MEAN AND COVARIANCE BASED ESTIMATION OF MULTIPLE ACCESS UWB CHANNELS

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ABSTRACT

Traditional pulse position modulation based ultra-wideband (UWB) communication system transmits short pulses whose positions carry user's data information. Using different time-hopping sequences, multiple access is enabled. We address blind multiuser channel estimation after following a pulse-rate discrete-time channel model. The model can be regarded in a tri-linear structure and also resembles a code-division multiple access (CDMA) system with newly defined hopping-code dependent matrices for each user. Considering that either the mean or covariance of received signals contains sufficient information for all unknown channels, least squares and covariance matching ideas are successfully applied for channel estimation. Performance of each estimator is analyzed and also verified by computer simulations.

1. INTRODUCTION

Ultra-wideband (UWB) technology has received considerable attention recently due to its attractive features [1]. A conventional UWB system transmits trains of time-hopping (TH) short-duration pulses with a low duty cycle and uses pulse position modulation (PPM). Therefore, multipath down to path delay differentials in nanoseconds is resolvable at the receiver, significantly mitigating multipath distortion and providing path diversity [2]. Meanwhile, its low probability of interception and detection property is of extreme importance for secure communication links [3].

A conventional UWB receiver is a RAKE receiver. It consists of waveform correlators [1]. To fully capture the signal energy spread over multiple paths, the receiver needs to know channel parameters when correlation is performed. However, in a dense multipath wireless environment, channel information is not known *a priori*. Although single-user maximum likelihood (ML) channel estimation methods have been proposed [2], [4], multiple access interference (MAI) is approximated as a Gaussian process which may not be accurate and has to be considered explicitly for performance improvement. A training based channel estimation method has been proposed in [5]. Channel parameters are also required by some existing methods in design of multiuser receivers either in time domain [6], [7] or frequency domain [8].

In this paper, we study blind multiuser channel estimation from statistics of received signals in a multiple access UWB system. For low complexity and easy implementation, only first order and second order statistics (SOS) are employed. First, we adopt pulse-rate discrete-time channel model developed in [6] which makes blind channel estimation possible. It is then observed that an UWB system resembles a direct sequence (DS) code-division multiple access (CDMA) system. After clearly defining a matrix for each user from its unique time-hopping (TH) sequence, each matrix can be treated as a code matrix, similar to the code matrix constructed from spreading codes in a CDMA system [9]. But it is sparse and consists of only zeros and ones, indicating whether there exists a contribution to the received signal during a particular time interval from a multipath channel or not. Locations of zeros and ones are different for different users, capable of differentiating users. After such linear modeling, PPM is transformed to superimposed amplitude modulation that is easy to handle. The received signal exhibits non-zero mean. It is linearly parameterized by multiuser channels which can be estimated without ambiguity by the least-square (LS) criterion. Meanwhile, assisted by unique code matrices, a covariance matching (CM) idea can be applied to estimate each channel up to a phase ambiguity after defining a rank-one channel dependent matrix for each user [9]. It is based on new data with mean subtracted instead of directly received data since non-zero mean incurs channel cross products in the autocorrelation of directly received data and complicates estimation. Although chip-rate sampling induced multiple-input multiple-output (MIMO) model can be used for symbol detection for given channel parameters [6], it creates multiple sub-channels corresponding to the same code matrix and same propagation channel for each user. Under such a modeling, LS method can only yield an estimate of linearly imposed sub-channels, while CM method can only provide an estimate with a unitary matrix ambiguity. Therefore, only pulse-rate sampling is adopted in this paper.

Notations: Following common practice, we denote Kronecker product by $\otimes$, Hadamard (element-wise) product by

$\odot$, complex conjugate (*) transpose (^{T}) by ^{H}, inverse by $^{-1}$. $E\{\cdot\}$ represents expectation of a random variable, $\boldsymbol{I}_a$ an identity matrix of degree a whose ith column is denoted by $\boldsymbol{e}_{a,i}$. $\mathbf{1}_a$ is a vector of length a with all elements equal one. An estimate of a quantity (scalar, vector or matrix) is denoted by putting a $\widehat{}$ over it, and correspondingly, the estimation error by preceding the quantity with a δ, such as $\widehat{\boldsymbol{x}}$ and $\delta\boldsymbol{x}$ for vector $\boldsymbol{x}$ respectively. We define distributive Kronecker product $\boldsymbol{X}\diamond\boldsymbol{Y} = [\boldsymbol{X}\otimes\boldsymbol{Y}(:,1), \boldsymbol{X}\otimes\boldsymbol{Y}(:,2), \cdots]$ based on columns of $\boldsymbol{Y}$.

2. SYSTEM MODEL

Consider a multiple access (MA) TH UWB system with K users. The transmitted baseband UWB signal from user k can be described by [6]

$$\alpha_k(t) = \sqrt{\mathcal{P}_k}\sum_{i=-\infty}^{\infty} w(t - iT_f - c_k(i)T_c - \tau_{I_k(\lfloor i/N_f\rfloor)}) \quad (1)$$

where $\mathcal{P}_k$ is the kth user's transmission power, $w(t)$ is the baseband monopulse, T_f is the frame duration, N_f is the number of frames over which an M-ary PPM symbol repeats, $c_k(i) \in [0, N_c - 1]$ is a periodic hopping sequence with period equal to one symbol period. Each chip has duration T_c. $I_k(\lfloor i/N_f\rfloor) \in [0, M-1]$ is the kth user's information bearing symbol during the ith frame, $\tau_{I_k(\lfloor i/N_f\rfloor)} = I_k(\lfloor i/N_f\rfloor)\sigma$ is the corresponding modulation delay in multiples of σ seconds. Assume $T_f = N_cT_c$ and $T_c = M\sigma$. If we define $w_m(t) \stackrel{\Delta}{=} w(t - m\sigma)$ where $m = 0, \cdots, M-1$ and $s_{k,m}(\lfloor i/N_f\rfloor) = \delta\Big(I_k(\lfloor i/N_f\rfloor) - m\Big)$, then (1) may be expressed by linear modulation in a chip rate as [6]

$$\alpha_k(t) = \sqrt{\mathcal{P}_k}\sum_{i=-\infty}^{\infty}\sum_{m=0}^{M-1} u_{k,m}(i)w_m(t - iT_c) \quad (2)$$

where chip index has replaced frame index in (1),

$$u_{k,m}(i) = s_{k,m}\Big(\lfloor i/(N_cN_f)\rfloor\Big)\widetilde{c}_k(i),$$

$$\widetilde{c}_k(i) = \delta\Big(\lfloor i/N_c\rfloor N_c + c_k(\lfloor i/N_c\rfloor) - i\Big).$$

It is clear according to (2) that input $u_{k,m}(i)$ is modulated by waveform $w_m(t)$ at a chip rate. The transmitted signal $\alpha_k(t)$ propagates through a linear channel with impulse response $\bar{g}_k(t)$. At the receiver, the channel output is first passed through a matched filter matched to the monopulse $w(t)$. We can define a front-end effective channel including effects from modulated pulse at the transmitter, propagation channel and matched filter at the receiver by $g_{k,m}(t) = w_m(t) \star \bar{g}_k(t) \star w(-t)$ where $\star$ denotes convolution. Considering additive white Gaussian noise (AWGN) $v(t)$ and propagation delay d_k for user k, the output of the matched filter becomes

$$y(t) = \sum_{k,i_1,m}\sqrt{\mathcal{P}_k}u_{k,m}(i_1)g_{k,m}(t - i_1T_c - d_k) + v(t). \quad (3)$$

Assume each effective channel has length $q\sigma$. Then $y(t)$ is sampled every σ seconds to yield a discrete-time output $y(n) = y(t)|_{t=n\sigma}$. Using the discrete-time version of the effective channel and invoking $T_c = M\sigma$, we obtain a pulse-rate model

$$y(n) = \sum_{k,m}\sum_{i_2=0}^{q}\sqrt{\mathcal{P}_k}u_{k,m}(\frac{n - i_2}{M})g_{k,m}(i_2) + v(n). \quad (4)$$

Consider P symbol intervals of data samples with corresponding time instants $nMN_cN_f + p$ for $p = 1, \cdots, MPN_cN_f$ and collect them in a big vector $\boldsymbol{y}_n$ of length $\nu = MPN_cN_f$, After noticing our definition of $u_{k,m}(i)$, a vector form data model follows

$$\boldsymbol{y}_n = \sum_{k,m,l}\boldsymbol{C}_{k,l}\boldsymbol{T}_m\boldsymbol{g}_k s_{k,m}(n+l) + \boldsymbol{v}_n \quad (5)$$

where symbol index l takes all integers $-\lceil q/(MN_cN_f)\rceil, \cdots, P-1$, $\boldsymbol{g}_k$ is an unknown channel vector for user k which contains channel coefficients at the pulse rate and power factor $\sqrt{\mathcal{P}_k}$, $\boldsymbol{T}_m = [\mathbf{0}, \boldsymbol{I}_q, \mathbf{0}]^T$ is a tall selection matrix in order to obtain the mth subchannel from $\boldsymbol{g}_k$ (delayed in $m\sigma$ seconds or equivalently downshifted by m elements), $\boldsymbol{C}_{k,l}$ is a matrix constructed from corresponding $\widetilde{c}_k(i)$ and is uniquely determined by the TH sequence. It consists of only zeros and ones and repeats from symbol to symbol because the TH sequence has period equal to one symbol interval. This model can be compactly expressed in another form

$$\boldsymbol{y}_n = \sum_{k,l}\boldsymbol{H}_{k,l}\boldsymbol{s}_{k,n,l} + \boldsymbol{v}_n = \boldsymbol{H}\boldsymbol{s}_n + \boldsymbol{v}_n \quad (6)$$

after collecting M inputs in a vector

$$\boldsymbol{s}_{k,n,l} = [s_{k,0}(n+l), \cdots, s_{k,M-1}(n+l)]^T$$

defining a corresponding effective channel matrix

$$\boldsymbol{H}_{k,l} = [\boldsymbol{C}_{k,l}\boldsymbol{T}_0\boldsymbol{g}_k, \cdots, \boldsymbol{C}_{k,l}\boldsymbol{T}_{M-1}\boldsymbol{g}_k]$$

and successively stacking such matrices (or vectors) in $\boldsymbol{H}$ (or $\boldsymbol{s}_n$). The total number of symbols from K users is denoted by $L = K(P + \lceil q/(MN_cN_f)\rceil)$. By employing data model (5), all channels can be estimated based on the statistics (mean or covariance) of $\boldsymbol{y}_n$.

3. BLIND CHANNEL ESTIMATION

It is observed that all $\boldsymbol{g}_k$ are embedded in the statistics of the data vector. For low complexity, we consider either its mean or covariance, yielding LS or CM method accordingly.

3.1. LS approach

Let us denote the mean of $\boldsymbol{y}_n$ as $\bar{\boldsymbol{y}}$. From our definition, the mean of $\boldsymbol{s}_{k,n,l}$ is easily found to be $\frac{1}{M}\mathbf{1}_M$. Since noise has zero mean even after the matched filter, we have

$$\bar{\boldsymbol{y}} = \frac{1}{M}\sum_{k,m,l} \boldsymbol{C}_{k,l}\boldsymbol{T}_m\boldsymbol{g}_k = \sum_k \mathcal{C}_k\boldsymbol{g}_k = \mathcal{C}\boldsymbol{g} \quad (7)$$

where all channel vectors are stacked in a big vector $\boldsymbol{g}$. Assume $\bar{\boldsymbol{y}}$ is estimated from N data vectors by sample average $\widehat{\bar{\boldsymbol{y}}} = \frac{1}{N}\sum_{n=1}^{N}\boldsymbol{y}_n$. Then a LS criterion can be applied to estimate $\boldsymbol{g}$ as follows

$$\widehat{\boldsymbol{g}} = \arg\min ||\bar{\boldsymbol{y}} - \widehat{\bar{\boldsymbol{y}}}||^2. \quad (8)$$

Invoking (7), the solution to (8) has the following form

$$\widehat{\boldsymbol{g}} = \boldsymbol{W}\widehat{\bar{\boldsymbol{y}}}, \;\; \boldsymbol{W} = (\mathcal{C}^H\mathcal{C})^{-1}\mathcal{C}^H. \quad (9)$$

Then the estimate of $\boldsymbol{g}_k$ can be obtained from the corresponding subvector of $\widehat{\boldsymbol{g}}$ as $\widehat{\boldsymbol{g}}_k = (\boldsymbol{e}_{K,k}^T \otimes \boldsymbol{I}_q)\widehat{\boldsymbol{g}}$.

3.2. CM approach

Since $\boldsymbol{y}_n$ has non-zero mean, its autocorrelation is found to have cross terms $\boldsymbol{g}_{k_1}\boldsymbol{g}_{k_2}^H$ of users k_1 and k_2 and not convenient for channel estimation. Thus covariance is considered. Define a new zero-mean data vector from $\boldsymbol{y}_n$ as

$$\boldsymbol{z}_n = \boldsymbol{y}_n - \bar{\boldsymbol{y}} = \sum_{k,l} \boldsymbol{H}_{k,l}\boldsymbol{a}_{k,n,l} + \boldsymbol{v}_n = \boldsymbol{H}\boldsymbol{a}_n + \boldsymbol{v}_n \quad (10)$$

where $\boldsymbol{a}_{k,n,l} = \boldsymbol{s}_{k,n,l} - \frac{1}{M}\boldsymbol{1}_M$. For shorter notation, we denote the information symbol in $\boldsymbol{s}_{k,n,l}$ simply by I after ignoring its time and user dependence. It takes values $0,\cdots,M-1$ with equal probability $\frac{1}{M}$. Then

$$\boldsymbol{a}_{k,n,l} = [\delta(I),\cdots,\delta(I-(M-1))]^T - \frac{1}{M}\boldsymbol{1}_M^T. \quad (11)$$

To obtain the covariance of $\boldsymbol{z}_n$, it is necessary to find that of $\boldsymbol{a}_{k,n,l}$. We denote it by $\boldsymbol{A} = E\{\boldsymbol{a}_{k,n,l}\boldsymbol{a}_{k,n,l}^T\}$. According to the distribution of I, it can be found that

$$\boldsymbol{A} = \frac{1}{M}\sum_{i=1}^{M}(\boldsymbol{e}_{M,i} - \frac{1}{M}\boldsymbol{1}_M)(\boldsymbol{e}_{M,i} - \frac{1}{M}\boldsymbol{1}_M)^T.$$

After simplification, it becomes

$$\boldsymbol{A} = \frac{1}{M}(\boldsymbol{I}_M - \frac{1}{M}\boldsymbol{1}_M\boldsymbol{1}_M^T)$$

which is easily shown to have rank $M-1$ since $\frac{1}{\sqrt{M}}\boldsymbol{1}_M$ is a unitary vector. Its (m_1,m_2)th element is defined as a_{m_1,m_2}. The ideal covariance of $\boldsymbol{z}_n$ is then derived to follow

$$\boldsymbol{R} = E\{\boldsymbol{z}_n\boldsymbol{z}_n^H\} = \sum_{k,l}\boldsymbol{H}_{k,l}\boldsymbol{A}\boldsymbol{H}_{k,l}^H + \sigma_v^2\boldsymbol{I}_\nu.$$

After defining a rank-one matrix $\boldsymbol{G}_k = \boldsymbol{g}_k\boldsymbol{g}_k^H$, it becomes

$$\boldsymbol{R} = \sum_{k,l,m_1,m_2} a_{m_1,m_2}\boldsymbol{C}_{k,l}\boldsymbol{T}_{m_1}\boldsymbol{G}_k\boldsymbol{T}_{m_2}^H\boldsymbol{C}_{k,l}^H + \sigma_v^2\boldsymbol{I}_\nu.$$

As in [9], vectored form is convenient to handle in the CM context. Define

$$\boldsymbol{r} = vec(\boldsymbol{R}), \;\; \boldsymbol{x}_k = vec(\boldsymbol{G}_k), \;\; \boldsymbol{x} = [\boldsymbol{x}_1^T,\cdots,\boldsymbol{x}_K^T,\sigma_v^2]^T.$$

Using the property of vec [10], we obtain

$$\boldsymbol{r} = \boldsymbol{S}\boldsymbol{x}, \quad \boldsymbol{S} = [\boldsymbol{S}_1,\cdots,\boldsymbol{S}_K, vec(\boldsymbol{I}_\nu)]. \quad (12)$$

$$\boldsymbol{S}_k = \sum_{l,m_1,m_2} a_{m_1,m_2}(\boldsymbol{C}_{k,l}\boldsymbol{T}_{m_2})^* \otimes (\boldsymbol{C}_{k,l}\boldsymbol{T}_{m_1}).$$

Therefore, $\boldsymbol{r}$ can be matched with its estimate $\widehat{\boldsymbol{r}}$ from N data vectors in a vector form

$$\widehat{\boldsymbol{x}} = \arg\min ||\boldsymbol{r} - \widehat{\boldsymbol{r}}||^2, \;\; \widehat{\boldsymbol{r}} = vec(\widehat{\boldsymbol{R}}) \quad (13)$$

$$\widehat{\boldsymbol{R}} = \frac{1}{N}\sum_{n=1}^{N}(\boldsymbol{y}_n - \widehat{\bar{\boldsymbol{y}}})(\boldsymbol{y}_n - \widehat{\bar{\boldsymbol{y}}})^H, \;\; \widehat{\bar{\boldsymbol{y}}} = \frac{1}{N}\sum_{n=1}^{N}\boldsymbol{y}_n. \quad (14)$$

Considering (12), the solution to (13) is given by

$$\widehat{\boldsymbol{x}} = \boldsymbol{Q}\widehat{\boldsymbol{r}}, \;\; \boldsymbol{Q} = (\boldsymbol{S}^H\boldsymbol{S})^{-1}\boldsymbol{S}^H. \quad (15)$$

Once $\boldsymbol{x}$ is estimated, $\boldsymbol{x}_k$ can be extracted. Then $\boldsymbol{G}_k$ is reconstructed by the reverse vec operation. These operations can be described by

$$\widehat{\boldsymbol{G}}_k = [(\boldsymbol{e}_{q,1}^T \otimes \boldsymbol{I}_q)\widehat{\boldsymbol{x}}_k,\cdots,(\boldsymbol{e}_{q,q}^T \otimes \boldsymbol{I}_q)\widehat{\boldsymbol{x}}_k] \quad (16)$$

$$\widehat{\boldsymbol{x}}_k = [\boldsymbol{e}_{K,k}^T \otimes \boldsymbol{I}_{q^2}, \boldsymbol{0}_{q^2\times 1}]\widehat{\boldsymbol{x}}. \quad (17)$$

Using (15), we can relate $\widehat{\boldsymbol{G}}_k$ to $\widehat{\boldsymbol{r}}$,

$$\widehat{\boldsymbol{G}}_k = [\boldsymbol{A}_{k,1}\widehat{\boldsymbol{r}},\cdots,\boldsymbol{A}_{k,q}\widehat{\boldsymbol{r}}] \quad (18)$$

where

$$\boldsymbol{A}_{k,i} = (\boldsymbol{e}_{q,i}^T \otimes \boldsymbol{I}_q)[\boldsymbol{e}_{K,k}^T \otimes \boldsymbol{I}_{q^2}, \boldsymbol{0}_{q^2\times 1}]_{q^2\times(Kq^2+1)}\boldsymbol{Q}$$

for $i = 1,\cdots,q$. Once $\widehat{\boldsymbol{G}}_k$ is obtained, channel vector $\boldsymbol{g}_k$ can be estimated from its singular value decomposition (SVD) by finding the singular vector corresponding to its maximum singular value. That singular vector becomes an estimate of $\boldsymbol{g}_k$ up to a multiplicative scalar.

4. CHANNEL ESTIMATION PERFORMANCE

In this section, we study identifiability conditions, covariance and mean-square-error (MSE) of channel estimators.

4.1. Channel identifiability

Our solution (9) or (15) exists only when the corresponding matrix is invertible. Therefore, the LS channel estimator is unique if matrix $\mathcal{C}$ has full column rank. According to (7), it depends on time-hopping codes of all users. It can only be checked for any assigned set of hopping sequences. Similarly, the CM channel estimator is unique if matrix $\boldsymbol{S}$ has full column rank. According to (12), it depends on time-hopping codes of all users as well [9]. It is also interesting to compare these two methods in terms of the maximum number of channels (users) that can be estimated. Matrix

$\mathcal{C}$ has dimension $\nu \times Kq$. Then $K \leq \lfloor \nu/q \rfloor$. However, $\boldsymbol{S}$ has size $\nu^2 \times (Kq^2+1)$. Correspondingly, K approximately satisfies $K \leq \lfloor (\nu/q)^2 \rfloor$. Usually, $\nu >> q$. Hence, the CM method allows more simultaneous users than the LS method regarding channel estimation, but it has higher complexity since it uses SOS of the zero-mean data rather than easily estimated first-order statistics in the LS method. When symbol detection is of interest, a sufficient condition for all inputs to be detected is that a channel matrix which contains all columns of $\boldsymbol{H}_{k,l}$ ($\forall k,l$) has full column rank. It thus imposes one more condition on K. Due to limited space, symbol detection issues will be studied elsewhere.

4.2. Performance of the LS estimator

From (9), we obtain

$$\delta \boldsymbol{g} = \boldsymbol{W}(\frac{1}{N}\sum_{n=1}^{N}\boldsymbol{y}_n - \bar{\boldsymbol{y}}) = \frac{1}{N}\boldsymbol{W}\sum_{n=1}^{N}\boldsymbol{z}_n. \qquad (19)$$

Then covariance of $\delta\boldsymbol{g}$ becomes

$$\mathrm{COV}(\delta \boldsymbol{g}) = \frac{1}{N^2}\boldsymbol{W}(\sum_{n_1,n_2} E\{\boldsymbol{z}_{n_1}\boldsymbol{z}_{n_2}^H\})\boldsymbol{W}^H. \qquad (20)$$

Assume $\boldsymbol{z}_n$ constitutes a sequence of independent vectors. Then due to its zero mean, (20) is simplified as

$$\mathrm{COV}(\delta \boldsymbol{g}) = \frac{1}{N}\boldsymbol{W}\boldsymbol{R}\boldsymbol{W}^H. \qquad (21)$$

The covariance of $\delta\boldsymbol{g}_k$ is the kth diagonal subblock of this matrix and the MSE is the trace of the corresponding matrix.

4.3. Performance of the CM estimator

Channel estimation error is due to an estimation error in the data covariance $\boldsymbol{R}$ or equivalently $\boldsymbol{r}$. From our definition of $\boldsymbol{G}_k$, $\boldsymbol{g}_k$ is an eigenvector corresponding to its unique nonzero eigenvalue. If $\widehat{\boldsymbol{r}}$ has an estimation error $\delta\boldsymbol{r} = \widehat{\boldsymbol{r}} - \boldsymbol{r}$ due to finite N, then an error is introduced to $\widehat{\boldsymbol{G}}_k$. From (18), $\boldsymbol{G}_k$ is perturbed by $\delta\boldsymbol{G}_k$ as

$$\delta\boldsymbol{G}_k = [\boldsymbol{A}_{k,1}\delta\boldsymbol{r}, \cdots, \boldsymbol{A}_{k,q}\delta\boldsymbol{r}]. \qquad (22)$$

Then the first-order perturbation in its eigenvector $\boldsymbol{g}_k$ becomes [11]

$$\delta\boldsymbol{g}_k \approx \boldsymbol{\Pi}^{\perp}_{\boldsymbol{g}_k}\delta\boldsymbol{G}_k\boldsymbol{g}_k, \;\; \boldsymbol{\Pi}^{\perp}_{\boldsymbol{g}_k} = \boldsymbol{\Sigma}_k\boldsymbol{\Sigma}_k^H \qquad (23)$$

where $\boldsymbol{\Sigma}_k$ is in size of $q \times (q-1)$ and spans a $(q-1)$-dimensional subspace orthogonal to $\boldsymbol{g}_k$. Substituting (22) into (23), we obtain

$$\delta\boldsymbol{g}_k \approx \boldsymbol{\Gamma}_k\delta\boldsymbol{r}, \quad \boldsymbol{\Gamma}_k = \boldsymbol{\Pi}^{\perp}_{\boldsymbol{g}_k}\sum_{i=1}^{q} g_k(i)\boldsymbol{A}_{k,i}. \qquad (24)$$

Then the auto-covariance of channel estimate becomes

$$\mathrm{Cov}(\delta\boldsymbol{g}_\mathrm{k}, \delta\boldsymbol{g}_\mathrm{k}) = E\{\delta\boldsymbol{g}_k\delta\boldsymbol{g}_k^H\} \approx \boldsymbol{\Gamma}_k\boldsymbol{\Phi}(\delta\boldsymbol{r})\boldsymbol{\Gamma}_k^H, \qquad (25)$$

where $\boldsymbol{\Phi}(\delta\boldsymbol{r}) = E\{\delta\boldsymbol{r}\delta\boldsymbol{r}^H\}$ is the covariance of $\delta\boldsymbol{r}$. It depends on data model (6) and covariance estimation method in (14). According to eq. (12) in [12], applying multilinearity and additivity properties of cumulant [13], and properties of $\otimes$ [10], the following proposition can be proved. Noticing that M inputs in $\boldsymbol{a}_{k,n,l}$ are all real and dependent due to the same information, the last term in (12) of [12] does not diminish with real inputs.

Proposition: If channel model follows (6) and data covariance is estimated from N independent data vectors as (14), then for a real system (all quantities are real)

$$\begin{aligned}\boldsymbol{\Phi}(\delta\boldsymbol{r}) \;&=\; \frac{(N-1)^2}{N^3}\boldsymbol{K}_z + \frac{N-1}{N^2}\boldsymbol{R}\otimes\boldsymbol{R} \\ &+\; \frac{(N-1)^2}{N^3}\boldsymbol{B}_{1r}\odot\boldsymbol{B}_{1r}^T \\ &+\; \frac{1}{N^2}\boldsymbol{r}\boldsymbol{r}^T + \frac{N-1}{N^3}\boldsymbol{B}_{2r}\end{aligned}$$

while for complex channel and noise,

$$\begin{aligned}\boldsymbol{\Phi}(\delta\boldsymbol{r}) \;&=\; \frac{(N-1)^2}{N^3}\boldsymbol{K}_z + \frac{N-1}{N^2}\boldsymbol{R}^*\otimes\boldsymbol{R} \\ &+\; \frac{(N-1)^2}{N^3}\boldsymbol{B}_{1c}\odot\boldsymbol{B}_{1c}^H \\ &+\; \frac{1}{N^2}\boldsymbol{r}\boldsymbol{r}^H + \frac{N-1}{N^3}\boldsymbol{B}_{2c}\end{aligned}$$

where

$$\boldsymbol{B}_{1r} = (\boldsymbol{I}_\nu\otimes\boldsymbol{1}_\nu)\boldsymbol{R}(\boldsymbol{1}_\nu^T\otimes\boldsymbol{I}_\nu), \quad \boldsymbol{B}_{2r} = \boldsymbol{R}\diamond\boldsymbol{R},$$

$$\boldsymbol{B}_{1c} = (\boldsymbol{I}_\nu\otimes\boldsymbol{1}_\nu)\sum_{k,l}(\boldsymbol{H}_{k,l}^*\boldsymbol{A}\boldsymbol{H}_{k,l}^H)(\boldsymbol{1}_\nu^T\otimes\boldsymbol{I}_\nu),$$

$$\boldsymbol{B}_{2c} = (\boldsymbol{H}^*\otimes\boldsymbol{H})(\mathcal{A}\diamond\mathcal{A})(\boldsymbol{H}^*\otimes\boldsymbol{H})^H, \;\; \mathcal{A} = \boldsymbol{I}_L\otimes\boldsymbol{A}$$

$\boldsymbol{K}_z$ is the cumulant matrix of $\boldsymbol{z}_n$ and is related to the cumulant matrix $\boldsymbol{K}_a$ of input vector $\boldsymbol{a}_{k,n,l}$ as

$$\boldsymbol{K}_z = \sum_{k,l}(\boldsymbol{H}_{k,l}^*\otimes\boldsymbol{H}_{k,l})\boldsymbol{K}_a(\boldsymbol{H}_{k,l}^*\otimes\boldsymbol{H}_{k,l})^H,$$

$$\begin{aligned}\boldsymbol{K}_a \;&=\; \frac{1}{M}\sum_{i=1}^{M}(\widetilde{\boldsymbol{e}}_{M,i}\widetilde{\boldsymbol{e}}_{M,i}^T)\otimes(\widetilde{\boldsymbol{e}}_{M,i}\widetilde{\boldsymbol{e}}_{M,i}^T) \\ &-\; vec(\boldsymbol{A})vec(\boldsymbol{A})^T - \boldsymbol{A}\otimes\boldsymbol{A} - \boldsymbol{B}_3\odot\boldsymbol{B}_3^T\end{aligned}$$

where $\widetilde{\boldsymbol{e}}_{M,i} = \boldsymbol{e}_{M,i} - \frac{1}{M}\boldsymbol{1}_M$

$$\boldsymbol{B}_3 = (\boldsymbol{I}_M\otimes\boldsymbol{1}_M)\boldsymbol{A}(\boldsymbol{1}_M^T\otimes\boldsymbol{I}_M).$$

5. SIMULATION

In our simulation, the system adopts binary PPM modulation with $N_c = 4$, $N_f = 8$ and four users with equal transmitting power. The monocycle pulse is chosen as normalized second derivative of the Gaussian pulse with pulse duration $D_g = 0.7ns$, as in [4]. Modulation delay parameter is set to be $\sigma = D_g$. Time hopping sequences are randomly generated. Multipath channels have time delay resolution of D_g with maximum delay spread to be one frame duration. Channel gains for different users are modeled as independent Gaussian random variables and weighted by linearly decreasing weights [7].

Averaged channel estimation errors over 100 independent realizations for one set of channel parameters are illustrated in Fig. 1. Either successive (with ISI) or independent (ISI free) data vectors can be collected. CM estimator is consistently better than LS estimator. But LS estimator can still achieve MSE of less than 1% when data length is over 400. Analytical curves are also plotted and show high consistency with their experimental counterparts (with independent vectors). Based on estimated channel coefficients from 1000 successive data vectors, we further examine bit error rate (BER) in Fig. 2 for each of Rake, ZF, conventional MMSE, and subspace MMSE receivers [14]. For comparison, corresponding ideal receivers assuming true channel parameters and correlation matrix, if applicable, are included. Receivers based on CM estimator are generally better than those based on LS estimator. Among four linear receivers, the ZF and subspace MMSE receivers show similar performance and outperform all others. Rake and ZF receivers can achieve performance close to their ideal ones. But for the conventional MMSE receiver, a clear gap is observed between experimental and ideal ones at high SNR mainly due to amplification of noise in practical conditions. However, its subspace variant almost achieves its ideal performance.

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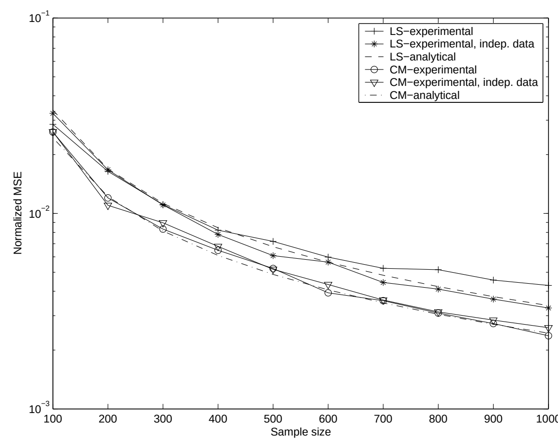

Fig. 1. Channel estimation error under SNR=15dB.

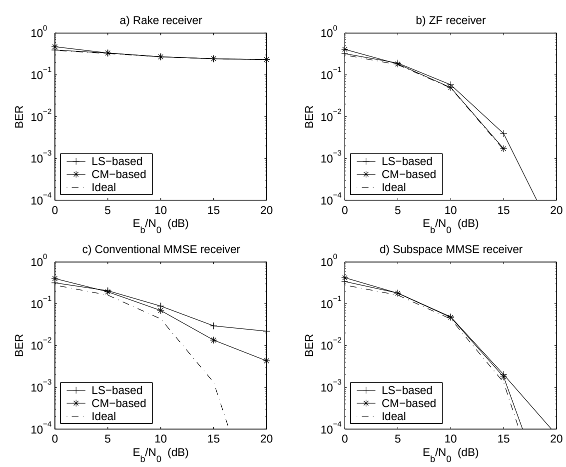

Fig. 2. Detection performance of different receivers.