FREQUENCY-DOMAIN ESTIMATION OF MULTIPLE ACCESS ULTRA-WIDEBAND SIGNALS

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ABSTRACT

In a time-hopping ultra-wideband (UWB) system, a receiver is required to operate over a large bandwidth. Conventional receivers consist of sliding correlators to correlate received signal with a template signal. Such a time-domain waveform detection scheme is essentially a single-user detection method, whose performance has been observed to degrade in a multiple access environment. In this paper, estimation of all users' signals is carried out in the frequency domain. First, Fourier transform (FT) is applied to the received time-domain signal. Then users' information in terms of modulation delays is estimated by frequency-domain multiuser detection (MUD) methods. Frequency diversity at the receiver can be explored for performance enhancement by generating multiple sinusoidal waveforms at different frequencies and processing transformed data in parallel. Its promising feature for combating narrow band interference is expected.

1. INTRODUCTION

Recently there emerges increasing research interest in time-hopping ultra-wideband (UWB) wireless systems due to their appealing features [1] and recent release of the spectral mask from the Federal Communications Commission [2]. However, data/carrier modulation is not specified. Although it remains unclear whether carrier modulation will be used or not in the future, a baseband UWB system can communicate directly by short-duration pulses. It does not require high-frequency modulation/demodulation and can lead to low-cost implementation. Meanwhile, its low probability of interception and detection property is of extreme importance for secure communications.

With on-going widespread deployment of UWB systems [3], reliable signal detection is crucial. Typically, an UWB receiver is required to operate over a large bandwidth [4]. Most existing approaches employ time-domain correlators to correlate the received signal with a template signal [1], [5]. This technique treats multiuser interference (MUI) as Gaussian random process. Thus it does not attempt to explicitly suppress MUI, resulting in performance loss when compared with the optimal detector. In addition, it has been observed that energy capture is highly sensitive to the number of single-path correlators [6]. However, it is very unlikely that hundreds of correlators can be employed due to practical constraints, causing significant performance degradation.

In order to achieve satisfactory performance while maintaining reasonable complexity, we investigate frequency-domain detection methods. Different frequency-domain signal processing techniques have been applied in timing acquisition of UWB transmissions [7], [8], performance enhancement with carrier interferometry pulse shaping [9], or spectral analysis in the presence of timing jitter [4]. In this paper, estimation of all users' signals is studied in the frequency domain by multiuser detection (MUD) which has been rigorously studied in the time domain [10], [11]. First, Fourier transform (FT) is applied to the received continuous-time signals. It can be implemented by correlating the received signal with locally generated sinusoidal waveforms for integration. Since in an UWB system, pulse position modulation (PPM) is currently a typical modulation format [1], each user's information is carried in the delay of the pulse. As is known, FT translates time delay to phase shift to the frequency-domain signal. Then delay estimation reduces to phase detection in the frequency domain. After performing FT and employing temporal diversity of hopping codes, a vector form input/output model pertaining to one symbol interval can be derived. Then typical linear receivers such as minimum mean-square-error (MMSE), or zero-forcing (ZF) receivers can be designed. In order to enhance detector's performance, frequency diversity can be explored by generating multiple sinusoidal waveforms at different frequencies [9], coupled with least-squares fitting from various estimated frequency-domain signals. Bit-error-rate performance of the proposed detector is studied. If high sampling rate is achievable, FT can be replaced by fast FT (FFT) in order to obtain different frequency samples.

2. UWB SYSTEM MODEL

Consider a multiple access time-hopping UWB system with K users. The transmitted baseband UWB signal from user k can be described by [1]

$$s_k(t) = \sqrt{\mathcal{P}_k} \sum_{n=-\infty}^{\infty} w(t - nT_f - c_k(n)T_c - \tau_{I_k(\lfloor n/N_f \rfloor)}) \quad (1)$$

where $\mathcal{P}_k$ is the kth user's transmission power, $w(t)$ is the baseband monopulse, T_f is the frame duration, N_f is the number of frames over which an M-ary PPM symbol repeats, $c_k(n) \in [0, N_c - 1]$ is a pseudorandom (PN) hopping sequence with period N_f, $I_k(\lfloor n/N_f \rfloor) \in [0, M-1]$ is the kth user's information bearing symbol during the nth frame, $\tau_{I_k(\lfloor n/N_f \rfloor)} = I_k(\lfloor n/N_f \rfloor)\sigma$ is the corresponding modulation delay in terms of modulation parameter σ. Assume the transmitted signal $s_k(t)$ propagates through a flat channel with propagation gain α_k. Then the received signal corrupted by additive white Gaussian noise (AWGN) with double-

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side power spectral density $N_0/2$ has the following form:

$$r(t) = \sum_{k=1}^{K} \alpha_k s_k(t) + v(t). \tag{2}$$

Our objective is to estimate all modulation delays $\tau_{I_k(\lfloor n/N_f \rfloor)}$. It is observed that each user's information embedded in the pulse position is inconvenient for processing except some direct methods to capture energy like correlation method. However, once we transform the signal to frequency domain, the problem can be obviated. The nonlinear modulation model (1) even exhibits a linear form. Thus, we will proceed to estimate $\tau_{I_k(\lfloor n/N_f \rfloor)}$ in the frequency domain.

3. FREQUENCY-DOMAIN ESTIMATION

In an UWB system, a symbol repeats N_f frames, and time-hopping codes change from frame to frame but have period N_f. Let us examine received signals within one symbol interval. After applying FT to the corresponding signal (2) over one frame, we obtain

$$\begin{aligned} R_n(f) &= \int_{\text{one frame}} r(t) e^{-j2\pi f t} dt \\ &= \sum_{k=1}^{K} \sqrt{\mathcal{P}_k} W(f) S_{n,k}(f) e^{-j2\pi f \tau_{I_k(\lfloor n/N_f \rfloor)}} + V_n(f) \end{aligned} \tag{3}$$

where

$$S_{n,k}(f) = \alpha_k e^{-j2\pi f c_k(n) T_c}$$

$W(f) = \mathcal{F}(w(t))$, $V(f) = \mathcal{F}(v(t))$, $\mathcal{F}(\cdot)$ represents FT. After collecting N_f frequency-domain signals $R_n(f), \cdots, R_{n+N_f-1}(f)$ in a vector $\boldsymbol{r}_n(f)$, we obtain

$$\boldsymbol{r}_n(f) = \boldsymbol{H}_n(f) \boldsymbol{a}_n(f) + \boldsymbol{v}_n(f), \tag{4}$$

$$\boldsymbol{a}_n(f) = [e^{-j2\pi f \tau_{I_1(\lfloor n/N_f \rfloor)}}, \cdots, e^{-j2\pi f \tau_{I_K(\lfloor n/N_f \rfloor)}}]^T$$

where $\boldsymbol{H}_n(f)$ can be viewed as a channel matrix of all users including hopping codes and propagation gains, $\boldsymbol{a}_n(f)$ includes all unknown modulation delays $\tau_{I_k(\lfloor n/N_f \rfloor)}$ in exponential forms. If we obtain $\boldsymbol{r}_n(f)$ symbol by symbol, then $\boldsymbol{H}_n(f)$ does not depend on n since time-hopping sequences $c_k(n)$ are periodic with period N_f. Then we can drop the subscript n during the rest part of this paper.

Notice that $\boldsymbol{r}_n(f)$ is not a zero mean vector, which will complicate our receiver structure later. We thus consider zero mean data vector after subtracting its mean $\bar{\boldsymbol{r}}_n(f)$

$$\widetilde{\boldsymbol{r}}_n(f) = \boldsymbol{r}_n(f) - \bar{\boldsymbol{r}}_n(f) = \boldsymbol{H}(f) \widetilde{\boldsymbol{a}}_n(f) + \boldsymbol{v}_n(f) \tag{5}$$

where

$$\widetilde{\boldsymbol{a}}_n(f) = \boldsymbol{a}_n(f) - m(f)\mathbf{1},$$

$$m(f) = E\left\{ e^{-j2\pi f \tau_{I_k(\lfloor n/N_f \rfloor)}} \right\} = \frac{\sin(\pi f \sigma M)}{M \sin(\pi f \sigma)} e^{-j\pi f \sigma (M-1)},$$

and $\mathbf{1}$ represents a vector with all elements equal to one. Our task is to estimate all modulation delays. If $\widetilde{\boldsymbol{a}}_n(f)$ is first treated as an unknown, then linear multiuser receivers in a matrix form $\boldsymbol{U}(f)$ can be designed to estimate all its elements, yielding outputs

$$\widehat{\widetilde{\boldsymbol{a}}}_n(f) = \boldsymbol{U}^H(f) \widetilde{\boldsymbol{r}}_n(f). \tag{6}$$

Here, two typical receivers are ZF and MMSE receivers [10]

$$\boldsymbol{U}_{ZF}(f) = \boldsymbol{H}(f) \left(\boldsymbol{H}^H(f) \boldsymbol{H}(f) \right)^{-1}$$

$$\begin{aligned} \boldsymbol{U}_{MMSE}(f) &= \boldsymbol{A}^{-1}(f) E\left\{ \widetilde{\boldsymbol{r}}_n(f) \widetilde{\boldsymbol{a}}_n^H(f) \right\} \\ &= \sin^2(\pi f \sigma) \boldsymbol{A}^{-1}(f) \boldsymbol{H}(f) \end{aligned} \tag{7}$$

where $\boldsymbol{A}(f) = E\left\{ \widetilde{\boldsymbol{r}}_n(f) \widetilde{\boldsymbol{r}}_n^H(f) \right\}$.

Once all elements in $\widetilde{\boldsymbol{a}}_n(f)$ are estimated, multiuser modulation delays can be determined by a least-squares method and data are demodulated. For example, estimate of kth user's information is obtained as follows

$$\widehat{I}_k(\lfloor n/N_f \rfloor) = \arg \min_{I_k(\lfloor n/N_f \rfloor)} \left| \widehat{\widetilde{a}}_{n,k}(f) - \widetilde{a}_{n,k}(f) \right|^2 \tag{8}$$

where we denote the kth element of $\widetilde{\boldsymbol{a}}_n(f)$ as $\widetilde{a}_{n,k}(f)$.

It is observed that our receivers and detectors are frequency dependent. A very natural question arises as which frequency should be used in this estimation method. To answer it, let us first revisit the transformed signal model (5). The signal power of user k is found to be

$$\begin{aligned} &\mathcal{P}_k \left| W(f) \right|^2 \sum_{n'=0}^{N_f-1} \left| S_{n+n',k}(f) \right|^2 E\{ \widetilde{a}_{n,k}(f) \widetilde{a}_{n,k}^*(f) \} \\ &= \alpha_k^2 N_f \mathcal{P}_k \sin^2(\pi f \sigma) \left| W(f) \right|^2 . \end{aligned} \tag{9}$$

It is clear that the strength of useful signal from each user is dependent on frequency while AWGN has constant power spectral density over all frequencies. Fig. 1 illustrates the normalized signal power in the frequency domain. Due to the term $\sin^2(\pi f \sigma)$, the peak of signal power has a small shift from that of $|W(f)|^2$. With this observation, we conclude if there is no other constraint, it is desired to choose frequencies around the point yielding the maximum signal power in order for high SNR. We will further address this issue later by simulation.

We need to point out that flexibility in choosing different frequencies for symbol detection provides possible solutions to avoid coexisting narrow-band interference. This is another major merit for applying frequency domain processing and constitutes a future research topic.

In order to enhance detection performance, we may explore frequency diversity by generating multiple sinusoidal waveforms at N different frequencies [9]. After obtaining $\widehat{\widetilde{\boldsymbol{a}}}_n(f_i)$ at different frequencies for $i = 1, \cdots, N$, estimate of $I_k(\lfloor n/N_f \rfloor)$ can be improved by the least-squares method as well. Considering (8), the criterion becomes

$$\widehat{I}_k(\lfloor n/N_f \rfloor) = \arg \min_{I_k(\lfloor n/N_f \rfloor)} \sum_{i=1}^{N} \left| \widehat{\widetilde{a}}_{n,k}(f_i) - \widetilde{a}_{n,k}(f_i) \right|^2 . \tag{10}$$

Because very short pulses in the order of nanoseconds are used in UWB communication systems, FFT requires analog-to-digital sampling rate of about several gigahertz and thus becomes prohibitive. Therefore, we apply FT operation on the continuous signal directly instead of FFT. To implement FT at a particular frequency f, we generate an exponential function $e^{-j2\pi f t}$ and integrate over one frame interval at the receiver. It is worth mentioning that although we use FT in the above discussion at this stage, we believe with advances of current technology, applying FFT in this method will become technically feasible in the near future [12].

4. PERFORMANCE ANALYSIS

To simplify analysis, we only consider $M = 2$ in this section. For short notation, we will use subscript i to express the dependency of those functions on a particular frequency f_i, if no ambiguity is introduced by doing so.

Without loss of generality, suppose user one is the desired user. Denote the corresponding receiver at frequency f_i as $\boldsymbol{u}_i$. For the nth symbol $I_1(n)$, using (6), the detection criterion (10) becomes

$$\widehat{I}_1(n) = \arg \min_{I_1(n)\in\{0,1\}} \sum_{i=1}^{N} \left| \boldsymbol{u}_i^H \widetilde{\boldsymbol{r}}_{n,i} - \widetilde{a}_{n,i,1} \right|^2 . \qquad (11)$$

$\widetilde{a}_{n,i,1}$ takes only two possible values. For notational convenience, they are defined as $b_{i,0} = 1 - m(f_i)$ when $I_1(n) = 0$, and $b_{i,1} = e^{-j2\pi f_i \sigma} - m(f_i)$ when $I_1(n) = 1$. We will derive bit error probability for given receiver $\boldsymbol{u}_i$ next.

Let's first consider "0" is transmitted. From (11), correct decision is made if

$$\sum_{i=1}^{N} \left| \boldsymbol{u}_i^H \widetilde{\boldsymbol{r}}_{n,i} - b_{i,0} \right|^2 < \sum_{i=1}^{N} \left| \boldsymbol{u}_i^H \widetilde{\boldsymbol{r}}_{n,i} - b_{i,1} \right|^2 . \qquad (12)$$

Here, $\widetilde{\boldsymbol{r}}_{n,i}$ is the data vector as in (5). We can break it down into the desired signal, MUI and noise

$$\widetilde{\boldsymbol{r}}_{n,i} = \boldsymbol{h}_{i,1} \widetilde{a}_{n,i,1} + \boldsymbol{\eta}_{n,i} = \boldsymbol{h}_{i,1} \widetilde{a}_{n,i,1} + \boldsymbol{H}_{i,int} \widetilde{\boldsymbol{a}}_{n,i,int} + \boldsymbol{v}_{n,i} \qquad (13)$$

$\boldsymbol{h}_{i,1}$ is the first column of $\boldsymbol{H}_i$ while $\boldsymbol{H}_{i,int}$ is the new matrix by deleting the first column of $\boldsymbol{H}_i$. $\boldsymbol{\eta}_{n,i}$ includes both MUI and noise. If we assume the combined effect of MUI and noise as a Gaussian random process in time domain [1], then $\boldsymbol{\eta}_{n,i}$ is a complex Gaussian random vector in frequency domain. Its mean is $\mathbf{0}$ and auto-covariance is $E\{\boldsymbol{\eta}_{n,i}\ \boldsymbol{\eta}_{n,i}^H\}$. Using (13), we can express the condition (12) as

$$\mu_n = \sum_{i=1}^{N} \left(\boldsymbol{g}_i^H \boldsymbol{\eta}_{n,i} + \boldsymbol{g}_i^T \boldsymbol{\eta}_{n,i}^* \right) < \sum_{i=1}^{N} e_i \qquad (14)$$

where

$$\boldsymbol{g}_i \triangleq (b_{i,1} - b_{i,0}) \boldsymbol{u}_i$$

$$e_i = |b_{i,1}|^2 - |b_{i,0}|^2 - 2\Re\left\{ b_{i,0}(b_{i,1}^* - b_{i,0}^*) \boldsymbol{u}_i^H \boldsymbol{h}_{i,1} \right\}$$

$\Re\{\cdot\}$ denotes the real part of a complex number.

Because $\boldsymbol{\eta}_{n,i}$ is Gaussian, μ_n is a real Gaussian random variable with zero mean and variance σ_μ^2, which can be shown to be

$$\sigma_\mu^2 = \sum_{i,j} 2\Re\left\{ \boldsymbol{g}_i^H E\left\{ \boldsymbol{\eta}_{n,i} \boldsymbol{\eta}_{n,j}^T \right\} \boldsymbol{g}_j^* + \boldsymbol{g}_i^H E\left\{ \boldsymbol{\eta}_{n,i} \boldsymbol{\eta}_{n,j}^H \right\} \boldsymbol{g}_j \right\} \qquad (15)$$

where

$$E\left\{ \boldsymbol{\eta}_{n,i} \boldsymbol{\eta}_{n,j}^T \right\} = \boldsymbol{H}_{i,int} E\left\{ \widetilde{\boldsymbol{a}}_{n,i,int} \widetilde{\boldsymbol{a}}_{n,j,int}^T \right\} \boldsymbol{H}_{j,int}^T + E\{ \boldsymbol{v}_{n,i} \boldsymbol{v}_{n,j}^T \} \qquad (16)$$

$$E\left\{ \boldsymbol{\eta}_{n,i} \boldsymbol{\eta}_{n,j}^H \right\} = \boldsymbol{H}_{i,int} E\left\{ \widetilde{\boldsymbol{a}}_{n,i,int} \widetilde{\boldsymbol{a}}_{n,j,int}^H \right\} \boldsymbol{H}_{j,int}^H + E\{ \boldsymbol{v}_{n,i} \boldsymbol{v}_{n,j}^H \} \qquad (17)$$

It is easy to find $E\left\{ \widetilde{\boldsymbol{a}}_{n,i,int} \widetilde{\boldsymbol{a}}_{n,j,int}^T \right\} = \alpha_{i,j} \boldsymbol{I}$ with $\alpha_{i,j} = m(f_i + f_j) - m(f_i)m(f_j)$ and $E\left\{ \widetilde{\boldsymbol{a}}_{n,i,int} \widetilde{\boldsymbol{a}}_{n,j,int}^H \right\} = \alpha'_{i,j} \boldsymbol{I}$, with $\alpha'_{i,j} = m(f_i - f_j) - m(f_i)m^*(f_j)$. $E\{\boldsymbol{v}_{n,i}\ \boldsymbol{v}_{n,j}^T\} = \gamma_{i,j} \boldsymbol{I}$ and $E\{\boldsymbol{v}_{n,i}\ \boldsymbol{v}_{n,j}^H\} = \gamma'_{i,j} \boldsymbol{I}$ with

$$\gamma_{i,j} = \frac{N_0 \sin\left(\pi(f_i + f_j)T_f\right)}{2\pi(f_i + f_j)} e^{-j\pi(f_i + f_j)T_f},$$

$$\gamma'_{i,j} = \frac{N_0 \sin\left(\pi(f_i - f_j)T_f\right)}{2\pi(f_i - f_j)} e^{-j\pi(f_i - f_j)T_f}.$$

Then σ_μ^2 can be obtained. With (14) and (15), we thus obtain the error probability when "0" is transmitted as $P_{E,0} = Q(\sum_i e_i/\sigma_\mu)$, where $Q(x) \triangleq \int_x^\infty \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx$. Similarly, we can find the error probability $P_{E,1}$ when "1" is transmitted. Because $b_{i,0} = -b_{i,1}$, it is easily shown that $P_{E,1} = P_{E,0}$. Suppose two information symbols have equal *a priori* probability. Then bit error rate (BER) can be calculated

$$P_E = \frac{1}{2}\left(P_{E,0} + P_{E,1} \right). \qquad (18)$$

This analytical result will be compared with experimental result in our simulation.

5. SIMULATION

We provide some numerical examples by simulation. Normalized second derivative of Gaussian function with duration of 0.7 nanoseconds is chosen as basic monocycle for the binary PPM system [1]. Modulation delay parameter is set to be $\sigma = 0.15$ nanoseconds. $N_c = 4$ and $N_f = 8$ [10]. Four users are active in the system with random time hopping codes.

We examine the effect of frequency diversity on estimation performance in Fig. 2. The first question to answer is what's the frequency range we should exploit to achieve the best performance. Fig. 2(a) illustrates BER with respect to bandwidth up to $1.6dB$ with frequency sample interval $12.2MHz$ and E_b/N_0 $10dB$. Here, the bandwidth is specified in terms of normalized signal power according to (9) (c.f. Fig. 1). Performance improves rapidly when frequency band increases from $0dB$ to $0.4dB$. However, such an improvement becomes gradual and the performance even slightly degrades when the bandwidth is above $0.6dB$ because more noisy data are included then.

Because we can only utilize finite frequency samples, frequency sample spacing is another design parameter we need to decide. Fig. 2(b) shows the result with respect to frequency resolution within fixed bandwidth $1.6dB$. The result reveals no significant improvement with finer resolution as long as time aliasing can be avoided during FT. This is meaningful during practical implementation because we can use fewer frequency samples within certain bandwidth.

In Fig. 3, we compare our proposed frequency domain detection method with conventional time domain correlator. We apply both the ZF receiver and the subspace-based MMSE receiver [13] for our frequency domain detection approach. We can see under typical operating SNR conditions, e.g., at $10dB$, our method outperforms the conventional method. When SNR increases, such an improvement becomes more significant. In addition, the analytical curve for the MMSE receiver from our analysis matches our experimental result. However, the large difference at high SNR between analytical and experimental results for the ZF receiver needs further investigation despite possible violation of the Gaussian assumption in our analysis.

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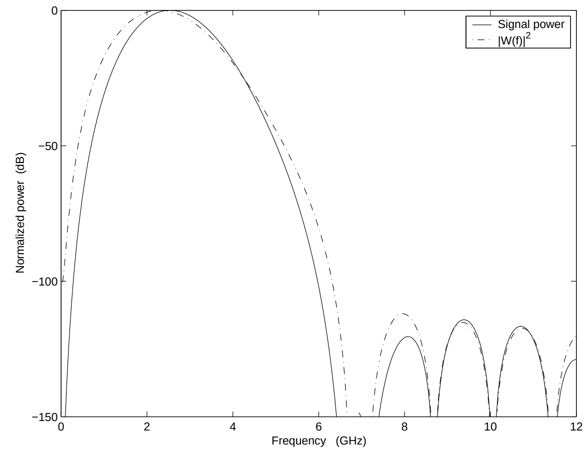

Fig. 1. Frequency-domain power distribution.

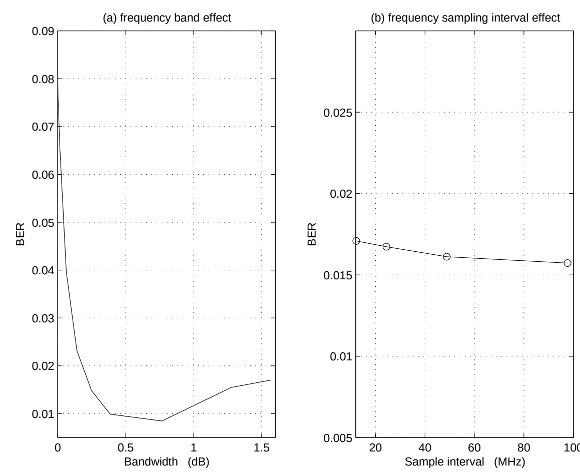

Fig. 2. Effect of frequency diversity on BER performance.

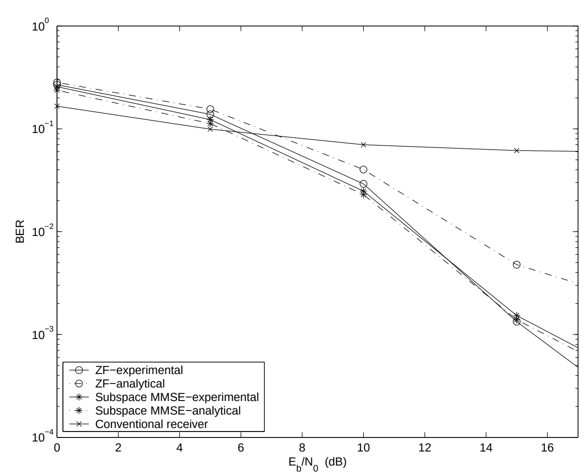

Fig. 3. BER performance of different detectors.