BLIND ADAPTIVE MULTIUSER DETECTION FOR UPLINK LONG-CODE CDMA SYSTEMS

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ABSTRACT

In the uplink of a long-code CDMA system, base station knows spreading codes of all serviced users. Given propagation delays, a blind adaptive CMA-based approach, with the aid of a set of MMSE-like constraints parameterized by channel-like vectors, is proposed to detect all users' symbols simultaneously. As by-products, the channel-like vectors are also obtained for all users. Since the constraints involve covariance of the received data, which is time varying and can not be obtained by traditional sample average, we thus propose to approximate it using estimated signature matrix and noise power. Simulation results show satisfactory performance of the proposed method.

1. INTRODUCTION

Study of long-code CDMA systems has attracted considerable attention in recent years. However, research for long-code uplink communications has been mainly limited to either training based solutions, or blind solutions but using first and second order statistics. Given pilot symbols of all users, least squares (LS) fitting or iterative maximum likelihood (ML) approaches have been reported [1], [2]. Blind channel estimation methods have been proposed using correlation matching techniques [3]. Blind space-time RAKE receivers are designed to maximize the output signal to interference plus noise ratio (SINR) [4]. Code decorrelating and subspace fitting are applied for both channel estimation and symbol detection in [5], [6]. Although higher order statistics (HOS) based approaches, by exploiting different constraints, have shown excellent performance in short code CDMA systems [7], [8], [9], [10], [11], [12], they are not applicable to long-code systems due to violation of cyclostationarity induced by time-varying codes.

In this paper, we show that, by imposing a set of MMSE-like constraints parameterized by conditional data covariance and channel-like vectors on receivers, constant modulus algorithm (CMA) can be applied to detect all users' symbols simultaneously in an uplink long-code system. The new constraints involve conditional data covariance, which can not be simply obtained by sample average as done in short code systems due to nonstationarity of the received signal, but can be estimated from estimated signature matrix and noise power each time. Adaptive implementation of the proposed method is developed. Convergence properties of the proposed algorithm are analyzed. Simulation results show satisfactory performance of the proposed approach.

2. UPLINK LONG-CODE CDMA SYSTEM MODEL

Consider a quasi-synchronous uplink CDMA system, where J users are communicating with a base station. The jth user has its symbol $w_j(n)$ first spread by aperiodic codes $c_{j,n}(k)$ ($k = 0, \dots, P-1$), and then transmitted through a multipath channel $g_j(m)$ with maximum order q. Then the chip-rate signal from J users arriving at the base station becomes $y(k)=\sum_{j=1}^{J}\sum_{m=0}^{q} g_j(m)\, s_j(k-m-d_j)+v(k)$, where d_j is the jth user's delay in chip periods and assumed to satisfy $0 \le d_j \ll P$, and $s_j(k) = \sum_{n=-\infty}^{\infty} w_j(n)c_{j,n}(k-nP)$. If our observation window spans ν symbol intervals, then the data vector containing $L = \nu P$ chip samples from time $nP - L + 1$ to nP can be expressed in the following form

$$\begin{aligned} \boldsymbol{y}_n &= \sum_{j=1}^{J}\sum_{\xi_j=0}^{\alpha_j-1} \boldsymbol{h}_j(n-\xi_j)w_j(n-\xi_j) + \boldsymbol{v}(n) \\ &= \boldsymbol{H}(n)\boldsymbol{w}(n) + \boldsymbol{v}(n) \end{aligned} \quad (1)$$

where

$$\boldsymbol{H}(n) = [\boldsymbol{h}_1(n)\dots,\boldsymbol{h}_1(n-\alpha_1+1),\dots,\boldsymbol{h}_J(n-\alpha_J+1)],$$

α_j is the number of the jth user's symbols contributed to $\boldsymbol{y}_n$. All columns of $\boldsymbol{H}(n)$ represent signature waveforms of different symbols. They are related to spreading codes and channel parameters as follows

$$\boldsymbol{h}_j(n-\xi_j) = \boldsymbol{C}_j(n-\xi_j)\boldsymbol{g}_j,$$

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$$C_j(n-\xi_j) = J^{P(\nu-1-\xi_j)+d_j}\widetilde{C}_j(n-\xi_j),$$

$\widetilde{C}_j(n)$ is a code filtering matrix with some zeros padded at the tail [11], g_j is the jth user's channel vector, $w(n)$ contains all user's symbols at different delays corresponding to $H(n)$, and $v(n)$ is AWGN vector with variance $\sigma_v^2 I$. J is a shift matrix of dimension $L \times L$ with all elements in the sub-diagonal below the main diagonal $1'$s, $J^{-1} \triangleq J^T$, $J^0 \triangleq I$. If we define $w_j(n)$ to be the desired signal for user j in the observation interval of $y(n)$, then its signature waveform becomes $h_j(n) = C_j(n)g_j$.

In the next section, we will develop an algorithm to detect all users' symbols simultaneously. For simplicity, we assume that $E\{|w_j(n)|^2\} = 1$ for $j = 1, \ldots, J$, since each user's symbol power can be migrated into the user's channel vector to form a new combined channel.

3. ADAPTIVE MULTIUSER DETECTION APPROACH

If we focus on linear solutions, then the MMSE receiver has been shown to be the optimal receiver in the sense it minimizes the mean square error. To simultaneously detect all users' symbols $w_j(n)$, we need to design J linear receivers. Given all users' codes, propagation delays, multipath channels and noise power, the MMSE receiver for user j can be directly constructed as

$$f_j(n) = R^{-1}(n)C_j(n)g_j \tag{2}$$

where $R(n) = H(n)H(n)^H + \sigma_v^2 I$ represents the data covariance conditioned on all long codes. Different from short code CDMA system, $R(n)$ is time varying. Although the MMSE receiver is the best linear receiver, direct construction of the MMSE receiver for each user based on (2) is impossible, since channel vectors and noise power are generally unknown. A blind method is thus desirable. Since CMA cost function has shown performance close to the MMSE solution, we propose to minimize the total CMA cost subject to a set of MMSE-like constraints on each user's receiver

$$\min_{f_1(n),\ldots,f_J(n)} \mathcal{J}_{CMA} = \sum_{j=1}^{J} E\{(|z_j(n)|^2 - 1)^2\},$$

$$\text{subject to } \; f_j(n) = R^{-1}(n)C_j(n)u_j, \;\; j = 1, \ldots, J \tag{3}$$

where $z_j(n) = f_j^H(n)y_n$ is the jth user's output, u_j is channel-like constraint vector for user j. The set of constraints aim at removing both delay and user ambiguities. Different from the approach for short code systems [11], conditional covariance matrix $R(n)$ is used in the constraints. If we directly apply the constraints to the jth output, then $z_j(n) = u_j^H C_j(n)^H R^{-1}(n)y_n$. In this way, the constrained optimization problem w.r.t. time-varying quantities $f_j(n)$ described by (3) is transformed to the following unconstrained one w.r.t. time-invariant constraint vectors

$$\min_{u_1,\ldots,u_J} \mathcal{J}(u_1, \ldots, u_J) \;\; = \;\; \sum_{j=1}^{J} E\{(u_j^H C_j^H(n)R^{-1}(n)y_n y_n^H R^{-1}(n)C_j(n)u_j - 1)^2\}. \tag{4}$$

The cost function in (4) is highly nonlinear. Closed-form solutions are impossible. Therefore, we apply gradient decent approach to obtain the solutions. Based on (4), u_j ($j = 1, \ldots, J$) is updated as the following,

$$\widehat{u}_j(n+1) = \widehat{u}_j(n) - \mu_j \nabla_{u_j(n)} \mathcal{J}(u_1, \ldots, u_J) \tag{5}$$

where μ_j is the step size for the jth user's constraint vector, $\nabla_{u_j(n)}\mathcal{J}$ is the gradient with respect to u_j and evaluated at time n. The gradient can be approximated by its instantaneous value as

$$\nabla_{u_j(n)}\mathcal{J} = 4[(|z_j(n)|^2 - 1]z_j^*(n)C_j^H(n)R^{-1}(n)y_n.$$

If we regard u_j as the jth user's channel vector, the jth user's receiver at time instant n is then constructed as $\widehat{f}_j(n)$ $=R^{-1}(n)C_1(n)u_j(n)$. Noticing that the conditional covariance matrix $R(n)$ is time-varying and can not be estimated by sample average as in short-code systems, we thus propose to approximate it directly from estimated signature matrix and noise as $\widehat{H}(n)\widehat{H}^H(n) + \widehat{\sigma}_v^2 I$. $\widehat{H}(n)$ can be constructed as (1) based on estimated channel vectors u_j ($j = 1, \ldots, J$) at each time instant. Since noise is stationary, its power is then estimated by sample average as $\widehat{\sigma}_v^2 = \beta\widehat{\sigma}_v^2 + (1-\beta)\widehat{v}(n)^H\widehat{v}(n)$, where the noise vector is approximated by

$$\widehat{v}(n) = y_n - \widehat{H}(n)\widehat{w}(n), \tag{6}$$

$\widehat{w}(n)$ contains estimated symbols at different delays. The most recent symbol is estimated by $\widehat{w}_j(n) = \widehat{f}_j^H(n)z(n)$ for each user j. All other past symbols with different delays have been directly obtained from previous iterations. On the other hand, although phase ambiguity exists in the estimated channel vectors, as will be shown in the next section, it can be easily verified that this ambiguity will not affect the combined term $\widehat{H}(n)\widehat{w}(n)$. Therefore $v(n)$ can be estimated correctly.

4. CONVERGENCE ANALYSIS

We first establish the global convergence of the proposed method based on (4). For convenience, assume all quantities are real and all inputs take values ± 1 with equal probability. However, results can be easily generalized to a complex system after subtle modification.

Proposition : When the covariance matrix $\boldsymbol{R}(n)$ is perfectly known, $\sigma_v^2 \to 0$, the optimal $\boldsymbol{u}_j$ obtained by the unconstrained optimization (4) will converge to the multipath channel $\boldsymbol{g}_j$ up to a phase ambiguity ± 1 for $j = 1, \ldots, J$.

Proof: If we denote $\mathcal{J}_j = E\{(\boldsymbol{u}_j^H \boldsymbol{C}_j^H(n)\boldsymbol{R}^{-1}(n)\boldsymbol{y}_n \boldsymbol{y}_n^H \boldsymbol{R}^{-1}(n)\boldsymbol{C}_j(n)\boldsymbol{u}_j - 1)^2\}$, and notice that $\mathcal{J}_j$ is positive, then (4) is equivalent to

$$\min_{\boldsymbol{u}_j} \mathcal{J}_j, \text{ for } j = 1, \ldots, J \tag{7}$$

which shows that joint optimization described by (4) is equivalent to individual optimization of $\mathcal{J}_j$ for each user j. Moreover, since $\boldsymbol{R}(n)$ is assumed known at each time n, then the optimization problem $\min_{\boldsymbol{u}_j} \mathcal{J}_j$ for user j becomes similar to the unconstrained approach in [11], which has been shown to globally converge to the jth user's channel vector up to ± 1. Therefore, the proposition holds. $\square$

Further convergence analysis after considering the estimated time-varying covariance matrix will be prohibitively difficult. Instead, we still assume that the conditional covariance is known, and turn to the convergence property of the adaptive implementation. Let the jth user's channel estimation error at time instant n be denoted as $\boldsymbol{e}_j(n) = \boldsymbol{u}_j(n) - \boldsymbol{g}_j$. According to (7), we have

$$\boldsymbol{e}_j(n+1) = \boldsymbol{e}_j(n) - \mu_j \nabla_{\boldsymbol{u}_j(n)} \mathcal{J}_j. \tag{8}$$

Since at the optimal point $\nabla_{\boldsymbol{g}_j} \mathcal{J}_j = 0$, then

$$\begin{aligned} \boldsymbol{e}_j(n+1) &= \boldsymbol{e}_j(n) - \mu_j[\nabla_{\boldsymbol{u}_j(n)}\mathcal{J}_j - \nabla_{\boldsymbol{g}_j}\mathcal{J}_j] \\ &= \boldsymbol{e}_j(n) - \mu_j \nabla^2_{\boldsymbol{g}_j+\alpha\boldsymbol{e}_j(n)}\mathcal{J}_j \boldsymbol{e}_j(n) \\ &= (\boldsymbol{I} - \mu_j \nabla^2_{\boldsymbol{g}_j+\alpha\boldsymbol{e}_j(n)}\mathcal{J}_j)\boldsymbol{e}_j(n) \end{aligned} \tag{9}$$

where $0 < \alpha < 1$, $\nabla^2_{\boldsymbol{g}_j+\alpha\boldsymbol{e}_j(n)}\mathcal{J}_j)$ is the Hessian matrix of the cost function evaluated at $\boldsymbol{u}_j(n) = \boldsymbol{g}_j + \alpha\boldsymbol{e}_j(n)$. $\nabla^2_{\boldsymbol{u}_j}\mathcal{J}_j$ can be shown to have a form [11]

$$\begin{aligned} \nabla^2_{\boldsymbol{u}_j}\mathcal{J}_j &= -24\sum_{i=1}^{M}[(\boldsymbol{u}_j^T\boldsymbol{C}_j(n)\boldsymbol{R}^{-1}(n)\boldsymbol{h}_i)^2\boldsymbol{C}_j(n)^H \\ &\qquad \boldsymbol{R}^{-1}(n)\boldsymbol{h}_i\boldsymbol{h}_i^H\boldsymbol{R}^{-1}(n)\boldsymbol{C}_j(n)] \\ &+ (12\boldsymbol{u}_j^H\boldsymbol{A}_j(n)\boldsymbol{u}_j - 4)\boldsymbol{A}_j(n) \\ &+ 24\boldsymbol{A}_j(n)\boldsymbol{u}_j\boldsymbol{u}_j^H\boldsymbol{A}_j(n) \end{aligned} \tag{10}$$

where dependence of $\boldsymbol{h}_i$ on time has been dropped for notational convenience, M is the number of columns of $\boldsymbol{H}(n)$, and $\boldsymbol{A}_j(n) = \boldsymbol{C}_j(n)^H\boldsymbol{R}^{-1}(n)\boldsymbol{C}_j(n)$. Therefore, the adaptive algorithm converges to the optimal point if and only if

$$0 < \mu_j < \frac{2}{\max_n \lambda_{max}(\nabla^2_{\boldsymbol{g}_j+\alpha\boldsymbol{e}_j(n)}\mathcal{J}_j)} \tag{11}$$

where $\lambda_{max}(\nabla^2_{\boldsymbol{g}_j+\alpha\boldsymbol{e}_j(n)}\mathcal{J}_j)$ denotes the largest eigenvalue of the Hessian matrix at time instant n. Next, we follow [13] to further analyze the effect of users' codes and noise power on the convergence, where (11) is assumed to be satisfied. Then $\boldsymbol{g}_j + \alpha\boldsymbol{e}_j(n)$ can be viewed as the jth user's channel estimate but with an estimation error $\alpha\boldsymbol{e}_j(n)$. For large n, $\boldsymbol{e}_j(n)$ has converged to the steady-state channel estimation error, which has been shown to be at the order of $O(\sigma_v^4)$ [11]. Therefore, omitting the higher order terms, $\nabla^2_{\boldsymbol{g}_j+\alpha\boldsymbol{e}_j(n)}\mathcal{J} \approx 8\boldsymbol{A}_j(n)$, and (11) is simplified as

$$0 < \mu_j < \frac{0.25}{\max_n \lambda_{max}(\boldsymbol{A}_j(n))}. \tag{12}$$

Eq. (12) shows that the convergence of the proposed algorithm is affected by all users' codes, each user's channel vector, transmission power and noise power.

5. SIMULATION RESULT

We consider a long-code CDMA system with 4 asynchronized equal powered users, each transmitting BPSK signals. The parameters are set to $P = 16$, $q = 3$, $J = 4$, $\nu = 2$, $\mu_j = 0.0008$, $\beta = 0.995$. In choosing μ_j, (12) has been used as a guideline. Each user's propagation delay is uniformly distributed from 0 to $P/2$, while path coefficient is assumed Gaussian distributed with unit variance. Once generated, they are fixed for all 100 realizations. But users' long spreading codes are randomly generated over different iterations and realizations. Input SNR is set to be 15dB. Without loss of generality, user 1 is assumed the desired user. We first plot in Fig. 1 the average output SINR. It can be observed that the average output SINR achieves a satisfactory level of 12 dB, which is very close to the theoretical line (straight line) computed from ideal MMSE receiver. Since, the proposed approach also provides channel estimate, we further illustrate the average channel estimation mean square error in Fig. 2. It can be observed that the MSE converges to a level of 10^{-2} after about 1000 iterations. Fig. 3 further plots the BER performance of user 1 over different SNRs. The BER is computed after 3000 iterations. We can see that the proposed method yields a very satisfactory BER performance.

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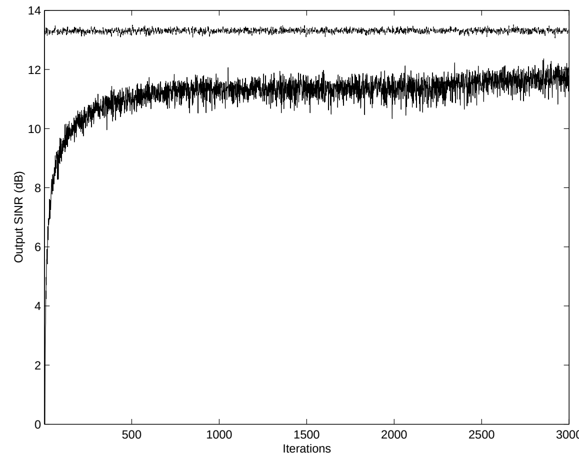

Fig. 1. Average output SINR.

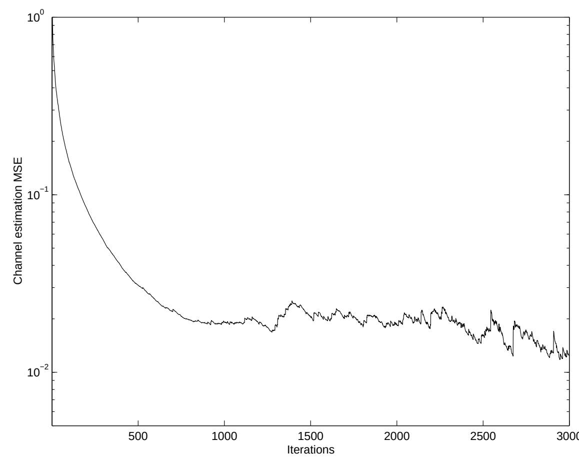

Fig. 2. Channel estimation MSE.

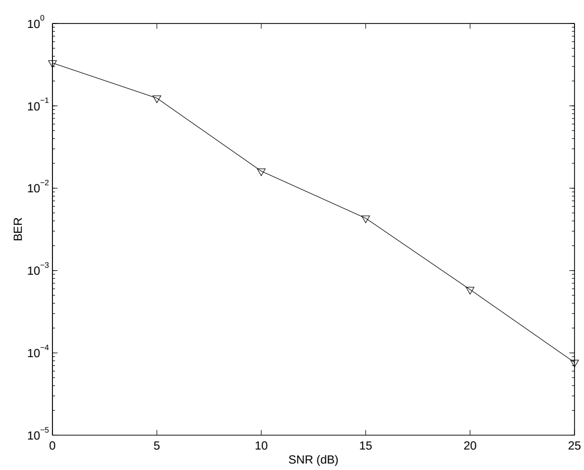

Fig. 3. BER performance.