Regularization Enhanced Fast Convergence for POR-Based Multiuser Detection

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***Abstract*— Recently, a power of R (POR) technique that is originated from a minimum variance method is proposed for multiuser detection in code division multiple access (CDMA) systems. It raises power of an inversed data covariance matrix in the MV cost function to a positive integer m to approximate the noise subspace component in order to bridge a gap between the MV method and a widely studied subspace method. The POR method asymptotically converges to the subspace one and also shows more robust performance in practical conditions with finite noisy data record. However, the convergence rate depends on m and signal to noise ratio (SNR). In particular, a wide gap is observed in the low SNR region for a small or moderate m. This work proposes a regularized POR (termed as RPOR) technique by introducing a regularization parameter to the POR cost function for fast convergence and better detection performance for low SNR CDMA systems. The parameter aims to compensate the noise effect and concurrently enhance the interference mitigation capability of the receiver.** [1]

I. Introduction

Code-division multiple access (CDMA) spread spectrum technology has become one of the leading solutions for multiuser communication systems beyond third generation [1] due to its unique capabilities of simultaneous spectrum sharing, mitigation of jamming, interception and multipath fading. However, multiuser interference (MUI) and multipath distortion remain continuously as major obstacles to reliable symbol detection [2]. Tremendous efforts have focused on developing multiuser detection techniques.

Among all blind solutions, the minimum variance (MV) (or minimum output energy - MOE) method can combat those adverse effects with aid of only the desired user's spreading codes [3]. It not only provides a multiuser receiver, but also a blind channel estimator obtained from the corresponding optimal constraint vector. No subspace decomposition of the data covariance matrix is required. The method shows closeness in performance to the subspace one such as [4]. The MV receiver's detection performance approaches that of the non-blind minimum mean-square-error (MMSE) receiver. Different adaptive implementations are easily realized [5]. However, penalties exist in both channel estimation and detection performance as functions of the signal to noise ratio (SNR) and interference structure. Therefore, modifications have been made to the MV cost function to alleviate the noise effect and ensure reduced interference plus noise power by properly selecting a regularization parameter. The noise contribution can be partially subtracted from the cost function if its power is unknown [6] or completely eliminated if its power can be estimated [7]. An alternative to combat interference plus noise is to raise the power of the data covariance matrix in the MV cost function to a positive integer m, yielding a so-called power of R (POR) technique [8]. Different from previously mentioned improved methods that still yield residual performance penalty, this method provides a channel estimator asymptotically convergent to the subspace channel estimator [4] either at high SNR or with a large m. Meanwhile, in terms of detection, it produces an optimal blind MMSE solution. The POR technique shows very robust performance in practical conditions with finite number of noisy observations. It has been found that parameter m suffices to take a number as small as two for a better complexity/performance tradeoff. If m equals one, the POR method reduces to the MV method.

Although performance improvement and asymptotic convergence of the POR method to the subspace method are guaranteed, its convergence rate and extent of improvement in detection are limited by SNR and m. It will be demonstrated that enhancement can be made after jointly considering regularization and the POR technique mentioned above, i.e., following a similar way, a regularization parameter is introduced to the POR cost function. Consequently, the channel estimation error is significantly reduced for any fixed m, while superior detection performance is achieved at moderate to low SNRs. Some simulation examples further validate our analysis.

II. System Modeling

Consider a CDMA system with J users. User j ($j = 1, \cdots, J$) is assigned spreading codes $c_j(k)$ ($k = 0, \ldots, P-1$) of length P to spread its information symbol $w_j(n)$. Let the chip sequence be transmitted through a linear channel with a baseband chip rate discrete-time impulse response $g_j(n)$. Then the received discrete-time signal $y_j(n)$ at the chip-rate receiver

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due to user j is [3]

$$y_j(n) = \sum_{l=-\infty}^{\infty} w_j(l) h_j(n - d_j - lP), \quad (1)$$

$$h_j(n) = \sum_{m=-\infty}^{\infty} g_j(m) c_j(n - m), \quad (2)$$

where $w_j(n)$ is assumed to have zero-mean and variance $\sigma_{w_j}^2$, $h_j(n)$ is a waveform sequence lumping the convolution effect of spreading codes and propagation channel, d_j is a propagation delay of user j in chip periods. After considering all J users and zero-mean additive white Gaussian noise (AWGN) $v(n)$ with variance σ_v^2, the received signal becomes

$$y(n) = \sum_{j=1}^{J} y_j(n) + v(n). \quad (3)$$

This discrete-time model can be easily formulated into a matrix/vector representation. Assume the receiver is synchronized to our desired user - user 1 ($d_1 = 0$) as in [3] and [8]. After collecting measurements corresponding to at least one symbol period, a vector $\boldsymbol{y}_n$ can be obtained as follows [3]

$$\boldsymbol{y}_n = \boldsymbol{h}_1 w_1(n) + \boldsymbol{H}_{int} \boldsymbol{w}_{int}(n) + \boldsymbol{v}(n) \quad (4)$$

where $\boldsymbol{h}_1 = \boldsymbol{C}_1 \boldsymbol{g}_1$, $\boldsymbol{C}_1$ is a code filtering matrix of the desired user constructed from its spreading codes [3], $\boldsymbol{g}_1$ is its unknown channel vector containing all its channel coefficients,

$$\boldsymbol{C}_1 = \begin{bmatrix} c_1(0) & & 0 \\ \vdots & \ddots & c_1(0) \\ c_1(P-1) & & \vdots \\ 0 & \ddots & c_1(P-1) \\ \boldsymbol{0} & \cdots & \boldsymbol{0} \end{bmatrix}, \quad \boldsymbol{g}_1 = \begin{bmatrix} g_1(0) \\ \vdots \\ g_1(q) \end{bmatrix}, \quad (5)$$

$\boldsymbol{w}_{int}(n)$ is the interference vector including MUI and inter-symbol interference, $\boldsymbol{H}_{int}$ is the corresponding signature matrix, $\boldsymbol{v}(n)$ is the noise component. The number of trailing zeros in matrix $\boldsymbol{C}_1$ depends on the size of the observation window.

III. Review of Relevant Methods

Before presenting our method, let's briefly review the MV method, POR and a few regularization methods proposed before. The objective of a blind receiver is to detect $w_1(n)$ without knowing channel vector $\boldsymbol{g}_1$, but only its spreading codes. The MV approach minimizes the output variance (or power) of the receiver $\boldsymbol{f}_{mv}$ subject to parameterized code constraints collected in a vector $\boldsymbol{g}_{mv}$ [3]. The optimal constraint vector can be further obtained after maximizing the residual power. The criterion can be described jointly as

$$(\boldsymbol{g}_{mv}, \boldsymbol{f}_{mv}) = \arg \max_{\boldsymbol{g}} \min_{\boldsymbol{f}} \boldsymbol{f}^H \boldsymbol{R} \boldsymbol{f}, \quad (6)$$

$$\text{subject to } \boldsymbol{C}_1^H \boldsymbol{f} = \boldsymbol{g}, \quad ||\boldsymbol{g}|| = 1,$$

where $\boldsymbol{R}$ is the autocorrelation of $\boldsymbol{y}_n$ as $\boldsymbol{R} = E\{\boldsymbol{y}_n \boldsymbol{y}_n^H\}$, and has the following expression according to (4)

$$\boldsymbol{R} = \sigma_{w_1}^2 \boldsymbol{h}_1 \boldsymbol{h}_1^H + \boldsymbol{H}_{int} E\{\boldsymbol{w}_{int}(n) \boldsymbol{w}_{int}^H(n)\} \boldsymbol{H}_{int}^H + \sigma_v^2 \boldsymbol{I}. \quad (7)$$

The unit norm constraint on the constraint vector is to avoid a trivial solution. The optimal constraint vector can be used as a channel estimator as analyzed in [3] and is given by

$$\boldsymbol{g}_{mv} = \arg \min_{\|\boldsymbol{g}\|=1} \boldsymbol{g}^H \boldsymbol{C}_1^H \boldsymbol{R}^{-1} \boldsymbol{C}_1 \boldsymbol{g}. \quad (8)$$

After ignoring a positive scalar, the MV receiver takes a form $\boldsymbol{f}_{mv} = \boldsymbol{R}^{-1} \boldsymbol{C}_1 \boldsymbol{g}_{mv}$. Since $\boldsymbol{g}_{mv}$ is different from $\boldsymbol{g}_1$ according to perturbation analysis in [3], an error in channel estimation causes degraded detection performance when $\boldsymbol{f}_{mv}$ is compared with the MMSE receiver $\boldsymbol{f}_{mmse} = \boldsymbol{R}^{-1} \boldsymbol{C}_1 \boldsymbol{g}_1$. After regularization, it is improved as [7]

$$\boldsymbol{g}_{rmv} = \arg \min_{\|\boldsymbol{g}\|=1} \boldsymbol{g}^H \boldsymbol{C}_1^H (\boldsymbol{R} - \alpha \boldsymbol{I})^{-1} \boldsymbol{C}_1 \boldsymbol{g}. \quad (9)$$

The positive parameter α aims at alleviating the noise effect, and should be chosen to satisfy $\alpha < \sigma_v^2$ such that the partially denoised matrix $\boldsymbol{R} - \alpha \boldsymbol{I}$ is positive semidefinite [7]. It is easily found that if $\alpha = 0$, then this regularized MV method reduces to the MV method. As an alleviation alternative, the POR method adopts the following cost function [8]

$$\boldsymbol{g}_{por} = \arg \min_{\|\boldsymbol{g}\|=1} \boldsymbol{g}^H \boldsymbol{C}_1^H \boldsymbol{R}^{-m} \boldsymbol{C}_1 \boldsymbol{g}, \quad (10)$$

where m is a positive integer. If $m = 1$, then it also reduces to the MV method. Both the regularized MV method and POR method have shown significant improvements over the MV method. They are closely related to the subspace one [4] which yields a perfect channel estimate under some identifiability conditions. However, the former method shows an error floor in channel estimation and a detection performance loss, while convergence rate of the latter highly depends on SNR and parameter m.

IV. Regularized POR Method and Its Performance

In this work, we combine merits of those two methods and modify the POR cost function by introducing a positive regularization parameter α as in [7]

$$\boldsymbol{g}_{rpor} = \arg \min_{\|\boldsymbol{g}\|=1} \boldsymbol{g}^H \boldsymbol{C}_1^H (\boldsymbol{R} - \alpha \boldsymbol{I})^{-m} \boldsymbol{C}_1 \boldsymbol{g}. \quad (11)$$

Define $\boldsymbol{A} = \boldsymbol{C}_1^H (\boldsymbol{R} - \alpha \boldsymbol{I})^{-m} \boldsymbol{C}_1$. Then, $\boldsymbol{g}_{rpor}$ is the minimum eigenvector of $\boldsymbol{A}$ associated with its minimum eigenvalue γ_{rpor}. The corresponding regularized POR (termed as RPOR) receiver takes a form $\boldsymbol{f}_{rpor} = \boldsymbol{R}^{-1} \boldsymbol{C}_1 \boldsymbol{g}_{rpor}$. Following similar procedures as in [8] and considering (7), it can be shown that for small $(\sigma_v^2 - \alpha)$ the channel estimation error is given by

$$\begin{aligned} \boldsymbol{g}_{rpor} - \boldsymbol{g}_1 & \\ \approx \quad & -\boldsymbol{A}_0^{\dagger} \boldsymbol{C}_1^H \boldsymbol{U}_s diag\{(\tfrac{\sigma_v^2 - \alpha}{\lambda_i^2 + \sigma_v^2 - \alpha})^m\} \boldsymbol{U}_s^H \boldsymbol{C}_1 \boldsymbol{g}_1 \\ \approx \quad & -(\sigma_v^2 - \alpha)^m \boldsymbol{A}_0^{\dagger} \boldsymbol{C}_1^H \boldsymbol{U}_s diag\{(\tfrac{1}{\lambda_i^2})^m\} \boldsymbol{U}_s^H \boldsymbol{C}_1 \boldsymbol{g}_1 \end{aligned} \quad (12)$$

where † denotes pseudo-inverse, $\boldsymbol{A}_0 = \boldsymbol{C}_1^H \boldsymbol{U}_n \boldsymbol{U}_n^H \boldsymbol{C}_1$, $\boldsymbol{U}_n$, $\boldsymbol{U}_s$, λ_i depend on the eigenvalue decomposition of $\boldsymbol{R}$

$$\boldsymbol{R} = \boldsymbol{U}_s diag\{\lambda_i^2 + \sigma_v^2\}\boldsymbol{U}_s^H + \sigma_v^2 \boldsymbol{U}_n \boldsymbol{U}_n^H. \tag{13}$$

According to (12), it is clear that the error is reduced compared with the MV method ($m = 1$, $\alpha = 0$), the regularized MV method ($m = 1$, $\alpha > 0$), and the existing POR method ($m > 1$, $\alpha = 0$). This error decreases approximately at a rate of $(\sigma_v^2 - \alpha)^m$ as SNR increases. Channel estimation performance is significantly improved especially when α approaches σ_v^2 from the left. Performance of the corresponding receiver is anticipated to improve as well, as argued for the POR method in [8]. The output signal to interference plus noise ratio (SINR) of the proposed receiver can be used for a performance measure and similarly defined as (see [8])

$$\text{SINR}_{rpor} = \frac{\sigma_{w_1}^2 |\boldsymbol{f}_{rpor}^H \boldsymbol{h}_1|^2}{\boldsymbol{f}_{rpor}^H \boldsymbol{R} \boldsymbol{f}_{rpor} - \sigma_{w_1}^2 |\boldsymbol{f}_{rpor}^H \boldsymbol{h}_1|^2}. \tag{14}$$

In practice, a residual channel estimation error exists for any $\alpha < \sigma_v^2$ and finite m according to (12). Since σ_v^2 is the minimum eigenvalue of $\boldsymbol{R}$, α can be chosen according to its eigenvalue decomposition. Under the condition $\alpha < \sigma_v^2$, the larger this parameter α is, the smaller the channel estimation error and thus the better the detection performance. As α approaches σ_v^2, the channel estimation error approaches zero. Consequently, the corresponding receiver converges to the MMSE receiver. The effect of α will be thoroughly studied in the simulation next.

Performance analysis of channel estimator and receiver with finite noisy data samples can be similarly carried out as in [8]. Define $\bar{\boldsymbol{R}} = \boldsymbol{R} - \alpha \boldsymbol{I}$. Due to sample size effect, resulting errors in interesting quantities are denoted as $\delta \boldsymbol{g}$, $\delta \boldsymbol{f}$, $\delta \boldsymbol{R}$. Perturbation $\delta \boldsymbol{R}$ arises in the estimated data correlation matrix when it is estimated from N data vectors

$$\widetilde{\boldsymbol{R}} = \frac{1}{N} \sum_{n=1}^{N} \boldsymbol{y}_n \boldsymbol{y}_n^H. \tag{15}$$

Then

$$\delta \boldsymbol{g} \approx \sum_{k=1}^{m} \boldsymbol{T}_k \delta \boldsymbol{R} \boldsymbol{t}_k \tag{16}$$

where $\boldsymbol{T}_k$ and $\boldsymbol{t}_k$ are deterministic quantities

$$\boldsymbol{T}_k = (\boldsymbol{A} - \gamma_{por} \boldsymbol{I})^{\dagger} \boldsymbol{C}_1^H \bar{\boldsymbol{R}}^{-k}, \quad \boldsymbol{t}_k = \bar{\boldsymbol{R}}^{-(m-k)} \boldsymbol{f}_{rpor}.$$

The covariance of $\delta \boldsymbol{g}$ becomes

$$\text{Cov}_g \approx \sum_{k_1, k_2} \boldsymbol{T}_{k_1} E\{\delta \boldsymbol{R} \boldsymbol{t}_{k_1} \boldsymbol{t}_{k_2}^H \delta \boldsymbol{R}\} \boldsymbol{T}_{k_2}^H \tag{17}$$

and the channel mean-square-error is equal to the trace of Cov_g. Both are dependent on the weighted covariance of $\delta \boldsymbol{R}$ which has been derived in [9] but restated next.

Proposition: If the channel model follows (4) with L inputs and ν outputs, and data covariance is estimated from N independent data vectors by (15), then for a real system, the covariance of $\delta \boldsymbol{R}$ weighted by a constant matrix $\boldsymbol{Z}$ satisfies

$$\begin{aligned} NE\{\delta \boldsymbol{R} \boldsymbol{Z} \delta \boldsymbol{R}\} &= \kappa_{4w} \boldsymbol{H}[\boldsymbol{I}_L \odot (\boldsymbol{H}^T \boldsymbol{Z} \boldsymbol{H})] \boldsymbol{H}^T \\ &+ tr(\boldsymbol{R}\boldsymbol{Z})\boldsymbol{R} + \boldsymbol{R}\boldsymbol{Z}^T \boldsymbol{R}, \end{aligned} \tag{18}$$

and for a complex system, it satisfies

$$\begin{aligned} NE\{\delta \boldsymbol{R} \boldsymbol{Z} \delta \boldsymbol{R}\} &= \kappa_{4w} \boldsymbol{H}[\boldsymbol{I}_L \odot (\boldsymbol{H}^H \boldsymbol{Z} \boldsymbol{H})] \boldsymbol{H}^H \\ &+ tr(\boldsymbol{R}\boldsymbol{Z})\boldsymbol{R} \end{aligned} \tag{19}$$

where κ_{4w} is the fourth order cumulant of input, $\boldsymbol{H} = [\boldsymbol{h}_1, \boldsymbol{H}_{int}]$, "$tr$" represents trace of a matrix, and "$\odot$" represents element wise matrix product. □

Replacing $\boldsymbol{t}_{k_1} \boldsymbol{t}_{k_2}^H$ in (17) by $\boldsymbol{Z}$, the covariance can be easily evaluated.

Receiver's performance under such a perturbation is related to perturbation $\delta \boldsymbol{f}$ as follows

$$\widetilde{\text{SINR}}_{rpor} \approx \sigma_{w_1}^2 \frac{\|\boldsymbol{f}_{rpor}^H \boldsymbol{h}_1\|^2 + E\{\delta \boldsymbol{f}^H \boldsymbol{h}_1 \boldsymbol{h}_1^H \delta \boldsymbol{f}\}}{\boldsymbol{f}_{rpor}^H \boldsymbol{R}_i \boldsymbol{f}_{rpor} + E\{\delta \boldsymbol{f}^H \boldsymbol{R}_i \delta \boldsymbol{f}\}} \tag{20}$$

where $\boldsymbol{R}_i = \boldsymbol{R} - \sigma_{w_1}^2 \boldsymbol{h}_1 \boldsymbol{h}_1^H$ is an interference matrix. In (20), $E\{\delta \boldsymbol{f}^H \boldsymbol{h}_1 \boldsymbol{h}_1^H \delta \boldsymbol{f}\}$ and $E\{\delta \boldsymbol{f}^H \boldsymbol{R}_i \delta \boldsymbol{f}\}$ are required. However, a general form $E\{\delta \boldsymbol{f}^H \boldsymbol{X} \delta \boldsymbol{f}\}$ can be first evaluated and then replace $\boldsymbol{X}$ by either $\boldsymbol{h}_1 \boldsymbol{h}_1^H$ or $\boldsymbol{R}_i$. From the expression of the receiver, $\delta \boldsymbol{f}$ is related to $\delta \boldsymbol{R}$ by

$$\delta \boldsymbol{f} \approx \sum_{k=1}^{m} \boldsymbol{Q}_k \delta \boldsymbol{R} \boldsymbol{t}_k - \boldsymbol{R}^{-1} \delta \boldsymbol{R} \boldsymbol{f}_{rpor} \tag{21}$$

where

$$\boldsymbol{Q}_k = \boldsymbol{R}^{-1} \boldsymbol{C}_1 \boldsymbol{T}_k.$$

Then we can find that

$$\begin{aligned} E\{\delta \boldsymbol{f}^H \boldsymbol{X} \delta \boldsymbol{f}\} &\approx \sum_{k_1, k_2} \boldsymbol{t}_{k_1}^H E\{\delta \boldsymbol{R} \boldsymbol{Q}_{k_1}^H \boldsymbol{X} \boldsymbol{Q}_{k_2} \delta \boldsymbol{R}\} \boldsymbol{t}_{k_2} \\ &+ \boldsymbol{f}_{rpor}^H E\{\delta \boldsymbol{R} \boldsymbol{R}^{-1} \boldsymbol{X} \boldsymbol{R}^{-1} \delta \boldsymbol{R}\} \boldsymbol{f}_{rpor} \\ &- \sum_{k} \boldsymbol{t}_k^H E\{\delta \boldsymbol{R} \boldsymbol{Q}_k^H \boldsymbol{X} \boldsymbol{R}^{-1} \delta \boldsymbol{R}\} \boldsymbol{f}_{rpor} \\ &- \boldsymbol{f}_{rpor}^H \sum_{k} E\{\delta \boldsymbol{R} \boldsymbol{R}^{-1} \boldsymbol{X} \boldsymbol{Q}_k \delta \boldsymbol{R}\} \boldsymbol{t}_k \end{aligned} \tag{22}$$

Invoking *Proposition*, evaluation of (22) follows and finally (20).

V. Simulation

Performance of the proposed regularized POR method is tested and compared with its precedents: MV, regularized MV and POR methods in terms of channel MSEs and SINRs of receivers. A CDMA system with 6 users is simulated. Each user is assigned a Gold spreading sequence of length 31 and transmits ±1 signals through an individual multipath channel. The desired user is synchronized while all interfering users have propagation delays uniformly distributed over one symbol period. For each user, an equally powered Gaussian channel of length 15 is generated in each realization. The average results from 500 independent channel realizations for a large range of input SNRs are recorded. For convenience, define $\alpha = \beta\sigma_v^2$. Then β is allowed to take values in $[0, 1)$. Figs. 1 and 2 show performance of the MV [3] and regularized MV [7] channel estimators and receivers. In Fig. 1, dashed lines are for the analytical results, dashed-dotted line for the MV method (without regularization, i.e., $\beta = 0$), and totally 6 solid lines are for the regularized MV method corresponding to different βs defined in the figure caption. It is observed that the experimental results converge to the analytical ones at higher SNR for larger β for the regularized MV method since $(\sigma_v^2 - \alpha)$ becomes smaller. For comparison in Fig. 2, SINR of the MMSE receiver is also presented. The dashed line is for the MMSE receiver, dashed-dotted line for the MV receiver, and solid lines are for the regularized MV receivers corresponding to different βs. Improvements with increasing β is clear. For the POR and proposed regularized POR methods with $m = 2$, Fig. 3 shows the channel MSEs and Fig. 4 presents the receivers' performance. Solid lines are for the proposed regularized POR method with different βs, dashed-dotted lines are for the POR method [8], dashed lines are either for the analytical MSEs or for the SINR of the MMSE receiver. To gain more insights, results for $m = 3$ are plotted in Figs. 5 and 6 respectively. Some observations can be made from these figures. Firstly, regularization significantly improves performance of a receiver in each category. As β (or equivalently α) increases, the corresponding receiver tends to converge to the MMSE receiver. Without regularization, there is a significant gap for all SNRs for the POR receiver for low SNRs with small ms currently adopted. Secondly, similarly as predicted for POR receivers in [8], improvement with respect to increasing m becomes gradual for $m > 2$ when Fig. 4 and Fig. 6 are compared. However, significant improvement is observed when m increases from 1 to 2. Thirdly, the point at which convergence to the MMSE receiver occurs shifts to low SNRs either for a fixed β when m increases from 1 to 3, or for a fixed m when β increases.

VI. Conclusion

A regularized POR method for channel estimation and multiuser detection is proposed. It is based on the previously presented MV, regularized MV and POR methods. By regularization, the current method exhibits much better performance and fast convergence to either the subspace channel estimator or the MMSE receiver, especially in the low SNR region.

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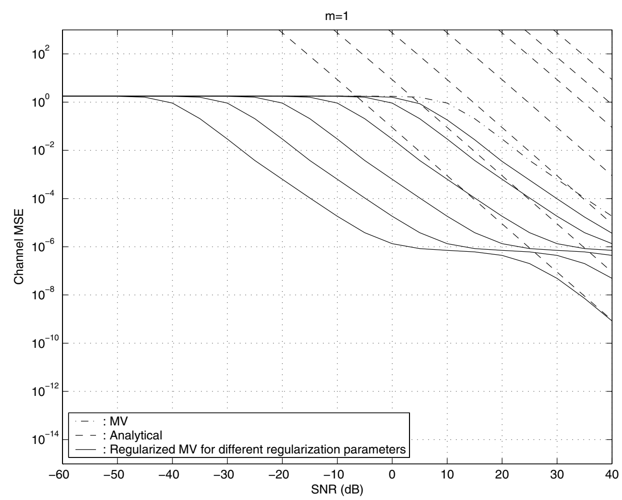

Fig. 1. Channel estimation performance of MV and regularized MV methods. Solid lines from right to left correspond to $\beta = 0.7$, 1-10^{-1}, 1-10^{-2}, 1-10^{-3}, 1-10^{-4}, 1-10^{-5} successively.

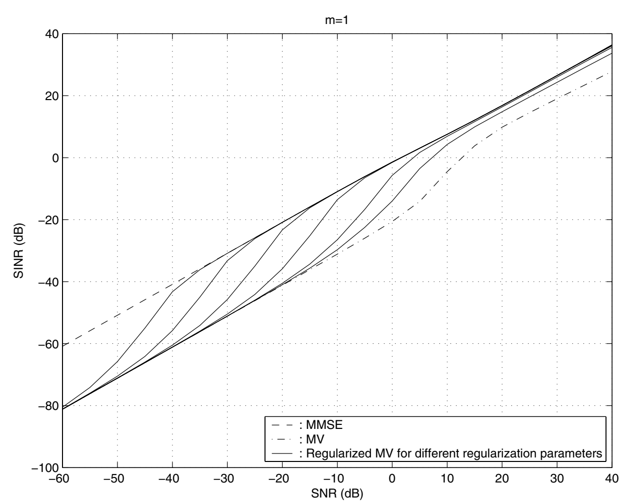

Fig. 2. Performance of MV, regularized MV and MMSE receivers. Correspondence of solid lines to β is described in Fig. 1.

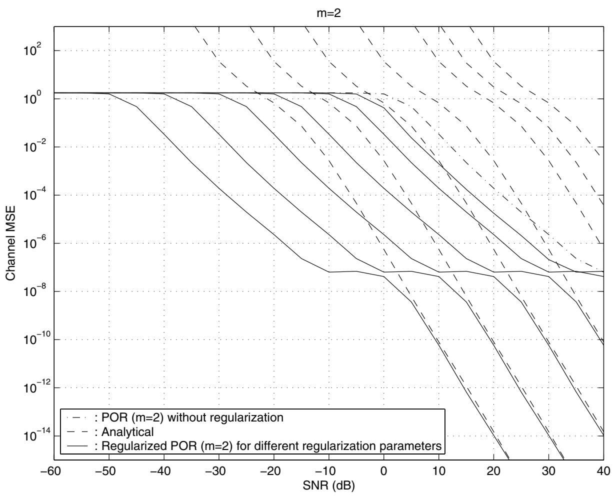

Fig. 3. Channel estimation performance of POR and regularized POR methods with $m = 2$. Correspondence of solid lines to β is described in Fig. 1.

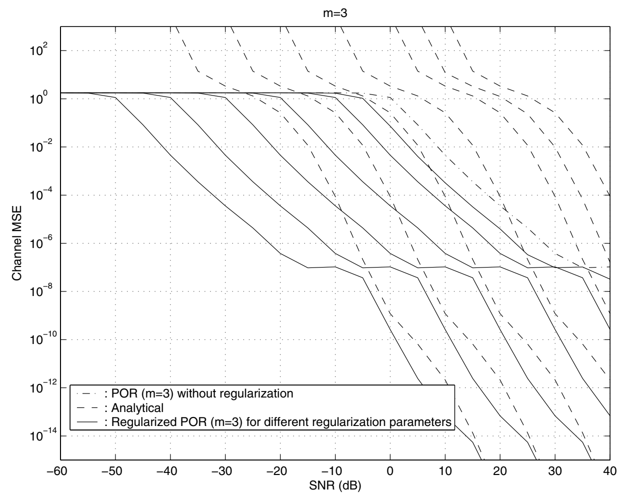

Fig. 5. Channel estimation performance of POR and regularized POR methods with $m = 3$. Correspondence of solid lines to β is described in Fig. 1.

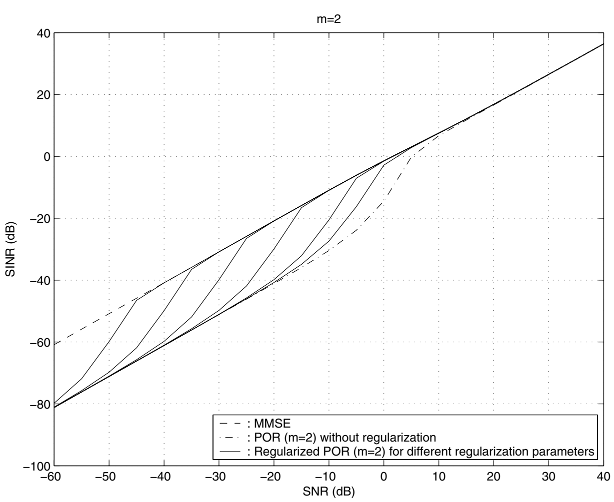

Fig. 4. Performance of POR, regularized POR for $m = 2$ and MMSE receivers. Correspondence of solid lines to β is described in Fig. 1.

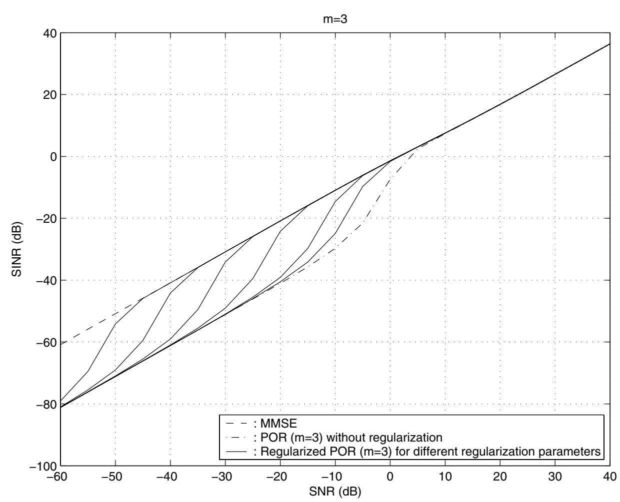

Fig. 6. Performance of POR, regularized POR for $m = 3$ and MMSE receivers. Correspondence of solid lines to β is described in Fig. 1.