol{R}$–in the cost function. For convenience, define $\lambda_m = \lambda_{max}^2 = \max_i \lambda_i^2$. Then for any $\boldsymbol{b}$, $\gamma(\boldsymbol{b})$ satisfies

$$\gamma(\boldsymbol{b}) \geq \frac{\sigma_v^2\boldsymbol{b}^H\boldsymbol{C}_1^H\boldsymbol{U}_s\boldsymbol{U}_s^H\boldsymbol{C}_1\boldsymbol{b}}{\lambda_{max}^2 + \sigma_v^2}.$$

Using the fact that $\boldsymbol{U}_s\boldsymbol{U}_s^H = \boldsymbol{I} - \boldsymbol{U}_n\boldsymbol{U}_n^H$, we obtain

$$\gamma(\boldsymbol{b}) \geq \frac{\sigma_v^2(\boldsymbol{b}^H\boldsymbol{C}_1^H\boldsymbol{C}_1\boldsymbol{b} - \xi_s)}{\lambda_{max}^2 + \sigma_v^2}.$$

Therefore based on (10), we have

$$\sigma_v^2\xi_c \geq \frac{\sigma_v^2\boldsymbol{b}^H\boldsymbol{C}_1^H\boldsymbol{C}_1\boldsymbol{b} + \lambda_{max}^2\xi_s}{\lambda_{max}^2 + \sigma_v^2},$$

from which ξ_c can be found to satisfy

$$\xi_s \leq \beta(\xi_c - \alpha\boldsymbol{b}^H\boldsymbol{C}_1^H\boldsymbol{C}_1\boldsymbol{b}), \quad (11)$$

where β and α are constants

$$\beta = \frac{\lambda_{max}^2 + \sigma_v^2}{\lambda_{max}^2}\sigma_v^2,\ \ \alpha = \frac{1}{\lambda_{max}^2 + \sigma_v^2}. \quad (12)$$

Result in (11) shows that the subspace cost function is upper-bounded by a quadratic function in its neighborhood. This function is different from the constrained cost function by a term $\alpha\boldsymbol{b}^H\boldsymbol{C}_1^H\boldsymbol{C}_1\boldsymbol{b}$ (again scalar β will not affect the solution for the constraint vector and is ignored). It is obvious that the constrained cost function (corresponding to $\alpha = 0$) is larger than the upper bound in (11) since $\alpha > 0$. It is thus not surprising that subspace method outperforms the constrained optimization method.

Motivated by these results, we modify the constrained cost function by

$$\begin{aligned}\xi(\boldsymbol{b}) &= \beta(\xi_c - \alpha\boldsymbol{b}^H\boldsymbol{C}_1^H\boldsymbol{C}_1\boldsymbol{b}) \\ &= \beta\boldsymbol{b}^H(\boldsymbol{C}_1^H\boldsymbol{R}^{-1}\boldsymbol{C}_1 - \alpha\boldsymbol{C}_1^H\boldsymbol{C}_1)\boldsymbol{b}, \quad (13)\end{aligned}$$

where α is a weighting parameter. From (8) and (12) it can be found that α is equal to the reciprocal of the maximum eigenvalue of $\boldsymbol{R}$. Due to (11), the matrix $\boldsymbol{C}_1^H\boldsymbol{R}^{-1}\boldsymbol{C}_1 - \alpha\boldsymbol{C}_1^H\boldsymbol{C}_1$ is guaranteed to be positive definite. By minimizing $\xi(\boldsymbol{b})$, the constraint vector can be obtained. It is thus the eigenvector of matrix $\boldsymbol{C}_1^H\boldsymbol{R}^{-1}\boldsymbol{C}_1 - \alpha\boldsymbol{C}_1^H\boldsymbol{C}_1$ corresponding to its minimum eigenvalue. Its uniqueness can be similarly argued as in [13]. This eigenvector $\bar{\boldsymbol{b}}$ is then used as the estimate for the channel vector $\boldsymbol{g}_1$. With this estimate, the detector can be constructed based on the MMSE criterion

$$\boldsymbol{f} = \boldsymbol{R}^{-1}\boldsymbol{C}_1\bar{\boldsymbol{b}}. \quad (14)$$

Its performance will be evaluated together with the channel estimation error next.

IV. ASYMPTOTIC PERFORMANCE

In this section we will analyze the asymptotic performance of the improved method as $\sigma_v^2 \to 0$ in terms of channel estimation error and the output SINR of our detector. Similar procedures as in [13] will be taken and some results from there will be borrowed. We will use notations with "¯" above to represent the analytical results from the improved method.

Our solution $\bar{\boldsymbol{b}}$ for the constraint vector is the eigenvector of the objective matrix $\beta \boldsymbol{C}_1^H(\boldsymbol{R}^{-1} - \alpha \boldsymbol{I})\boldsymbol{C}_1$ associated with its minimum eigenvalue. This objective matrix can be expressed as a power series of σ_v^2. Based on definitions of α and β given in (12), and using the fact that $\boldsymbol{I} = \boldsymbol{U}_s\boldsymbol{U}_s^H + \boldsymbol{U}_n\boldsymbol{U}_n^H$, we can easily obtain [13]

$$\begin{aligned}\beta(\boldsymbol{R}^{-1} - \alpha \boldsymbol{I}) =& \boldsymbol{U}_n\boldsymbol{U}_n^H + \sigma_v^2 \boldsymbol{U}_s(\boldsymbol{\Lambda}_s^{-1} - \lambda_m^{-1}\boldsymbol{I})\boldsymbol{U}_s^H \\ &+ \sigma_v^4 \boldsymbol{U}_s(\lambda_m^{-1}\boldsymbol{\Lambda}_s^{-1} - \boldsymbol{\Lambda}_s^{-2})\boldsymbol{U}_s^H + \mathbf{O}(\sigma_v^6).\end{aligned}$$

Therefore the objective matrix has the following expression

$$\beta \boldsymbol{C}_1^H(\boldsymbol{R}^{-1} - \alpha \boldsymbol{I})\boldsymbol{C}_1 = \bar{\boldsymbol{A}}_0 + \sigma_v^2 \bar{\boldsymbol{A}}_1 + \sigma_v^4 \bar{\boldsymbol{A}}_2 + \mathbf{O}(\sigma_v^6), \quad (15)$$

where

$$\begin{aligned}\bar{\boldsymbol{A}}_0 &= \boldsymbol{C}_1^H\boldsymbol{U}_n\boldsymbol{U}_n^H\boldsymbol{C}_1, \\ \bar{\boldsymbol{A}}_1 &= \boldsymbol{C}_1^H\boldsymbol{U}_s\boldsymbol{\Lambda}_s^{-1}\boldsymbol{U}_s^H\boldsymbol{C}_1 - \lambda_m^{-1}\boldsymbol{C}_1^H\boldsymbol{U}_s\boldsymbol{U}_s^H\boldsymbol{C}_1, \\ \bar{\boldsymbol{A}}_2 &= -\boldsymbol{C}_1^H\boldsymbol{V}_s\boldsymbol{\Lambda}_s^{-2}\boldsymbol{V}_s^H\boldsymbol{C}_1 + \lambda_m^{-1}\boldsymbol{C}_1^H\boldsymbol{U}_s\boldsymbol{\Lambda}_s^{-1}\boldsymbol{U}_s^H\boldsymbol{C}_1.\end{aligned}$$

It can be seen from (15) that this matrix becomes the objective matrix $\boldsymbol{C}_1^H\boldsymbol{U}_n\boldsymbol{U}_n^H\boldsymbol{C}_1$ in the subspace method as $\sigma_v^2 \to 0$. In this case the constraint vector is the unique null vector of $\boldsymbol{C}_1^H\boldsymbol{U}_n\boldsymbol{U}_n^H\boldsymbol{C}_1$ which is the channel vector. However, due to the presence of noise, $\bar{\boldsymbol{b}}$ becomes a perturbed channel vector. Based on perturbation theory (see references and derivation in [13]), the first order perturbation of $\bar{\boldsymbol{b}}$ is given by

$$\Delta \bar{\boldsymbol{b}} \simeq -\sigma_v^2 \bar{\boldsymbol{A}}_0{}^{\dagger} \bar{\boldsymbol{A}}_1 \frac{\boldsymbol{g}_1}{\|\boldsymbol{g}_1\|}, \quad (16)$$

where $\dagger$ denotes pseudo-inverse and we have used $\boldsymbol{U}_s\boldsymbol{U}_s^H = \boldsymbol{I} - \boldsymbol{U}_n\boldsymbol{U}_n^H$ and $\boldsymbol{U}_n^H\boldsymbol{C}_1\boldsymbol{g}_1 = \mathbf{0}$. With this perturbation, the output SINRs of the proposed detector and the ideal MMSE detector have the following relationship [13]

$$\frac{\text{SINR}_{mmse}}{\text{SINR}(\bar{\boldsymbol{b}})} \to 1 + \bar{\delta} \;\; \text{as} \;\; \sigma_v^2 \to 0 \quad (17)$$

where $\bar{\delta}$ is positive representing the performance loss of the proposed detector

$$\bar{\delta} = \frac{\boldsymbol{g}_1^H \bar{\boldsymbol{A}}_1{}^H \bar{\boldsymbol{A}}_0{}^{\dagger} \bar{\boldsymbol{A}}_1 \boldsymbol{g}_1}{\boldsymbol{g}_1^H \boldsymbol{C}_1^H \boldsymbol{V}_s \boldsymbol{\Lambda}_s^{-2} \boldsymbol{V}_s^H \boldsymbol{C}_1 \boldsymbol{g}_1}.$$

It is interesting to compare $\bar{\delta}$ with the performance loss δ of the constrained MOE detector [13]. Based on the way we develop the modified cost function, it is our conjecture that $\bar{\delta} < \delta$. From a large number of computer experiments, it is observed that this statement holds. However theoretical proof of this conjecture has not been available and is still under investigation.

In the batch approach, it is computationally expensive to compute $\boldsymbol{R}^{-1}$ and the eigen-pair of the objective matrix using conventional EVD method. Motivated by the fact that subspace tracking algorithm [16] is developed to reduce complexity of the canonical subspace method, we will derive our adaptive method to reduce computations as well in the next section.

V. ADAPTIVE IMPLEMENTATION

In the proposed method, only those quantities are required such as the largest eigenvalue of $\boldsymbol{R}$ and the smallest eigen-pair of $\boldsymbol{A} = \boldsymbol{C}_1^H\boldsymbol{R}^{-1}\boldsymbol{C}_1 - \alpha\boldsymbol{C}_1^H\boldsymbol{C}_1$. It is thus not necessary to obtain the entire space by eigen-decomposition. There are some efficient methods to track a few eigen-pairs [27]. In an extreme case with the interest of only the maximum or minimum eigen-pair, power method has been shown to be more efficient [25], [26]. In our approach, $\frac{1}{\alpha}$ is the maximum eigenvalue of $\boldsymbol{R}$ which can be recursively updated by the power method. In matrix $\boldsymbol{A}$, $\boldsymbol{R}^{-1}$ can be updated directly from data by recursive least squares (RLS) [28]. Based on α and $\boldsymbol{R}^{-1}$, $\boldsymbol{A}$ can be computed at each time.

Our next task is to find the minimum eigen-pair of $\boldsymbol{A}$. It is generally believed that it is easier to obtain the maximum eigen-pair of a matrix by directly using power method than to obtain its minimum eigen-pair. As suggested in [26], we may apply power method to find the maximum eigen-pair of $(\nu_2\boldsymbol{I} - \boldsymbol{A})$ first, where ν_2 is the maximum eigenvalue of $\boldsymbol{A}$ similarly found by the power method. Assume this pair is $(\nu_3, \boldsymbol{x})$. Then the minimum eigen-pair of $\boldsymbol{A}$ will be $(\nu_2 - \nu_3, \boldsymbol{x})$ and $\boldsymbol{x}$ is our estimated channel vector. This adaptive algorithm is summarized in Table 1, where n represents the time index in terms of bit periods, $\boldsymbol{u}_1$ is the eigenvector of $\boldsymbol{R}$ corresponding to its maximum eigenvalue, and $\boldsymbol{u}_2$ is the eigenvector of $\boldsymbol{A}$ corresponding to its maximum eigenvalue. The convergence rate of the algorithm is well known to depend on the eigenvalue spread of $\boldsymbol{R}$ and $\boldsymbol{A}$. It is observed in our simulation that the algorithm converges within several hundred iterations.

We may gain an insight into how complex this algorithm is by counting total multiplications in different steps. The complexity to obtain weighting factor is $O((P+q)^2)$, while the complexity to update either the maximum eigenvalue of $\boldsymbol{A}$ or the constraint vector is $O((q+1)^2)$. Therefore the total complexity to obtain the channel estimate at each time is about $O((P+q)^2 + (q+1)^2)$. It is thus significantly reduced compared with the complexity $O((P+q)^3 + (q+1)^3)$ required by our batch method. In the case of small q, matrix $\boldsymbol{A}$ has a relatively small dimension $(q+1) \times (q+1)$ compared with that of $\boldsymbol{R}^{-1}$. We may consider eigen-decomposition directly on $\boldsymbol{A}$ to obtain the corresponding eigenvector without introducing significant complexity. Once the constraint vector is obtained, it can be used as an estimate for channel vector $\boldsymbol{g}_1$. Therefore a MMSE detector can be constructed to detect input symbols [16], [17].

One may wonder if joint update of the constraint vector and the detector can be performed to achieve some optimality similarly as in [14]. This interesting topic still remains open. Some difficulties may arise in obtaining a joint cost function of the constraint vector and the detector.

VI. NUMERICAL EXAMPLES

We simulate a 5-user CDMA system by computers. Each user has bits $\{+1, -1\}$ to transmit. Different Gold sequences

Table 1. Adaptive algorithm to update the constraint vector and construct the detector.

Step 1: Initialize $\boldsymbol{R}(0) = \varepsilon \boldsymbol{I}$, $\boldsymbol{R}^{-1}(0) = \frac{1}{\varepsilon}\boldsymbol{I}$, $\boldsymbol{b}(0) \neq \boldsymbol{0}$, $\boldsymbol{u}_1(0) \neq \boldsymbol{0}$, $\boldsymbol{u}_2(0) \neq \boldsymbol{0}$, and choose forgetting factor η for the RLS method.

Step 2: For $n = 1, 2, \cdots$

(1) obtain weighting factor α

$$\boldsymbol{R}(n) = \eta \boldsymbol{R}(n-1) + \boldsymbol{y}(n)\boldsymbol{y}^H(n)$$
$$\boldsymbol{u}_1(n) = \boldsymbol{R}(n)\boldsymbol{u}_1(n-1)$$
$$\frac{\boldsymbol{u}_1(n)}{\|\boldsymbol{u}_1(n)\|} \longrightarrow \boldsymbol{u}_1(n)$$
$$\alpha(n) = \frac{1}{\boldsymbol{u}_1^H(n)\boldsymbol{R}(n)\boldsymbol{u}_1(n)}$$

(2) update $\boldsymbol{R}^{-1}$

$$\boldsymbol{k}_n = \frac{\boldsymbol{R}^{-1}(n-1)\boldsymbol{y}(n)}{\eta + \boldsymbol{y}^H(n)\boldsymbol{R}^{-1}(n)\boldsymbol{y}(n)}$$
$$\boldsymbol{R}^{-1}(n) = \frac{\boldsymbol{R}^{-1}(n-1) - \boldsymbol{k}_n \boldsymbol{y}^H(n)\boldsymbol{R}^{-1}(n-1)}{\eta}$$

(3) obtain ν_2 - maximum eigenvalue of $\boldsymbol{A}$

$$\boldsymbol{A}(n) = \boldsymbol{C}_1^H \boldsymbol{R}^{-1}(n)\boldsymbol{C}_1 - \alpha(n)\boldsymbol{C}_1^H\boldsymbol{C}_1$$
$$\boldsymbol{u}_2(n) = \boldsymbol{A}(n)\boldsymbol{u}_2(n-1)$$
$$\frac{\boldsymbol{u}_2(n)}{\|\boldsymbol{u}_2(n)\|} \longrightarrow \boldsymbol{u}_2(n)$$
$$\nu_2(n) = \frac{1}{\boldsymbol{u}_2^H(n)\boldsymbol{A}(n)\boldsymbol{u}_2(n)}$$

(4) update $\boldsymbol{b}$

$$\boldsymbol{b}(n) = [\nu_2(n)\boldsymbol{I} - \boldsymbol{A}(n)]\boldsymbol{b}(n-1),$$
$$\frac{\boldsymbol{b}(n)}{\|\boldsymbol{b}(n)\|} \longrightarrow \boldsymbol{b}(n)$$

(5) construct the detector

$$\boldsymbol{f}(n) = \boldsymbol{R}^{-1}(n)\boldsymbol{C}_1\boldsymbol{b}(n)$$

of length 31 are used as spreading sequences for different users. The multipath channel coefficients $[\boldsymbol{g}_1, \boldsymbol{g}_2, \boldsymbol{g}_3, \boldsymbol{g}_4, \boldsymbol{g}_5]$ for all users are randomly selected as follows

$$\begin{bmatrix} 0.3091 & -0.4393 & -0.5342 & 0.3041 & -0.5645 \\ 0.5769 & 0.1316 & 0.4665 & -0.5841 & 0.1581 \\ -0.7011 & 0.5846 & 0.6548 & 0.0581 & 0.4315 \\ 0.2830 & -0.6693 & 0.2612 & -0.7503 & -0.6857 \end{bmatrix}.$$

First, the data length effect on the channel estimation error and the output SINR are tested and compared with the MOE method [13] in order to examine the improvement. The proposed improved detector is also compared with the MMSE detector (with exact knowledge of the channel in the current context) and the subspace based method (e.g., [17]) to gain an insight into how good the proposed method is. Assume user 1 is the desired user and all users have equal power. Since the difference between the proposed and the MOE method is expected to be evident for higher noise level (larger σ_v^2), we let the background noise have the same power (0 dB). The near-far effect and the noise effect will be investigated in detail later. We also assume the detector is synchronized to the desired user (user 1) in order to construct the matrix $\boldsymbol{C}_1$. Totally data samples corresponding to 1000 bit periods are collected and processed. 50 realizations are performed to obtain the average results. Delays for other users are randomly generated from 0 to 30 chip periods in each realization. The MSEs of channel estimate are presented in Fig. 1. The dashed-dotted line represents the MOE method [13], the solid line for the proposed, and dashed line for the subspace method [17]. As can be seen that the improvement over [13] is significant with

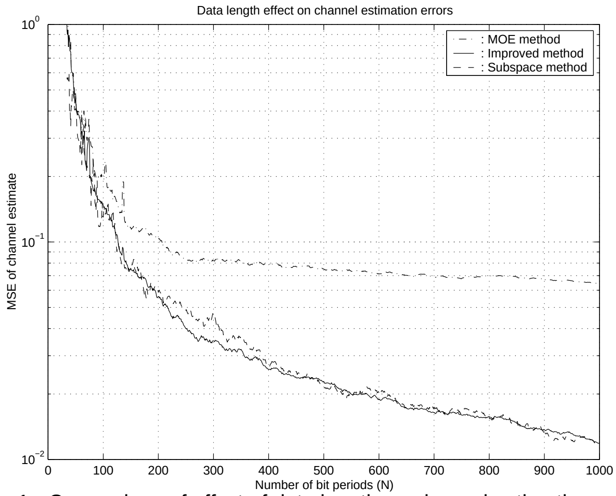

Fig. 1. Comparison of effect of data length on channel estimation errors for different methods.

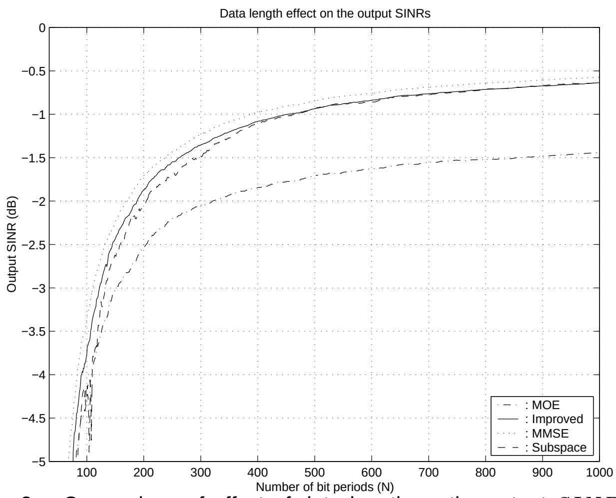

Fig. 2. Comparison of effect of data length on the output $SINR$ for different detectors.

longer data records. The performance of the proposed method and the subspace method is almost indistinguishable. Slightly better results from the proposed method are obtained from 200 to 400 bit periods. The performance in channel estimation can be reflected in the output SINRs which are plotted in Fig. 2. Detectors based on these methods are also compared with the MMSE detector denoted by colons (:). Similar conclusions can be made about the performance of the proposed method, [13] and [17]. Also, the proposed method and [17] are very close to the MMSE method. This is not surprising since these two methods provide more accurate channel estimates.

Next, the near-far effect on the performance of the MOE method and the proposed method is tested where the true autocorrelation matrix is used. 10 dB noise is added to the system. All interfering users have equal power. The signal to interference ratio (SIR) varies from -20 dB to 20 dB. The MSEs of channel estimate are plotted in Fig. 3. Both methods give satisfactory results under severe near-far situation. The MSEs are still below 10^{-3} when $SIR < -10$ dB. However, significant improvement of the proposed method over [13] can be observed when $SIR > 0$ dB. Even under power control to make all users

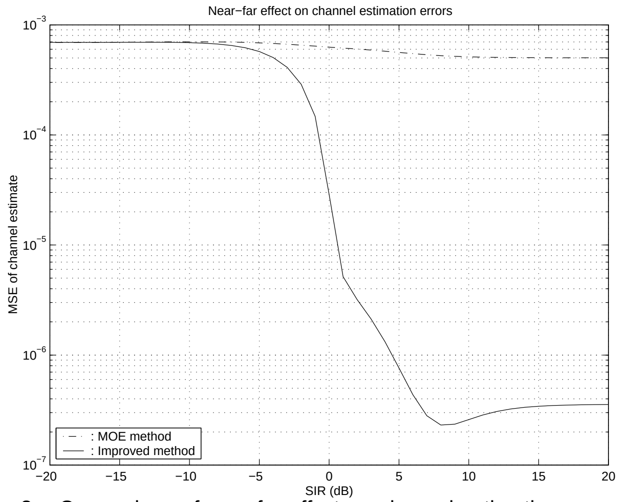

Fig. 3. Comparison of near-far effect on channel estimation errors for different methods.

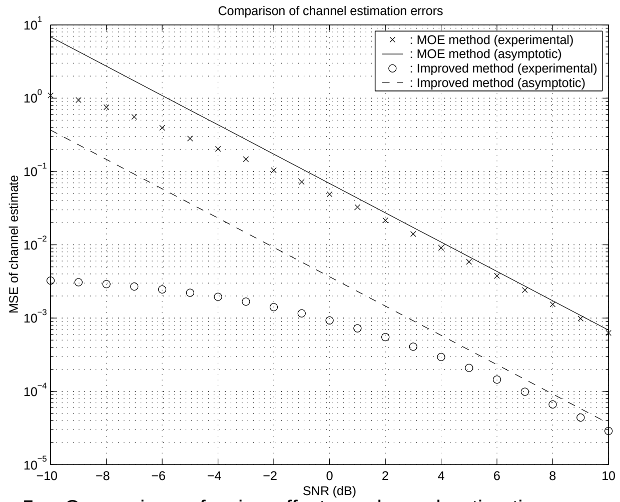

Fig. 5. Comparison of noise effect on channel estimation errors and asymptotic performance for different methods.

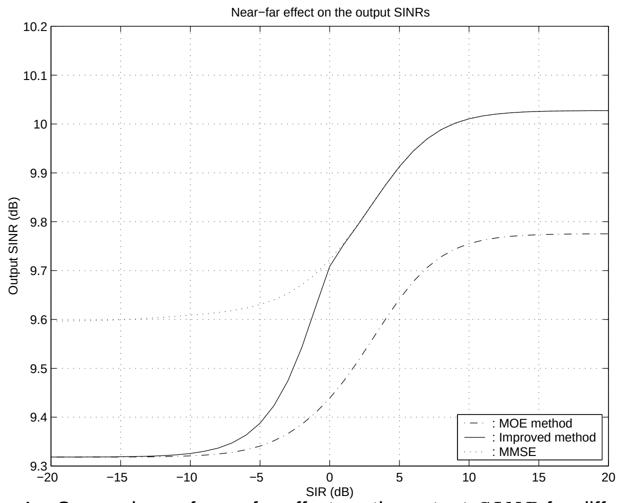

Fig. 4. Comparison of near-far effect on the output $SINR$ for different detectors.

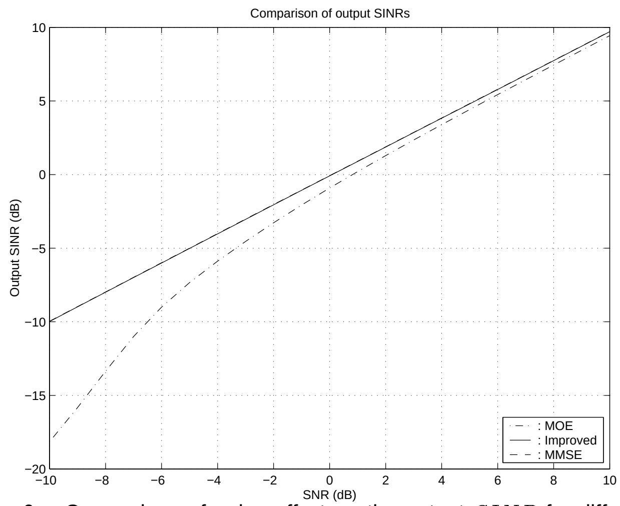

Fig. 6. Comparison of noise effect on the output $SINR$ for different detectors.

have equal power, more than 10 dB gain is achieved by the proposed method (3×10^{-5} vs. 6×10^{-4}). With those channel estimates, the corresponding detectors are constructed. Their output SINRs are also compared with the MMSE detector in Fig. 4. It is clear that with much interference in the system, two methods are almost the same and lose 0.3 dB compared with the MMSE detector. However, the improved detector approaches the MMSE detector as SIR increases. It also goes about 0.25 dB higher than the MOE detector.

Similarly, the noise effect is also investigated. All users have equal power. The channel estimation errors are shown in Fig. 5 for both the MOE and the improved methods in "x" and "o" respectively. The corresponding analytical results are also plotted by solid line and dashed line respectively. We can see that as SNR increases, experimental results from both methods are highly consistent with our asymptotic analysis. Secondly, much lower MSEs based on the improved method are achieved for the entire SNR region from -10 dB to 10 dB. The improvement can also be observed in Fig. 6 in terms of output SINRs of the corresponding detectors. For comparison, the output SINRs from the MMSE detector is presented in dashed line, but it is overlapped with the solid line which represents the results for the improved detector. Comparison is also made based on the ratio of SINRs and the SINR of the MMSE detector in Fig. 7. The asymptotic SINR ratios $\frac{SINR}{SINR_{mmse}}$ are shown in dashed-dotted line for the MOE detector and in dashed line for the improved detector. It is observed again that the performance loss of the proposed detector is negligible. It is less than that of the MOE detector. Both detectors approach their asymptotic performance as SNR increases.

The channel order also affects the performance of the method. We plot the average channel estimation errors for the proposed method and MOE approach from 100 realizations in Fig. 8. For a 5-user system with 10 dB noise, the channel parameters are fixed for interfering users in all realizations. Signals from the desired user propagate through two different paths with equal power. Channel coefficients of the desired user are randomly generated in different realizations. It is observed that when channel order of the desired user increases from 1 to 16, performance of both channel estimators degrades. However, the proposed method always provides better channel es-

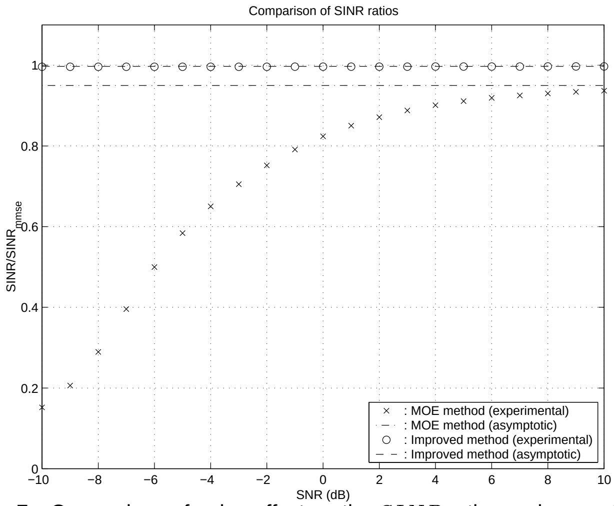

Fig. 7. Comparison of noise effect on the $SINR$ ratios and asymptotic performance for different detectors.

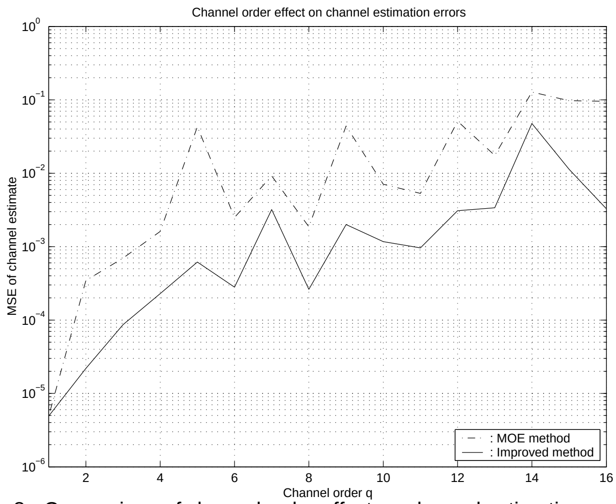

Fig. 8. Comparison of channel order effect on channel estimation errors.

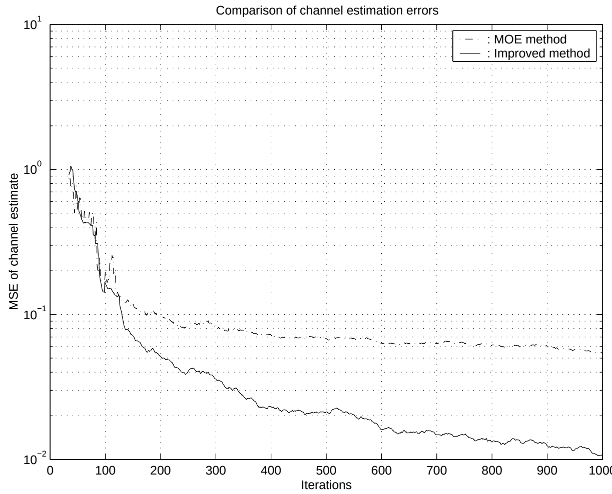

Fig. 9. Channel estimation errors based on adaptive implementation.

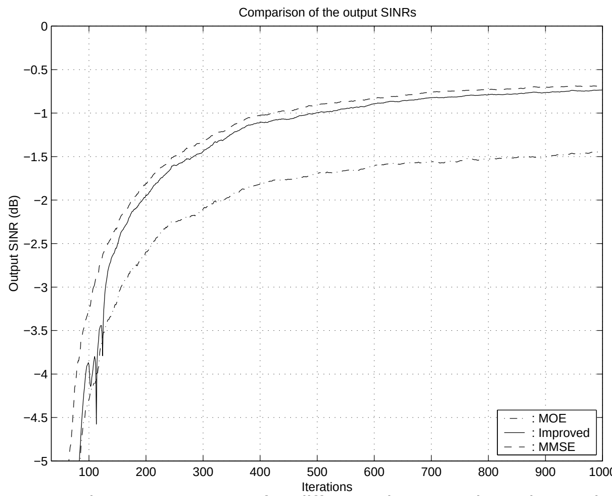

Fig. 10. The output $SINR$ for different detectors based on adaptive implementation.

timate than the MOE approach. In the next experiment, we test the performance of the proposed adaptive method. We set $SNR = 0$ dB. Initialization is as follows: $\varepsilon = 10^{-3}$, $\eta = 0.998$, $\boldsymbol{u}_1(0) = [1, 0, \cdots, 0]^T$, $\boldsymbol{u}_2(0) = \boldsymbol{b}(0) = [1, 0, \cdots, 0]^T$. The average MSEs of channel estimate from 50 independent realizations is compared with the MOE method in Fig. 9. One iteration corresponds to one bit period. Smaller errors are achieved once more by the improved method. The average SINRs of both detectors are compared with the MMSE detector in Fig. 10. We can observe that the proposed method is better than the MOE and is closer to the MMSE. All of them converge after several hundred iterations.

VII. CONCLUSIONS

In this paper, constraint vector is improved compared with the MOE multiuser detection method in the presence of multipath. Our method is derived based on the relationship among the proposed cost function, the MOE cost function and the subspace cost function. The proposed method is shown to have better performance than the MOE method especially in the low SNR region, and performance much closer to the MMSE detector. Adaptive implementation of the method is also presented by employing power method.

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