SUBSPACE MULTIUSER RECEIVERS FOR UWB COMMUNICATION SYSTEMS

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ABSTRACT

In a typical impulse radio ultra-wideband (UWB) communication environment, transmitted signal carries user's information in pulse positions and undergoes fading from a number of propagation paths. Conventional RAKE receiver may explore path diversity for better performance, but is unable to suppress multiple access interference. Multiuser receiver can significantly improve detection performance. However, it requires channel state information. To seek this information, we adopt a channel input/output model that exhibits a structure similar to a code-division multiple access system. In particular, code matrices for each user can be defined from its unique time-hopping sequence. Then subspace technique is applied to estimate each channel after some necessary modification due to particular formats of defined inputs. Subsequently, zero-forcing and minimum mean-square-error receivers are designed, applicable for both uplink and downlink, and different from existing multiuser detection (MUD) methods that assume perfect channel knowledge.

1. INTRODUCTION

With recent release of spectral mask from the Federal Communications Commission, communication society has witnessed an increasing interest in impulse radio (IR) ultra-wideband (UWB) technology [1]. A conventional IR system transmits trains of time-hopping (TH) short-duration pulses with a low duty cycle and uses pulse position modulation (PPM). Therefore, multipath down to path delay differentials in nanosecond is resolvable, significantly mitigating multipath distortion and providing path diversity [2].

In an UWB system, a conventional RAKE receiver consists of multiple waveform correlators [1]. To fully capture signal energy spread over multiple paths, the receiver needs to know channel parameters when correlation is performed. However, in a dense multipath wireless environment, channel information is not known *a priori*. Although channel parameters can be estimated by maximum likelihood (ML) methods [2], [3], multiple access interference (MAI) is approximated as a Gaussian process which may not be accurate. New channel estimation methods with explicit consideration of MAI need to be developed for construction of multiuser receivers [4].

In this paper, we focus on multiple access channel estimation based on the second order statistics (SOS) of the received signal in order to construct linear receivers. SOS can be easily estimated from data with low complexity and fast convergence. They have been employed in acquisition of the arrival time of the first path of UWB channels [5], linear detection of input symbols when channels are given [4], [6], [7]. First, an UWB system is shown to follow a similar model as a direct sequence (DS) code-division multiple access (CDMA) system [4]. Multiple (M corresponding to the modulation level) inputs originated from the same user information can be regarded as a rate-M user in a multirate system. Code matrices can be clearly defined for each user from its unique time-hopping (TH) sequence, like code matrices constructed from spreading codes in a multirate CDMA system [8]. But they consist of only zeros and ones, indicating existence of path contributions to the received signal from a multipath channel. Locations of zeros and ones are different for different users. Then, using a subspace concept [8], [9] aided by unique code matrices, corresponding channel vectors for all users can be estimated. Since " multi-rate" signals are dependent under such a modeling and received signal has non-zero mean, novel adaptation of subspace technique is necessary. Corresponding channel estimation performance is studied in detail when SOS are estimated from finite data samples. With estimated channels, linear receivers are then designed such as zero-forcing (ZF) and mean-square-error (MMSE) receivers. In building a MMSE receiver, either direct matrix inversion (DMI) or subspace receiver can be considered [9]. However, subspace MMSE receiver shows better performance in general in practical conditions. Bit-error-rate of a linear receiver is derived for a M-ary PPM.

Some notations following common practice are adopted throughout the paper. We denote Kronecker product [10] by $\otimes$, complex conjugate (*) transpose (^{T}) by ^{H}, inverse by $^{-1}$, pseudo-inverse by †, trace of a matrix by $tr()$, determinant by $det()$. $Re\{\cdot\}$ represents real part, $E\{\cdot\}$ expectation, $\boldsymbol{I}_a$ an identity matrix of degree a whose ith column is denoted by $\boldsymbol{e}_{a,i}$. $\mathbf{1}_a$ is a vector of length a with all elements equal to one. An estimate of a quantity (scalar, vector or matrix) is denoted by putting a $\widehat{}$ over it, and correspondingly, the estimation error by preceding the quantity with a δ, such as $\widehat{\boldsymbol{X}}$ and $\delta\boldsymbol{X}$ for matrix $\boldsymbol{X}$ respectively. Meanwhile without confusing, we also use $\delta(\cdot)$ to represent a discrete-time unit impulse function. A Q function $Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^\infty e^{\frac{-t^2}{2}}dt$ will be used in analyzing detection performance.

2. DISCRETE-TIME UWB SYSTEM

Assume there are K users simultaneously sharing the spectrum in a multiple access (MA) TH UWB system. The transmitted baseband UWB signal from user k can be described by [4]

$$\alpha_k(t) = \sqrt{\mathcal{P}_k} \sum_{i=-\infty}^{\infty} w(t - iT_f - c_k(i)T_c - \tau_{I_k(\lfloor i/N_f \rfloor)}) \quad (1)$$

where $\mathcal{P}_k$ is the kth user's transmission power, $w(t)$ is the baseband monopulse, T_f is the frame duration, N_f is the number of frames over which an M-ary PPM symbol repeats, $c_k(i) \in [0, N_c - 1]$ is a periodic hopping sequence with period equal to one symbol period. Each chip has duration T_c. $I_k(\lfloor i/N_f \rfloor) \in [0, M-1]$ is the kth user's information bearing symbol during the ith frame, $\tau_{I_k(\lfloor i/N_f \rfloor)} = I_k(\lfloor i/N_f \rfloor)\sigma$ is the corresponding modulation delay in a multiples of σ seconds. Assume $T_f = N_c T_c$ and $T_c = M\sigma$. If we define $w_m(t) \triangleq w(t - m\sigma)$ where $m = 0, \cdots, M-1$ and $s_{k,m}(\lfloor i/N_f \rfloor) = \delta\Big(I_k(\lfloor i/N_f \rfloor) - m\Big)$, then (1) may be expressed by linear modulation in a chip rate as [4]

$$\alpha_k(t) = \sqrt{\mathcal{P}_k} \sum_{i=-\infty}^{\infty} \sum_{m=0}^{M-1} u_{k,m}(i) w_m(t - iT_c) \quad (2)$$

where chip index has replaced frame index in (1),

$$u_{k,m}(i) = s_{k,m}\Big(\lfloor i/(N_c N_f) \rfloor\Big) \widetilde{c}_k(i),$$

$$\widetilde{c}_k(i) = \delta\Big(\lfloor i/N_c \rfloor N_c + c_k(\lfloor i/N_c \rfloor) - i\Big).$$

It is clear according to (2) that input $u_{k,m}(i)$ is modulated by waveform $w_m(t)$ at a chip rate. The transmitted signal $\alpha_k(t)$ propagates through a linear channel with channel $\bar{g}_k(t)$. At the receiver, the channel output is first passed through a matched filter matched to the monopulse $w(t)$. We can define a front-end effective channel including effects from modulated pulse at the transmitter, propagation channel and matched filter at the receiver by $g_{k,m}(t) = w_m(t) \star \bar{g}_k(t) \star w(-t)$ where $\star$ denotes convolution. Considering additive white Gaussian noise (AWGN) $v(t)$ and propagation delay d_k for user k, the output of the matched filter becomes

$$y(t) = \sum_{k,i_1,m} \sqrt{\mathcal{P}_k} u_{k,m}(i_1) g_{k,m}(t - i_1 T_c - d_k) + v(t). \quad (3)$$

Assume each effective channel has length $q\sigma$. Then $y(t)$ is sampled every σ seconds to yield a discrete-time output $y(n) = y(t)|_{t=n\sigma}$. Using the discrete-time version of the effective channel and invoking $T_c = M\sigma$, we obtain a pulse-rate model

$$y(n) = \sum_{k,m} \sum_{i_2=0}^{q} \sqrt{\mathcal{P}_k} u_{k,m}\Big(\frac{n - i_2}{M}\Big) g_{k,m}(i_2) + v(n). \quad (4)$$

Consider P symbol intervals of data samples with corresponding time instants $nMN_cN_f + p$ for $p = 1, \cdots, MPN_cN_f$ and collect them in a big vector $\boldsymbol{y}_n$ of length $\nu = MPN_cN_f$, After noticing our definition of $u_{k,m}(i)$, a vector form data model follows

$$\boldsymbol{y}_n = \sum_{k,m,l} \boldsymbol{C}_{k,l} \boldsymbol{T}_m \boldsymbol{g}_k s_{k,m}(n+l) + \boldsymbol{v}_n \quad (5)$$

where symbol index l takes all integers $-\lceil q/(MN_cN_f) \rceil, \cdots, P-1$, $\boldsymbol{g}_k$ is an unknown channel vector for user k which contains channel coefficients at the pulse rate and power factor $\sqrt{\mathcal{P}_k}$, $\boldsymbol{T}_m = [\boldsymbol{0}, \boldsymbol{I}_q, \boldsymbol{0}]^T$ is a tall selection matrix in order to obtain the mth subchannel from $\boldsymbol{g}_k$ (delayed in $m\sigma$ seconds or equivalently downshifted by m elements), $\boldsymbol{C}_{k,l}$ is a matrix constructed from corresponding $\widetilde{c}_k(i)$ and is uniquely determined by the TH sequence. It consists of only zeros and ones, and repeats from symbol to symbol because the TH sequence has period equal to one symbol interval. This model can be compactly expressed in another form

$$\boldsymbol{y}_n = \sum_{k,l} \boldsymbol{H}_{k,l} \boldsymbol{s}_{k,n,l} + \boldsymbol{v}_n = \boldsymbol{H}\boldsymbol{s}_n + \boldsymbol{v}_n \quad (6)$$

after collecting M inputs in a vector

$$\boldsymbol{s}_{k,n,l} = [s_{k,0}(n+l), \cdots, s_{k,M-1}(n+l)]^T$$

defining a corresponding effective channel matrix

$$\boldsymbol{H}_{k,l} = [\boldsymbol{C}_{k,l}\boldsymbol{T}_0\boldsymbol{g}_k, \cdots, \boldsymbol{C}_{k,l}\boldsymbol{T}_{M-1}\boldsymbol{g}_k]$$

and successively stack such matrices (or vectors) in $\boldsymbol{H}$ (or $\boldsymbol{s}_n$). By employing either structure of (5) or this structure, all channels can be estimated based on a multirate subspace concept [8].

3. SUBSPACE CHANNEL ESTIMATION AND SYMBOL DETECTION

3.1. Zero-mean data

Let us denote the mean of $\boldsymbol{y}_n$ as $\bar{\boldsymbol{y}}$ which can be easily found from our definition of $\boldsymbol{s}_{k,n,l}$. Since noise has zero mean even after the matched filter, we have

$$\bar{\boldsymbol{y}} = \frac{1}{M} \sum_{k,m,l} \boldsymbol{C}_{k,l} \boldsymbol{T}_m \boldsymbol{g}_k = \sum_k \mathcal{C}_k \boldsymbol{g}_k = \mathcal{C}\boldsymbol{g} \quad (7)$$

where all channel vectors are stacked in a big vector $\boldsymbol{g}$. Due to non-zero mean, the autocorrelation of $\boldsymbol{y}_n$ has cross terms $\boldsymbol{g}_{k_1}\boldsymbol{g}_{k_2}^H$ of users k_1 and k_2 and is not convenient for channel estimation. Thus covariance is considered. Define a new zero-mean data vector from $\boldsymbol{y}_n$ as

$$\boldsymbol{z}_n = \boldsymbol{y}_n - \bar{\boldsymbol{y}} = \sum_{k,l} \boldsymbol{H}_{k,l} \boldsymbol{a}_{k,n,l} + \boldsymbol{v}_n = \boldsymbol{H}\boldsymbol{a}_n + \boldsymbol{v}_n \quad (8)$$

where $\boldsymbol{a}_{k,n,l} = \boldsymbol{s}_{k,n,l} - \frac{1}{M}\boldsymbol{1}_M$ all of which are stacked in a big vector $\boldsymbol{a}_n$ with corresponding effective channel matrix defined as $\boldsymbol{H}$. For shorter notation, we denote the information symbol in $\boldsymbol{s}_{k,n,l}$ simply by I after ignoring its time and user dependence. It takes values $0, \cdots, M-1$ with equal probability $\frac{1}{M}$. Then

$$\boldsymbol{a}_{k,n,l} = [\delta(I), \cdots, \delta(I-(M-1))]^T - \frac{1}{M}\boldsymbol{1}_M^T. \quad (9)$$

Denote the covariance of $\boldsymbol{a}_{k,n,l}$ by $\boldsymbol{A} = E\{\boldsymbol{a}_{k,n,l}\boldsymbol{a}_{k,n,l}^T\}$. According to the distribution of I, it can be found that

$$\boldsymbol{A} = \frac{1}{M}\sum_{i=1}^{M} \widetilde{\boldsymbol{e}}_{M,i}\widetilde{\boldsymbol{e}}_{M,i}^T, \quad \widetilde{\boldsymbol{e}}_{M,i} = \boldsymbol{e}_{M,i} - \frac{1}{M}\boldsymbol{1}_M.$$

After simplification, it becomes $\boldsymbol{A} = \frac{1}{M}(\boldsymbol{I}_M - \frac{1}{M}\boldsymbol{1}_M\boldsymbol{1}_M^T)$ which is easily shown to have rank $M-1$ since $\frac{1}{\sqrt{M}}\boldsymbol{1}_M$ is a unitary vector. Thus its eigenvalue decomposition (EVD) has a form $\boldsymbol{A} = \boldsymbol{B}_a\boldsymbol{\Lambda}_a^2\boldsymbol{B}_a^H$ where $\boldsymbol{B}_a$ is of $M \times (M-1)$ and $\boldsymbol{\Lambda}_a$ is a $(M-1)\times(M-1)$ diagonal matrix with all positive entries.

3.2. Channel estimator

The ideal covariance of $\boldsymbol{z}_n$ is then derived to follow

$$\boldsymbol{R} = E\{\boldsymbol{z}_n\boldsymbol{z}_n^H\} = \sum_{k,l}\boldsymbol{H}_{k,l}\boldsymbol{A}\boldsymbol{H}_{k,l}^H + \sigma_v^2\boldsymbol{I}_\nu.$$

Meanwhile, $\boldsymbol{a}_{k,n,l}$ can be whitened and correspondingly (8) becomes

$$\boldsymbol{z}_n = \sum_{k,l}\boldsymbol{H}_{k,l}\boldsymbol{B}_a\boldsymbol{\Lambda}_a\widetilde{\boldsymbol{a}}_{k,n,l} + \boldsymbol{v}_n \quad (10)$$

where $\widetilde{\boldsymbol{a}}_{k,n,l}$ has identity covariance. Assume $\boldsymbol{R}$ is decomposed by EVD as

$$\boldsymbol{R} = \begin{bmatrix} \boldsymbol{U}_s & \boldsymbol{U}_n \end{bmatrix} \begin{bmatrix} \boldsymbol{\Lambda}_s + \sigma_v^2\boldsymbol{I}_\xi & \boldsymbol{0} \\ \boldsymbol{0} & \sigma_v^2\boldsymbol{I}_{\nu-\xi} \end{bmatrix} \begin{bmatrix} \boldsymbol{U}_s^H \\ \boldsymbol{U}_n^H \end{bmatrix} \quad (11)$$

where $\boldsymbol{\Lambda}_s = diag\{\lambda_1^2, \ \ldots \ , \lambda_\xi^2\}$, $\boldsymbol{U}_s$ and $\boldsymbol{U}_n$ represent the signal and noise subspaces respectively. Based on orthogonality principle, $\boldsymbol{U}_n^H\boldsymbol{H}_{k,l}\boldsymbol{B}_a\boldsymbol{\Lambda}_a = \boldsymbol{0}$ or equivalently $\boldsymbol{U}_n^H\boldsymbol{H}_{k,l}\boldsymbol{B}_a = \boldsymbol{0}$ for all possible k and l. Denoting the (i,j)th element of $\boldsymbol{B}_a$ by $b_{i,j}$. Then we have

$$\boldsymbol{U}_n^H[\boldsymbol{C}_{k,l}\boldsymbol{T}_0\boldsymbol{g}_k, \cdots, \boldsymbol{C}_{k,l}\boldsymbol{T}_{M-1}\boldsymbol{g}_k]\boldsymbol{B}_a = \boldsymbol{0}$$

which can be expanded column by column as

$$\boldsymbol{U}_n^H\boldsymbol{D}_{k,l,j}\boldsymbol{g}_k = \boldsymbol{0}, \quad j = 1, \cdots, M-1 \quad (12)$$

where $\boldsymbol{D}_{k,l,j} = \sum_{i=1}^M b_{i,j}\boldsymbol{C}_{k,l}\boldsymbol{T}_{i-1}$. Therefore we can design the following channel estimation criterion for user k by minimizing total projection error

$$\widehat{\boldsymbol{g}}_k = \arg\min \sum_{l,j} ||\boldsymbol{U}_n^H\boldsymbol{D}_{k,l,j}\boldsymbol{g}_k||^2. \quad (13)$$

After defining $\boldsymbol{X}_k = \sum_{l,j}\boldsymbol{D}_{k,l,j}^H\boldsymbol{U}_n\boldsymbol{U}_n^H\boldsymbol{D}_{k,l,j}$, (13) becomes

$$\widehat{\boldsymbol{g}}_k = \arg\min \sum_{l,j}\boldsymbol{g}_k^H\boldsymbol{X}_k\boldsymbol{g}_k. \quad (14)$$

$\widehat{\boldsymbol{g}}_k$ is the minimum eigenvector of $\boldsymbol{X}_k$.

3.3. Linear receivers

In order to detect input symbol in $\boldsymbol{a}_{k,n,l}$ which has only one maximum while all others are smaller, we need to design M receivers $\boldsymbol{f}_i$ $(i = 1, \cdots, M)$ with each one corresponding to each element in $\boldsymbol{a}_{k,n,l}$. Then outputs of M receivers are compared and the index of the maximum element is determined. Considering I takes values $0, \cdots, M-1$, our symbol detection criterion can be described as follows

$$I = \arg\max_{i\in\{1,\cdots,M\}} Re\{\boldsymbol{f}_i^H\boldsymbol{z}_n\} - 1.$$

Receiver takes different forms for different types. Consider the current symbol ($l = 0$) and collect all M receivers in a matrix $\boldsymbol{F}_k$ for user k. Based on (8), RAKE receivers are given by

$$\boldsymbol{F}_{k,rake} = \boldsymbol{H}_{k,0}.$$

ZF receivers take forms

$$\boldsymbol{F}_{k,zf} = \boldsymbol{H}(\boldsymbol{H}^H\boldsymbol{H})^{-1}[\boldsymbol{e}_{\nu,2(K-1)M+1}, \ldots, \boldsymbol{e}_{\nu,2(K-1)M+M}].$$

The DMI MMSE receivers can be found as follows after considering the covariance of $\boldsymbol{a}_{k,n,l}$ is $\boldsymbol{A}$

$$\boldsymbol{F}_{k,mmse-dmi} = \boldsymbol{R}^{-1}\boldsymbol{H}_{k,0}\boldsymbol{A}.$$

The subspace MMSE receivers can also be easily derived [9]. Since $\boldsymbol{U}_n^H\boldsymbol{H}_{k,0}\boldsymbol{A} = \boldsymbol{0}$, according to (11), we obtain

$$\boldsymbol{F}_{k,mmse-sub} = \boldsymbol{U}_s(\boldsymbol{\Lambda}_s + \sigma_v^2\boldsymbol{I}_\xi)^{-1}\boldsymbol{U}_s^H\boldsymbol{H}_{k,0}\boldsymbol{A}.$$

Performance of subspace channel estimator and receivers will be studied next.

4. PERFORMANCE ANALYSIS

It is found that both channel estimator and receivers depend on the data covariance matrix $\boldsymbol{R}$. In practical conditions, together with the mean of received data vector, it is often estimated from N data vectors as follows

$$\widehat{\boldsymbol{R}} = \frac{1}{N}\sum_{n=1}^{N}(\boldsymbol{y}_n - \widehat{\boldsymbol{y}})(\boldsymbol{y}_n - \widehat{\boldsymbol{y}})^H, \ \widehat{\boldsymbol{y}} = \frac{1}{N}\sum_{n=1}^{N}\boldsymbol{y}_n. \quad (15)$$

The sample size N will determine the accuracy of the subspace estimate, thus affect the performance of the estimator. An estimation error occurs $\delta\boldsymbol{R} = \widehat{\boldsymbol{R}} - \boldsymbol{R}$. For large N, it can be regarded as a small perturbation, making perturbation technique applicable [11].

4.1. Channel estimation performance

For notational convenience, let $\boldsymbol{Z}$ be the noise-free data covariance matrix $\boldsymbol{Z} = \boldsymbol{R} - \sigma_v^2 \boldsymbol{I}_\nu = \boldsymbol{U}_s \boldsymbol{\Lambda}_s \boldsymbol{U}_s^H$. Then perturbation of $\boldsymbol{U}_s$ has the following form [11]

$$\delta \boldsymbol{U}_n \approx -\boldsymbol{Z}^\dagger \delta \boldsymbol{R} \boldsymbol{U}_n. \tag{16}$$

Because channel estimate is the minimum eigenvector of $\boldsymbol{X}_k$, $\delta \boldsymbol{U}_n$ causes an error $\delta \boldsymbol{X}_k$. Then $\delta \boldsymbol{g}$ has the following form [8], [11]

$$\delta \boldsymbol{g}_k \approx -\boldsymbol{X}_k^\dagger \delta \boldsymbol{X}_k \boldsymbol{g}_k. \tag{17}$$

According to our definition of $\boldsymbol{X}_k$, $\delta \boldsymbol{X}_k$ is given by

$$\delta \boldsymbol{X}_k \approx \sum_{l,j} \boldsymbol{D}_{k,l,j}^H [\delta \boldsymbol{U}_n \boldsymbol{U}_n^H + \boldsymbol{U}_n \delta \boldsymbol{U}_n^H] \boldsymbol{D}_{k,l,j}. \tag{18}$$

Substituting (16) in (18) then (18) in (17), and noticing (12), $\delta \boldsymbol{g}_k$ is related to random matrix $\delta \boldsymbol{R}$ by

$$\delta \boldsymbol{g}_k \approx \sum_{l,j} \boldsymbol{X}_k^\dagger \boldsymbol{D}_{k,l,j}^H \boldsymbol{U}_n \boldsymbol{U}_n^H \delta \boldsymbol{R} \boldsymbol{Z}^\dagger \boldsymbol{D}_{k,l,j} \boldsymbol{g}_k. \tag{19}$$

The covariance becomes

$$\mathrm{COV}(\delta \boldsymbol{g}_k) \approx \sum_{l_1,l_2,j_1,j_2} \boldsymbol{M}_{l_1,j_1} E\{\delta \boldsymbol{R} \boldsymbol{Y} \delta \boldsymbol{R}\} \boldsymbol{M}_{l_2,j_2}^H \tag{20}$$

$$\boldsymbol{M}_{l,j} = \boldsymbol{X}_k^\dagger \boldsymbol{D}_{k,l,j}^H \boldsymbol{U}_n \boldsymbol{U}_n^H$$

$$\boldsymbol{Y} = \boldsymbol{Z}^\dagger \boldsymbol{D}_{k,l_1,j_1} \boldsymbol{g} \boldsymbol{g}^H \boldsymbol{D}_{k,l_2,j_2}^H \boldsymbol{Z}^\dagger.$$

The mean-square-error (MSE) is then $tr(\mathrm{COV}(\delta \boldsymbol{g}_k))$. To evaluate either of them, it suffices to determine a general-form quantity $E\{\delta \boldsymbol{R} \boldsymbol{\Theta} \delta \boldsymbol{R}\}$ for an arbitrary matrix $\boldsymbol{\Theta}$. Although results for a system with white inputs have been derived in [12], unfortunately our current inputs do not satisfy that condition. Therefore, new results need to be derived by following procedures therein. For convenience, let us partition matrix $\boldsymbol{H}$ in (8) into L sub-blocks as $\boldsymbol{H} = [\boldsymbol{H}_1, \cdots, \boldsymbol{H}_L]$ where each sub-block corresponds to one symbol irrespective of user. Then $L = K(P + \lceil q/(MN_cN_f) \rceil)$. We present the following results while omitting tedious but straightforward proof due to lack of space.

Proposition: If channel model follows (6) and data covariance is estimated from N independent data vectors as (15), then for a real system (all quantities are real),

$$\begin{aligned}
E\{\delta \boldsymbol{R} \boldsymbol{\Theta} \delta \boldsymbol{R}\} = & \frac{(N-1)^2}{N^3} \sum_{l=1}^{L} \frac{1}{M} \sum_{j=1}^{M} (\widetilde{\boldsymbol{h}}_{l,j}^T \boldsymbol{\Theta} \widetilde{\boldsymbol{h}}_{l,j}) \widetilde{\boldsymbol{h}}_{l,j} \widetilde{\boldsymbol{h}}_{l,j}^T \\
& - \frac{(N-1)^2}{N^3} \sum_{l=1}^{L} tr(\boldsymbol{H}_l \boldsymbol{A} \boldsymbol{H}_l^T \boldsymbol{\Theta}) \boldsymbol{H}_l \boldsymbol{A} \boldsymbol{H}_l^T \\
& - \frac{(N-1)^2}{N^3} \sum_{l=1}^{L} \boldsymbol{H}_l \boldsymbol{A} (\boldsymbol{H}_l^T \boldsymbol{\Theta} \boldsymbol{H}_l + \boldsymbol{H}_l^T \boldsymbol{\Theta}^T \boldsymbol{H}_l) \boldsymbol{A} \boldsymbol{H}_l^T \\
& + \frac{N-1}{N^2} [tr(\boldsymbol{R} \boldsymbol{\Theta}) \boldsymbol{R} + \boldsymbol{R} \boldsymbol{\Theta}^T \boldsymbol{R}] + \frac{1}{N^2} \boldsymbol{R} \boldsymbol{\Theta} \boldsymbol{R}
\end{aligned}$$

while for complex channel and noise,

$$\begin{aligned}
E\{\delta \boldsymbol{R} \boldsymbol{\Theta} \delta \boldsymbol{R}\} = & \frac{(N-1)^2}{N^3} \sum_{l=1}^{L} \frac{1}{M} \sum_{j=1}^{M} (\widetilde{\boldsymbol{h}}_{l,j}^H \boldsymbol{\Theta} \widetilde{\boldsymbol{h}}_{l,j}) \widetilde{\boldsymbol{h}}_{l,j} \widetilde{\boldsymbol{h}}_{l,j}^H \\
& - \frac{(N-1)^2}{N^3} \sum_{l=1}^{L} tr(\boldsymbol{H}_l \boldsymbol{A} \boldsymbol{H}_l^H \boldsymbol{\Theta}) \boldsymbol{H}_l \boldsymbol{A} \boldsymbol{H}_l^H \\
& - \frac{(N-1)^2}{N^3} \sum_{l=1}^{L} \boldsymbol{H}_l \boldsymbol{A} (\boldsymbol{H}_l^H \boldsymbol{\Theta} \boldsymbol{H}_l + \boldsymbol{H}_l^T \boldsymbol{\Theta}^T \boldsymbol{H}_l^*) \boldsymbol{A} \boldsymbol{H}_l^H \\
& + \frac{N-1}{N^2} [tr(\boldsymbol{R} \boldsymbol{\Theta}) \boldsymbol{R} + \boldsymbol{H} \mathcal{A} \boldsymbol{H}^T \boldsymbol{\Theta}^T \boldsymbol{H}^* \mathcal{A} \boldsymbol{H}^H] \\
& + \frac{1}{N^2} \boldsymbol{R} \boldsymbol{\Theta} \boldsymbol{R}
\end{aligned}$$

where $\widetilde{\boldsymbol{h}}_{l,j} = \boldsymbol{H}_l \widetilde{\boldsymbol{e}}_{M,j}$ for shorter notation.

It can be observed that the above results are different from [12] because of different distributions of inputs.

4.2. Detection performance

Without loss of generality, assume user 1 is the desired user and its information $I = 0$ is transmitted. We only focus on a real system although we will still use ^{H} instead of ^{T} for consistency next. Our data vector becomes $\boldsymbol{z}_n = \widetilde{\boldsymbol{h}}_{1,1} + \boldsymbol{u}_n$. $\boldsymbol{u}_n$ includes all interference with covariance $\boldsymbol{R}_u$ and is approximated as a Gaussian process for convenience of analysis. Denote M receivers simply by $\boldsymbol{f}_j$ for $j = 1, \cdots, M$. It can represent any linear receiver presented before. Then the event of right detection becomes

$$\{\boldsymbol{f}_1^H \boldsymbol{z}_n > \boldsymbol{f}_j^H \boldsymbol{z}_n, j = 2, \cdots, M\} = \{\Delta \boldsymbol{f}_j^H \boldsymbol{z}_n > 0\}$$

where $\Delta \boldsymbol{f}_j = \boldsymbol{f}_1 - \boldsymbol{f}_j$. Define a $(M-1)$ dimensional random vector $\boldsymbol{x}_n = \Delta \boldsymbol{F}^H \boldsymbol{z}_n$ where $\Delta \boldsymbol{F}$ contains all $\Delta \boldsymbol{f}_j$ as columns. Since $\boldsymbol{z}_n$ is assumed Gaussian distributed, $\boldsymbol{x}_n$ is also Gaussian. We can find its probability density function

$$f_{\boldsymbol{x}} = \frac{e^{-\frac{1}{2}(\boldsymbol{x}_n - \Delta \boldsymbol{F}^H \widetilde{\boldsymbol{h}}_{1,1})^H (\mathrm{COV}(\boldsymbol{x}))^{-1} (\boldsymbol{x}_n - \Delta \boldsymbol{F}^H \widetilde{\boldsymbol{h}}_{1,1})}}{\sqrt{(2\pi)^{M-1} det(\mathrm{COV}(\boldsymbol{x}))}}$$

where $\mathrm{COV}(\boldsymbol{x}) = \Delta \boldsymbol{F}^H \boldsymbol{R}_u \Delta \boldsymbol{F}$ is the covariance of $\boldsymbol{x}_n$. Then, probability of detection error becomes $BER_0 = 1 - Prob\{\boldsymbol{x}_n > \boldsymbol{0}\} = 1 - \int \cdots \int_0^\infty f_{\boldsymbol{x}} d\boldsymbol{x}$. It can be numerically evaluated. Similarly, we can find the BER when other symbols $I = 1, \cdots, M-1$ are transmitted, denoted as $BER_1, \cdots, BER_{M-1}$. Then the average probability of error becomes $BER = \frac{1}{M} \sum_{m=0}^{M-1} BER_m$. In a special case when $M = 2$, results can be simplified as $BER_0 = Q(\frac{\Delta \boldsymbol{f}_1^H \widetilde{\boldsymbol{h}}_{1,1}}{\sigma_1})$ and $BER_1 = Q(\frac{\Delta \boldsymbol{f}_2^H \widetilde{\boldsymbol{h}}_{1,2}}{\sigma_2})$ where $\Delta \boldsymbol{f}_2 = \boldsymbol{f}_2 - \boldsymbol{f}_1 = -\Delta \boldsymbol{f}_1$, $\sigma_j^2 = \Delta \boldsymbol{f}_j^H \boldsymbol{R}_u \Delta \boldsymbol{f}_j$ for $j = 1, 2$. Since it can be shown that $\widetilde{\boldsymbol{h}}_{1,1} = -\widetilde{\boldsymbol{h}}_{1,2}$, we conclude that $BER_0 = BER_1$ as expected. Then $BER = \frac{1}{2}(BER_0 + BER_1)$.

5. SIMULATION

Performance of the proposed channel estimator and four linear receivers are evaluated. System parameters are set to be $N_c = 8$, $N_f = 4$, $M = 2$, $K = 8$. Each user's time hopping codes and 16-path Gaussian channel spread over one frame are randomly generated once and fixed for all realizations. The received signal is the second derivative of the Gaussian function with pulse width equal to 0.7 ns [1]. Simulation results are based on 100 independent realizations. User 1 is assumed to be the desired user, and the receiver is assumed to be synchronized to the desired user. Fig. 1 illustrates effect of data length N on channel MSE at 15dB SNR. As expected, MSE decreases as N increases, and the experimental MSE curve converges to the analytical one for large N. Fig. 2 (a) $\sim$ (d) show four linear receivers' BER performance with respect to SNR and data length N. Subspace MMSE receiver and ZF receiver have similar and the best performance. RAKE receiver shows poor performance since it can not remove multiple access interference effectively. On the other hand, larger N yields better BER performance than smaller N at every SNR examined and for each receiver. Moreover, except the DMI MMSE receiver, all other receivers with $N = 3000$ show very close performance to their ideal counterparts constructed from perfect channel information and noise power, and also close performance to their respective analytical BER bounds computed according to previous section. The data based DMI MMSE receivers show poor performance and diverge from both the ideal DMI MMSE and the corresponding analytical BER bound, due to large perturbation in inverse of the estimated covariance matrix.

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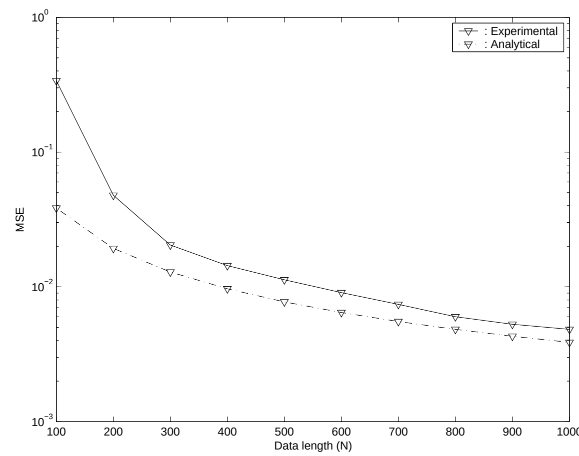

Fig. 1. Channel estimation error.

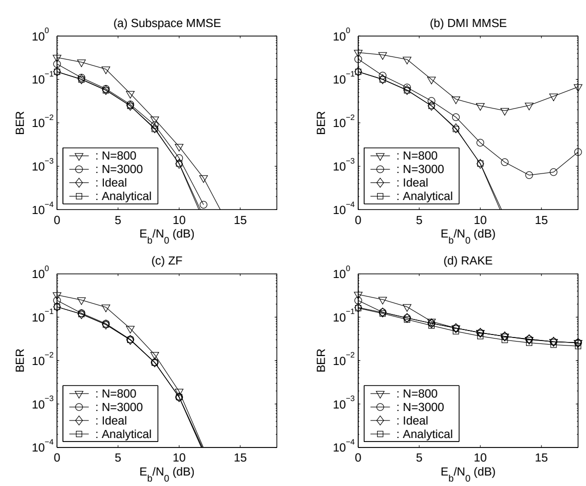

Fig. 2. BER performance for different receivers.