MINIMUM VARIANCE MULTIUSER DETECTION FOR IMPULSE RADIO UWB SYSTEMS

Ping Liu, Zhengyuan Xu, Jin Tang
Department of Electrical Engineering
University of California
Riverside, CA 92521
{pliu,dxu,jtang}@ee.ucr.edu

ABSTRACT

The minimum variance (MV) technique minimizes output variance of a receiver subject to a constraint which guarantees no cancellation of the desired signal while interference is mitigated. It has been successfully applied to multiuser detection for a direct sequence code-division multiple access (CDMA) system even when multipath channel is unknown. Its applicability in multiple access ultra-wideband (UWB) systems is investigated in this paper. First, similarities between a time-hopping (TH) UWB system using pulse position modulation (PPM) and a multirate CDMA system are explored where a code matrix for each user can be defined based on its unique TH sequence, similar to a spreading code matrix in a CDMA system. Meanwhile, modulation delays are transformed to amplitudes of pulses, yielding a virtually linear model. After following MV optimization procedures, the desired signal can be detected from outputs of a bank of receivers. Performance of the proposed detection criterion is analyzed and verified.

1. INTRODUCTION

Recently there emerges considerable interest in studying and deploying time-hopping (TH) ultra-wideband (UWB) communication systems due to their appealing features [1] and recent release of the spectral mask from the Federal Communications Commission [2]. UWB technology is an ideal candidate for secure low-power multiuser communications. It offers exceptional multipath resolvability, robustness to jamming, low probability of interception and detection [3].

With on-going widespread deployment of UWB systems, reliable signal detection is desirable. Most existing approaches employ correlators to correlate received signal with a template signal [4], [5]. This technique appears very powerful, but not so satisfactory in a multipath and multiple access channel. Therefore, both unknown multiple access interference (MAI) and multipath distortion need to be mitigated. Although multiuser detection (MUD) techniques can be directly applied to a direct sequence (DS) code-division multiple access (CDMA) based UWB system [6], their applicability is not trivial to an UWB system employing pulse position modulation (PPM). Some work has appeared for perfect channel [7] or multipath but known channel [8], [9], [10]. However no MUD method exists in the presence of previously mentioned unknown interference. Since modulation delay causes non-linearity of the system, which is not easy for signal processing, we thus first transform the UWB channel input/output model to a linearly modulated system following [8] at a pulse rate sampling. The new "linear" model casts the modulation delay into amplitudes of newly defined pulses. The TH sequence uniquely specifies a "code" matrix for each user that only contains zeros and ones to indicate whether contribution of the channel exists or not. Then received data is linearly dependent on amplitude, code matrix and channel in a tri-linear form. If we treat "code" matrix to be in a similar role as code matrix in a CDMA system [11], then the model will be shown to be similar to a multi-code multirate CDMA system [12], leading to possible application of the MV technique [11], [13]. By using "code" matrices to distinguish users, multirate receivers can be designed for the desired user without a need to know its channel parameters.

In this paper, we use lower case boldfaced letters for vectors, upper case boldfaced letters for matrices. Denote transpose by ^{T}, inverse by $^{-1}$, pseudo-inverse by † and determinant by $det(\cdots)$. $E\{\cdot\}$ represents expectation of a random variable, $\boldsymbol{I}_a$ an identity matrix of degree a whose ith column is denoted by $\boldsymbol{e}_{a,i}$. $\mathbf{1}_a$ is a vector of length a with all elements equal to one. $\delta(\cdot)$ is a discrete-time unit impulse function. $Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^\infty e^{\frac{-t^2}{2}}dt$ is a Q function. $\lfloor\cdot\rfloor$ stands for integer floor, while $\lceil\cdot\rceil$ for integer ceiling.

2. DISCRETE-TIME UWB MODEL

Assume there are K users simultaneously sharing the spectrum in a multiple access (MA) TH UWB system. The transmitted baseband UWB signal from user k can be described by [8]

$$\alpha_k(t) = \sqrt{\mathcal{P}_k}\sum_{i=-\infty}^{\infty} w(t-iT_f-c_k(i)T_c-\tau_{I_k(\lfloor i/N_f\rfloor)}) \quad (1)$$

where $\mathcal{P}_k$ is the kth user's transmission power, $w(t)$ is the baseband monopulse, T_f is the frame duration, N_f is the number of frames over which an M-ary PPM symbol repeats, $c_k(i) \in [0, N_c-1]$ is a periodic hopping sequence with period equal to one symbol period. Each chip has duration T_c. $I_k(\lfloor i/N_f\rfloor) \in [0, M-1]$ is the kth user's information bearing symbol during the ith frame, $\tau_{I_k(\lfloor i/N_f\rfloor)} = I_k(\lfloor i/N_f\rfloor)\sigma$ is the corresponding modulation delay in a multiples of σ seconds. Assume $T_f = N_c T_c$ and $T_c = M\sigma$. If we define $w_m(t) \triangleq w(t - m\sigma)$ where $m = 0, \cdots, M-1$ and $s_{k,m}(\lfloor i/N_f \rfloor) = \delta\Big(I_k(\lfloor i/N_f \rfloor) - m\Big)$, then (1) may be expressed by linear modulation in a chip rate as [8]

$$\alpha_k(t) = \sqrt{\mathcal{P}_k} \sum_{i=-\infty}^{\infty} \sum_{m=0}^{M-1} u_{k,m}(i) w_m(t - iT_c) \quad (2)$$

where chip index has replaced frame index in (1),

$$u_{k,m}(i) = s_{k,m}\Big(\lfloor i/(N_c N_f) \rfloor\Big) \widetilde{c}_k(i),$$

$$\widetilde{c}_k(i) = \delta\Big(\lfloor i/N_c \rfloor N_c + c_k(\lfloor i/N_c \rfloor) - i\Big).$$

It is clear according to (2) that input $u_{k,m}(i)$ is modulated by waveform $w_m(t)$ at a chip rate. The transmitted signal $\alpha_k(t)$ propagates through a linear channel with response $\bar{g}_k(t)$. At the receiver, the channel output is first passed through a matched filter matched to the monopulse $w(t)$. We can define a front-end effective channel including effects from modulated pulse at the transmitter, propagation channel and matched filter at the receiver by $g_{k,m}(t) = w_m(t) \star \bar{g}_k(t) \star w(-t)$ where $\star$ denotes convolution. Considering additive white Gaussian noise (AWGN) $v(t)$ and propagation delay d_k for user k, the output of the matched filter becomes

$$y(t) = \sum_{k,i_1,m} \sqrt{\mathcal{P}_k} u_{k,m}(i_1) g_{k,m}(t - i_1 T_c - d_k) + v(t). \quad (3)$$

Assume each effective channel has length $q\sigma$. Then $y(t)$ is sampled every σ seconds to yield a discrete-time output $y(n) = y(t)|_{t=n\sigma}$. Using the discrete-time version of the effective channel and invoking $T_c = M\sigma$, we obtain a pulse-rate model

$$y(n) = \sum_{k,m} \sum_{i_2=0}^{q} \sqrt{\mathcal{P}_k} u_{k,m}\Big(\frac{n - i_2}{M}\Big) g_{k,m}(i_2) + v(n). \quad (4)$$

Consider P symbol intervals of data samples with corresponding time instants $nMN_cN_f + p$ for $p = 1, \cdots, MPN_cN_f$ and collect them in a big vector $\boldsymbol{y}_n$ of length $\nu = MPN_cN_f$, After noticing our definition of $u_{k,m}(i)$, a vector form data model follows

$$\boldsymbol{y}_n = \sum_{k,m,l} \boldsymbol{C}_{k,l} \boldsymbol{T}_m \boldsymbol{g}_k s_{k,m}(n + l) + \boldsymbol{v}_n \quad (5)$$

where symbol index l takes all integers $-\lceil q/(MN_cN_f) \rceil$, $\cdots, P - 1$, $\boldsymbol{g}_k$ is an unknown channel vector for user k which contains channel coefficients at the pulse rate and power factor $\sqrt{\mathcal{P}_k}$, $\boldsymbol{T}_m = [\boldsymbol{0}, \boldsymbol{I}_q, \boldsymbol{0}]^T$ is a tall selection matrix in order to obtain the mth subchannel from $\boldsymbol{g}_k$ (delayed in $m\sigma$ seconds or equivalently downshifted by m elements), $\boldsymbol{C}_{k,l}$ is a matrix constructed from corresponding $\widetilde{c}_k(i)$ and is uniquely determined by the TH sequence. It consists of only zeros and ones and repeats from symbol to symbol because the TH sequence has period equal to one symbol interval. There are totally $L = K(P + \lceil q/(MN_cN_f) \rceil)$ symbols from all users which contribute to the received data. This model can be compactly expressed in another form

$$\boldsymbol{y}_n = \sum_{k,l} \boldsymbol{H}_{k,l} \boldsymbol{s}_{k,n,l} + \boldsymbol{v}_n = \boldsymbol{H}\boldsymbol{s}_n + \boldsymbol{v}_n \quad (6)$$

after collecting M inputs in a vector

$$\boldsymbol{s}_{k,n,l} = [s_{k,0}(n + l), \cdots, s_{k,M-1}(n + l)]^T$$

defining a corresponding effective channel matrix

$$\boldsymbol{H}_{k,l} = [\boldsymbol{C}_{k,l}\boldsymbol{T}_0\boldsymbol{g}_k, \cdots, \boldsymbol{C}_{k,l}\boldsymbol{T}_{M-1}\boldsymbol{g}_k]$$

and successively stacking such matrices (or vectors) in $\boldsymbol{H}$ (or $\boldsymbol{s}_n$). In the next section, we will show how to detect symbols $I_k(\lfloor i/N_f \rfloor$ using MV technique.

3. BLIND MULTIUSER DETECTION IN UNKNOWN MULTIPATH CHANNEL

The MV technique requires variance of the output signal. Let us denote the mean of $\boldsymbol{y}_n$ as $\bar{\boldsymbol{y}}$ which can be easily found from our definition of $\boldsymbol{s}_{k,n,l}$. Since noise has zero mean even after the matched filter, we have

$$\bar{\boldsymbol{y}} = \frac{1}{M} \sum_{k,m,l} \boldsymbol{C}_{k,l} \boldsymbol{T}_m \boldsymbol{g}_k. \quad (7)$$

In order to obtain the covariance of $\boldsymbol{y}_n$, define a new zero-mean data vector $\boldsymbol{z}_n = \boldsymbol{y}_n - \bar{\boldsymbol{y}}$. It has the following form

$$\boldsymbol{z}_n = \sum_{k,l,m} \boldsymbol{C}_{k,l} \boldsymbol{T}_m \boldsymbol{g}_k a_{k,m}(n+l) + \boldsymbol{v}_n = \boldsymbol{H}\boldsymbol{a}_n + \boldsymbol{v}_n \quad (8)$$

where $a_{k,m}(n+l) = s_{k,m}(n+l) - \frac{1}{M}$. For shorter notation, we denote the information symbol in $s_{k,m}(n + l)$ simply by I after ignoring its time and user dependence. It takes values $0, \cdots, M - 1$ with equal probability $\frac{1}{M}$. If we define $\boldsymbol{a}_{k,n,l} = \boldsymbol{s}_{k,n,l} - \frac{1}{M}\boldsymbol{1}_M$, then

$$\boldsymbol{a}_{k,n,l} = [\delta(I), \cdots, \delta(I - (M - 1))]^T - \frac{1}{M}\boldsymbol{1}_M^T. \quad (9)$$

To obtain the covariance of $\boldsymbol{z}_n$, it is necessary to find that of $\boldsymbol{a}_{k,n,l}$. We denote it by $\boldsymbol{A} = E\{\boldsymbol{a}_{k,n,l}\boldsymbol{a}_{k,n,l}^T\}$. According to the distribution of I, it can be found that

$$\boldsymbol{A} = \frac{1}{M} \sum_{i=1}^{M} \widetilde{\boldsymbol{e}}_{M,i} \widetilde{\boldsymbol{e}}_{M,i}^T, \quad \widetilde{\boldsymbol{e}}_{M,i} = \boldsymbol{e}_{M,i} - \frac{1}{M}\boldsymbol{1}_M.$$

After simplification, it becomes

$$\boldsymbol{A} = \frac{1}{M}(\boldsymbol{I}_M - \frac{1}{M}\boldsymbol{1}_M\boldsymbol{1}_M^T) \quad (10)$$

which is easily shown to have rank $M-1$ since $\frac{1}{\sqrt{M}}\mathbf{1}_M$ is a unitary vector. The ideal covariance of $\boldsymbol{z}_n$ is then derived as

$$\boldsymbol{R} = E\{\boldsymbol{z}_n\boldsymbol{z}_n^H\} = \sum_{k,l}\boldsymbol{H}_{k,l}\boldsymbol{A}\boldsymbol{H}_{k,l}^T + \sigma_v^2\boldsymbol{I}_\nu.$$

According to our previous analysis, inputs in $\boldsymbol{a}_{k,n,l}$ are correlated with covariance $\boldsymbol{A}$ in (10). They must be whitened before MV technique can be applied,

$$\boldsymbol{a}_{k,n,l} = \boldsymbol{B}_a\boldsymbol{\Lambda}_a\widetilde{\boldsymbol{a}}_{k,n,l} \tag{11}$$

where $\widetilde{\boldsymbol{a}}_{k,n,l}$ is the new $M-1$ uncorrelated input vector with identity correlation and $\boldsymbol{B}_a\boldsymbol{\Lambda}_a^2\boldsymbol{B}_a^H = \boldsymbol{A}$. Let $b_{i,m}$ denote the (i,m)th element of $\boldsymbol{B}_a$ and λ_m denote the mth diagonal element of $\boldsymbol{\Lambda}_a$. If we define $\boldsymbol{S}_{k,m,l} = \sum_{i=1}^M b_{i,m}\lambda_m\boldsymbol{C}_{k,l}\boldsymbol{T}_{i-1}$, then (8) can be rewritten as

$$\boldsymbol{z}_n = \sum_{k,l}\widetilde{\boldsymbol{H}}_{k,l}\widetilde{\boldsymbol{a}}_{k,n,l} + \boldsymbol{v}_n \tag{12}$$

where $\widetilde{\boldsymbol{H}}_{k,l} = [\boldsymbol{S}_{k,1,l}\boldsymbol{g}_k,\ \ldots,\ \boldsymbol{S}_{k,M-1,l}\boldsymbol{g}_k]$. (12) resembles a multi-rate CDMA system [12], where $\boldsymbol{S}_{k,m,l}$ can be treated as a code matrix for the mth input in $\widetilde{\boldsymbol{a}}_{k,n,m}$. From our derived model (12), each time there are $M-1$ virtual inputs in $\widetilde{\boldsymbol{a}}_{k,n,l}$ ($l=0$) to be detected. Therefore, we need to design $M-1$ receivers $\widetilde{\boldsymbol{f}}_{k,m}$ for user k. Since the $M-1$ virtual inputs of user k propagate through the same channel $\boldsymbol{g}_k$ after being spread by individual codes, we propose to minimize the total output power of all receivers subject to a common unknown constraint vector $\boldsymbol{u}_k$ [12]

$$(\widetilde{\boldsymbol{f}}_{k,1},\cdots,\widetilde{\boldsymbol{f}}_{k,M-1}) = \arg\min\sum_{m=1}^{M-1}\widetilde{\boldsymbol{f}}_{k,m}^T\boldsymbol{R}\widetilde{\boldsymbol{f}}_{k,m},$$

$$\text{subject to}\ \ \boldsymbol{S}_{k,m,0}^T\widetilde{\boldsymbol{f}}_{k,m} = \boldsymbol{u}_k. \tag{13}$$

The solution to (13) becomes

$$\widetilde{\boldsymbol{f}}_{k,m} = \boldsymbol{R}^{-1}\boldsymbol{S}_{k,m,0}(\boldsymbol{S}_{k,m,0}^T\boldsymbol{R}^{-1}\boldsymbol{S}_{k,m,0})^{-1}\boldsymbol{u}_k, \tag{14}$$

and the total output power parameterized by $\boldsymbol{u}_k$ becomes

$$\mathcal{J}_k = \boldsymbol{u}_k^T\boldsymbol{X}\boldsymbol{u}_k,\ \ \boldsymbol{X} = \sum_{m=1}^{M-1}(\boldsymbol{S}_{k,m,0}^T\boldsymbol{R}^{-1}\boldsymbol{S}_{k,m,0})^{-1}. \tag{15}$$

Since (15) is the resulting power after interference has been suppressed, the optimal constraint vector can be obtained by further maximizing this output power with respect to the constraint vector under constraint $||\boldsymbol{u}_k|| = 1$ to avoid trivial solutions [11]. Therefore, $\boldsymbol{u}_k$ is the maximum eigenvector of matrix $\boldsymbol{X}$. Once $\boldsymbol{u}_k$ is obtained, our detectors can be easily derived based on (14).

These $M-1$ receivers generate output vector as estimate for $\widetilde{\boldsymbol{a}}_{k,n,0}$, from which the M virtual inputs $\boldsymbol{a}_{k,n,0}$ can be obtained according to (11). However, we can define a new set of M receivers as

$$[\boldsymbol{f}_{k,1},\ldots,\boldsymbol{f}_{k,M}] \triangleq [\widetilde{\boldsymbol{f}}_{k,1},\ldots,\widetilde{\boldsymbol{f}}_{k,M-1}]\boldsymbol{\Lambda}_a\boldsymbol{B}_a^H,$$

which can be viewed as equivalent receivers generating estimate for $\boldsymbol{a}_{k,n,0}$ directly from $\boldsymbol{z}_n$. In the sequel, all analysis will be based on the M receivers.

It is observed that there is only one maximum element in $\boldsymbol{a}_{k,n,0}$ at time n while all others are equal and much smaller. In order to detect input symbol I embedded in $\boldsymbol{a}_{k,n,l}$, we compare outputs of M receivers and find the corresponding index of the maximum output. Considering I takes values $0,\cdots,M-1$, symbol detection criterion can be described as follows

$$I = \arg\max_{m\in\{1,\cdots,M\}}\boldsymbol{f}_{k,m}^T\boldsymbol{z}_n - 1. \tag{16}$$

Next we will analyze this detector's performance.

4. BER ANALYSIS

We first derive the BER of the proposed detector (16). Then we specifically study the effect of noise.

4.1. Performance of the detector

Without loss of generality, assume user 1 is the desired user and its information $I=0$ is transmitted. We partition $\boldsymbol{H}$ according to transmitted symbols from all users as $\boldsymbol{H} = [\boldsymbol{H}_1,\cdots,\boldsymbol{H}_L]$ and define $\widetilde{\boldsymbol{h}}_{l,j} = \boldsymbol{H}_l\widetilde{\boldsymbol{e}}_{M,j}$. Then from (8) we have $\boldsymbol{z}_n = \widetilde{\boldsymbol{h}}_{1,1} + \boldsymbol{w}_n$. $\boldsymbol{w}_n$ includes all interference with covariance $\boldsymbol{R}_w$, and is approximated as a Gaussian process for convenience of analysis. Denote M receivers simply by $\boldsymbol{f}_j$ for $j=1,\cdots,M$. Then according to (16), correct detection is equivalent to the following event

$$\{\boldsymbol{f}_1^T\boldsymbol{z}_n > \boldsymbol{f}_j^T\boldsymbol{z}_n, j=2,\cdots,M\} = \{\Delta\boldsymbol{f}_j^T\boldsymbol{z}_n > 0\} \tag{17}$$

where $\Delta\boldsymbol{f}_j = \boldsymbol{f}_1 - \boldsymbol{f}_j$. Define a $(M-1)$ dimensional random vector $\boldsymbol{x}_n = \Delta\boldsymbol{F}^T\boldsymbol{z}_n$ where $\Delta\boldsymbol{F}$ contains all $\Delta\boldsymbol{f}_j$ as columns. Then (17) becomes $\{\boldsymbol{x}_n > \boldsymbol{0}\}$. Since $\boldsymbol{z}_n$ is assumed Gaussian distributed, $\boldsymbol{x}_n$ is also Gaussian after linear transform. Its probability density function is given by

$$f_{\boldsymbol{x}} = \frac{e^{-\frac{1}{2}(\boldsymbol{x}_n-\Delta\boldsymbol{F}^T\widetilde{\boldsymbol{h}}_{1,1})^T(\mathrm{COV}(\boldsymbol{x}))^{-1}(\boldsymbol{x}_n-\Delta\boldsymbol{F}^T\widetilde{\boldsymbol{h}}_{1,1})}}{\sqrt{(2\pi)^{M-1}det(\mathrm{COV}(\boldsymbol{x}))}}$$

where $\mathrm{COV}(\boldsymbol{x}) = \Delta\boldsymbol{F}^T\boldsymbol{R}_w\Delta\boldsymbol{F}$ is the covariance of $\boldsymbol{x}_n$. Probability of detection error when $I=0$ is transmitted is

$$\begin{aligned} BER_0 &= 1 - Prob\{\boldsymbol{x}_n > \boldsymbol{0}\} \\ &= 1 - \int_0^\infty\cdots\int_0^\infty f_{\boldsymbol{x}}\,dx_1\cdots dx_{M-1} \end{aligned} \tag{18}$$

This is a $(M-1)$ dimensional integral and has no closed form in general. It can be numerically evaluated. Similarly, we can find the BERs when other symbols $I=1,\cdots,M-1$ are transmitted, resulting in $\boldsymbol{z}_n = \widetilde{\boldsymbol{h}}_{1,m+1} + \boldsymbol{w}_n$ ($m = 1,\cdots,M-1$) and correspondingly different vectors $\boldsymbol{x}_n$.

The BERs are denoted as $BER_1, \cdots, BER_{M-1}$. Then the average probability of detection error has a form

$$BER = \frac{1}{M}\sum_{m=0}^{M-1} BER_m.$$

In a special case when $M = 2$, the BERs can be simplified according to (18) as

$$BER_0 = 1 - Q(-\frac{\Delta \boldsymbol{f}_1^T \widetilde{\boldsymbol{h}}_{1,1}}{\sigma_1}) = Q(\frac{\Delta \boldsymbol{f}_1^T \widetilde{\boldsymbol{h}}_{1,1}}{\sigma_1})$$

$$BER_1 = 1 - Q(-\frac{\Delta \boldsymbol{f}_2^T \widetilde{\boldsymbol{h}}_{1,2}}{\sigma_2}) = Q(\frac{\Delta \boldsymbol{f}_2^T \widetilde{\boldsymbol{h}}_{1,2}}{\sigma_2})$$

where $\Delta \boldsymbol{f}_2 = \boldsymbol{f}_2 - \boldsymbol{f}_1 = -\Delta \boldsymbol{f}_1$, $\sigma_j^2 = \Delta \boldsymbol{f}_j^T(\boldsymbol{R} - \widetilde{\boldsymbol{h}}_{1,j}\widetilde{\boldsymbol{h}}_{1,j}^T)\Delta \boldsymbol{f}_j$ for $j = 1, 2$. This form is encountered for a typical communication system. It can be shown that $\widetilde{\boldsymbol{h}}_{1,1} = -\widetilde{\boldsymbol{h}}_{1,2}$. Therefore we conclude that $BER_0 = BER_1$ as expected. Then $BER = \frac{1}{2}(BER_0 + BER_1)$. It is observed that the BER depends on the output signal to interference plus noise ratio (SINR) of receivers $\Delta \boldsymbol{f}_j$ for $j = 1, 2$. The SINRs of these two receivers are same. Unfortunately, this conclusion can not be generalized to an arbitrary M since M virtual inputs may have different signature waveforms although two signature waveforms happen to be only inverted in sign for $M = 2$.

4.2. Effect of noise

In this subsection, we particularly analyze the effect of noise on the performance of receivers and constraint vector. Since each MV receiver depends on the optimal constraint vector $\boldsymbol{u}_1$ given by (15) which is determined by $\boldsymbol{R}$, let us decompose $\boldsymbol{R}$ by eigenvalue decomposition (EVD) to clearly real the noise effect as

$$\boldsymbol{R} = [\ \boldsymbol{U}_s \quad \boldsymbol{U}_n\ ] \begin{bmatrix} \boldsymbol{\Lambda}_s + \sigma_v^2 \boldsymbol{I}_\xi & \boldsymbol{0} \\ \boldsymbol{0} & \sigma_v^2 \boldsymbol{I}_{\nu-\xi} \end{bmatrix} \begin{bmatrix} \boldsymbol{U}_s^T \\ \boldsymbol{U}_n^T \end{bmatrix} \tag{19}$$

where $\boldsymbol{\Lambda}_s = diag\{\lambda_1^2, \ \ldots\ , \lambda_\xi^2\}$, $\boldsymbol{U}_s$ and $\boldsymbol{U}_n$ represent the signal and noise subspaces respectively, σ_v^2 is the noise power. Based on (19) and noticing that $\boldsymbol{S}_{1,m,0}\boldsymbol{g}_1$ is orthogonal to $\boldsymbol{U}_n$, it can be shown that $(\boldsymbol{S}_{k,m,0}^T \boldsymbol{R}^{-1} \boldsymbol{S}_{k,m,0})^{-1}$ in $\boldsymbol{X}$ can be expanded as a power series of σ_v^2 (details omitted)

$$(\boldsymbol{S}_{k,m,0}^T \boldsymbol{R}^{-1} \boldsymbol{S}_{k,m,0})^{-1} = \boldsymbol{g}_1 \boldsymbol{g}_1^T + \sigma_v^2 \boldsymbol{B}_m + \boldsymbol{O}(\sigma_v^4)$$

where

$$\begin{aligned}
\boldsymbol{B}_m &= (\boldsymbol{g}_1^T \boldsymbol{A}_{m,1} \boldsymbol{A}_{m,0}^\dagger \boldsymbol{A}_{m,1} \boldsymbol{g}_1 + \boldsymbol{g}_1^T \boldsymbol{A}_{m,2} \boldsymbol{g}_1)\boldsymbol{g}_1 \boldsymbol{g}_1^T \\
&+ \boldsymbol{A}_{m,0}^\dagger - \boldsymbol{g}_1 \boldsymbol{g}_1^T \boldsymbol{A}_{m,1} \boldsymbol{A}_{m,0}^\dagger - \boldsymbol{A}_{m,0}^\dagger \boldsymbol{A}_{m,1} \boldsymbol{g}_1 \boldsymbol{g}_1^T, \\
\boldsymbol{A}_{m,0} &= \boldsymbol{S}_{k,m,0}^T \boldsymbol{U}_n \boldsymbol{U}_n^T \boldsymbol{S}_{k,m,0}, \\
\boldsymbol{A}_{m,1} &= \boldsymbol{S}_{k,m,0}^T \boldsymbol{U}_s \boldsymbol{\Lambda}_s^{-1} \boldsymbol{U}_s^T \boldsymbol{S}_{k,m,0}, \\
\boldsymbol{A}_{m,2} &= \boldsymbol{S}_{k,m,0}^T \boldsymbol{U}_s \boldsymbol{\Lambda}_s^{-2} \boldsymbol{U}_s^T \boldsymbol{S}_{k,m,0}.
\end{aligned}$$

Then we can obtain

$$\widetilde{\boldsymbol{X}} = \frac{1}{M-1}\boldsymbol{X} = \boldsymbol{g}_1 \boldsymbol{g}_1^T + \frac{\sigma_v^2}{M-1}\sum_{m=1}^{M-1} \boldsymbol{B}_m + \boldsymbol{O}(\sigma_v^4).$$

The first order approximation of its maximum eigenvector due to perturbation in $\boldsymbol{X}$ by noise power has a form [14]

$$\boldsymbol{u}_1 \approx \boldsymbol{g}_1 + \frac{\sigma_v^2}{M-1}\boldsymbol{\Pi}_{\boldsymbol{g}_1}\sum_{m=1}^{M-1} \boldsymbol{B}_m \boldsymbol{g}_1, \ \ \boldsymbol{\Pi}_{\boldsymbol{g}_1} = \boldsymbol{\Sigma}_1 \boldsymbol{\Sigma}_1^T \tag{20}$$

where $\boldsymbol{\Sigma}_1$ is in size of $q \times (q-1)$ and spans a $(q-1)$-dimensional subspace orthogonal to $\boldsymbol{g}_1$. Using the expression of $\boldsymbol{B}_m$, noticing $\boldsymbol{A}_{m,0}^\dagger \boldsymbol{g}_1 = \boldsymbol{0}$ and $\boldsymbol{\Pi}_{\boldsymbol{g}_1}\boldsymbol{g}_1 = \boldsymbol{0}$, we obtain perturbation from (20) as

$$\delta \boldsymbol{u}_1 = \boldsymbol{u}_1 - \boldsymbol{g}_1 \approx -\frac{\sigma_v^2}{M-1}\boldsymbol{\Pi}_{\boldsymbol{g}_1}\sum_{m=1}^{M-1} \boldsymbol{A}_{m,0}^\dagger \boldsymbol{A}_{m,1} \boldsymbol{g}_1. \tag{21}$$

This result shows that $\boldsymbol{u}_1$ can be used as an estimate for $\boldsymbol{g}_1$ when noise is small. The estimation mean-square-error (MSE) is

$$MSE \approx \frac{\sigma_v^4}{(M-1)^2}\sum_{i,j=1}^{M-1} \boldsymbol{g}_1^T \boldsymbol{A}_{i,1}^T \boldsymbol{A}_{i,0}^\dagger \boldsymbol{\Pi}_{\boldsymbol{g}_1} \boldsymbol{A}_{j,0}^\dagger \boldsymbol{A}_{j,1} \boldsymbol{g}_1. \tag{22}$$

From our previous discussion, we know that the BER depends on the SINR of each of two receivers when $M = 2$. Their SINRs are equal. The SINR of the MV receiver corresponding to m has a form [11]

$$SINR_{m,mv} = \frac{1}{\frac{\boldsymbol{u}_1^T(\boldsymbol{S}_{k,m,0}^T \boldsymbol{R}^{-1} \boldsymbol{S}_{k,m,0})^{-1}\boldsymbol{u}_1}{||\boldsymbol{u}_1^T \boldsymbol{g}_1||^2} - 1}.$$

After using previous perturbation results on both $\boldsymbol{u}_1$ and $(\boldsymbol{S}_{k,m,0}^T \boldsymbol{R}^{-1} \boldsymbol{S}_{k,m,0})^{-1}$, we can find

$$SINR_{m.mv} \approx \frac{1}{\sigma_v^2(\boldsymbol{g}_1^T \boldsymbol{A}_{m,1} \boldsymbol{A}_{m,0}^\dagger \boldsymbol{A}_{m,1} \boldsymbol{g}_1 + \boldsymbol{g}_1^T \boldsymbol{A}_{m,2} \boldsymbol{g}_1)}.$$

Instead, the SINR of the MMSE receiver has a form [11]

$$SINR_{m,mmse} \approx \frac{1}{\sigma_v^2 \boldsymbol{g}_1^T \boldsymbol{A}_{m,2} \boldsymbol{g}_1}.$$

Penalty exists in the MV receiver when $SINR_{m,mv}$ is compared with $SINR_{m,mmse}$. Under Gaussian approximation of the MAI and using these SINR results, the analytical BER can be found based on the Q function and these SINRs when $M = 2$.

5. NUMERICAL EXAMPLES

We test performance of the propoed method by simulations in this section. Second derivative of Gaussian function with pulse width equal to 0.7 ns is assumed for the received signal [5]. Other system parameters are set to be $N_c = 8$,

$N_f = 4$, $M = 2$, $K = 8$. Each user's time hopping codes and 3-path Gaussian channel spread over one frame duration are randomly generated. Path delays of the desired user are assumed known. Data length is taken as infinite. The receiver is assumed to be synchronized to the desired user (i.e. user 1). Fig. 1 (a) and (b) illustrate the average channel MSE and receiver's output SINR ratio for 100 random generated channels at various SNR. The experimental MSE curve converges to the analytical one very well from 8dB SNR, and both curves decrease monotonically as SNR increases. The SINR ratio converges to a value very close to 1, indicating that the proposed receiver has similar performance to the MMSE receiver. Finally, Fig. 2 shows average BER performance over 100 realizations, where channels are fixed for all users. Clearly for either MOE or MMSE receiver, experimental BER curve is very consistent with its analytical counterpart. Moreover, the MOE receiver shows performance very close to the MMSE receiver.

6. ACKNOWLEDGEMENT

The authors thank Brian Sadler and Ananthram Swami of ARL for their encouragement.

7. REFERENCES

[1] H. F. Harmuth, *Transmission of Information by Orthogonal Functions*, Springer-Verlag, New York, 1969.

[2] Federal Communications Commission News Release, "Revision of Part 15 of the Commission's Rules Regarding Ultra-Wideband Transmission Systems," ET Docket 98-153, Washington D.C., February 14, 2002.

[3] R. Fontana, A. Ameti, E. Richley, L. Beard, and D. Guy, "Recent advances in ultra wideband communications systems," *Proc. 2002 UWBST*, May 2002, pp. 129-133.

[4] R. A. Scholtz, "Multiple access with time-hopping impulse modulation," *Proc. MILCOM'93*, October 1993, pp. 447-450.

[5] M. Z. Win and R. A. Scholtz, "Impulse radio: how it works," *IEEE Commun. Letters*, vol. 2, no. 2, pp. 36-38, Feb. 1998.

[6] Q. Li and L. A. Rusch, "Multiuser receivers for DS-CDMA UWB," *Proc. UWBST*, May 2002, pp. 163-167.

[7] Y. C. Yoon and R. Kohno, "Optimum multi-user detection in ultra-wideband (UWB) multiple-access communication systems," *Proc. ICC*, 2002, vol. 2, pp. 812-816.

[8] C. J. Le Martret and G. B. Giannakis, "All-digital impulse radio with multiuser detection for wireless cellular systems," *IEEE Trans. Commun.*, vol. 50, no. 9, pp. 1440-1450, Sept. 2002.

[9] L. Yang and G. B. Giannakis, "Multistage block-spreading for impulse radio multiple access through ISI channels," *IEEE J. Selected Areas Commun.*, vol. 20, no. 9, pp. 1767-1777, Dec. 2002.

[10] Z. Xu, J. Tang and P. Liu, "Frequency-domain estimation of multiple access ultra-wideband signals," *Proc. IEEE Workshop on Statistical Signal Processing*, Missouri, September 2003.

[11] M. Tsatsanis and Z. Xu, "Performance analysis of minimum variance CDMA receivers," *IEEE Trans. Signal Processing*, vol. 46, no. 11, pp. 3014-3022, Nov. 1998.

[12] M. Tsatsanis, Z. Xu and X. Lu, "Blind multiuser detectors for dual rate DS-CDMA systems over frequency selective channels," *Proc. EUSIPCO*, vol. 2, pp. 631-634, Tampere, Finland, September 5-8, 2000.

[13] H. Li, X. Lu and G. B. Giannakis, "Capon multiuser receiver for CDMA systems with space-time coding," *IEEE Transactions on Signal Processing*, vol. 50, no. 5, pp. 1193-1204, May 2002.

[14] Z. Xu, "Perturbation analysis for subspace decomposition with applications in subspace-based algorithms," *IEEE Trans. Signal Processing*, vol. 50, no. 11, pp. 2820-2830, Nov. 2002.

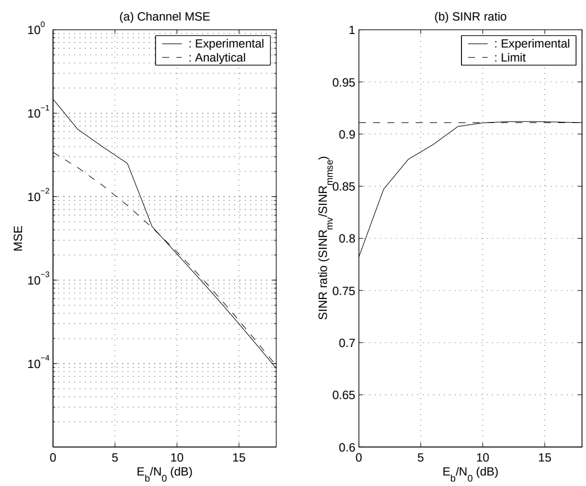

Fig. 1. Channel MSE and SINR ratio.

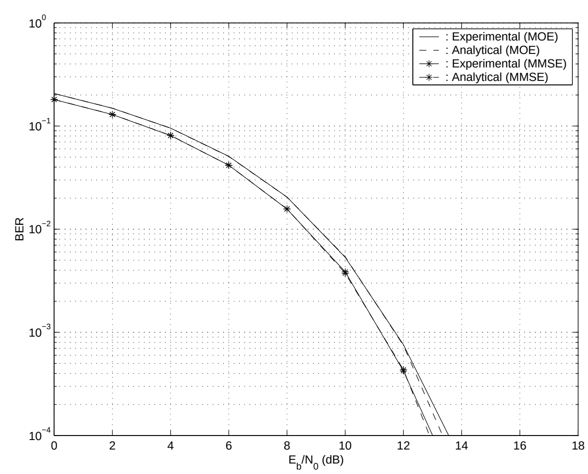

Fig. 2. BER performance.