On Performance Study of Data Based Multiuser Detection Methods for Uplink Long-Code CDMA Systems

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***Abstract*— In a long-code uplink CDMA system, subspace method is recently proposed to estimate all users' multipath parameters and noise power based on the correlation matrix of whitened data. In this paper, we focus on performance of three typical linear receivers, known as zero-forcing (ZF), minimum mean-square-error (MMSE), and RAKE receivers, constructed from the estimated channels and noise power. All analyses are performed from a perturbation perspective and verified by our numerical examples.**

I. Introduction

Study of long-code CDMA systems has attracted considerable attention in recent years. Tremendous efforts have been focused on developing solutions for downlink communications [1], [2]. Uplink communications pose new challenges due to different code assignment strategies. Given pilot symbols of all users, least squares (LS) fitting or iterative maximum likelihood (ML) approaches have been reported [3], [4]. Blind methods have also appeared using correlation techniques [5], or employing a space-time 2D RAKE receiver structure to maximize the output signal to interference plus noise ratio (SINR) [6], LS [7], or exploiting code correlation and subspace based channel estimation techniques [8].

In this paper, we analyze performance of three linear detectors: ZF, MMSE and RAKE receivers constructed from the estimated channels and noise by the method [8]. Since finite data samples cause channel estimate, noise power and finally the detectors to deviate from their ideal forms, to quantify the data effect, each receiver's SINR and bit-error-rate (BER) are derived from a perturbation perspective. Unlike previous work [9], [10], which studied the effect of random codes on the fluctuation of the SINR of receivers constructed from perfect signatures, our analysis considers the effect of imperfect signatures caused by finite data samples on the performance of receivers. High consistency is observed between our experimental and analytical results.

II. Uplink CDMA System Model

Consider a quasi-synchronous uplink CDMA system [5], where J mobile stations are communicating with a base station. The ith user's bit $w_i(n)$ is first spread by aperiodic codes $c_{i,n}(k)$ ($k = 0, \ldots, P-1$), and then transmitted through a multipath channel $g_i(m)$. All channels are assumed to have maximum order q ($q << P$). With quasi-synchronization, the ith user's delay d_i is assumed much less than P. Then, if we collect only $L = P - \mu$ samples in the nth bit interval into a vector $\boldsymbol{y}(n) = [y(nP + \mu), \ldots, y(nP + P - 1)]^T$ with $\mu = max\{q + d_i\}$, the received data vector follows a simple matrix form [8]

$$\boldsymbol{y}(n) = \mathcal{C}(n)\mathcal{G}\boldsymbol{w}(n) + \boldsymbol{v}(n) = \boldsymbol{H}(n)\boldsymbol{w}(n) + \boldsymbol{v}(n) \quad (1)$$

where

$$\begin{aligned} \mathcal{C}(n) &= [\boldsymbol{C}_1(n), \ldots, \boldsymbol{C}_J(n)], \\ \mathcal{G} &= diag\{\boldsymbol{g}_1, \ldots, \boldsymbol{g}_J\}, \\ \boldsymbol{H}(n) &= [\boldsymbol{C}_1(n)\boldsymbol{g}_1, \ldots, \boldsymbol{C}_J(n)\boldsymbol{g}_J] = \mathcal{C}(n)\mathcal{G}, \end{aligned} \quad (2)$$

$\boldsymbol{C}_j(n)$ is the truncated code filtering matrix of user j, $\boldsymbol{g}_i$ is the channel vector, $\boldsymbol{w}(n) = [w_1(n), \ldots, w_J(n)]^T$ and $\boldsymbol{v}(n) = [v(nP + \mu) , \ldots , v(nP + P - 1)]^T$ is an AWGN vector with variance $\sigma_v^2 \boldsymbol{I}$.

III. Symbol Detection

In [8], we have shown that the ith user's channel can be estimated if we first decorrelate the received data vector at each symbol as $\bar{\boldsymbol{u}}(n) = \mathcal{C}(n)^{\dagger}\boldsymbol{y}(n)$ where † represents pseudo-inverse, and then partition its covariance $\bar{\boldsymbol{R}} = \boldsymbol{G}\boldsymbol{G}^H + \sigma_v^2 \boldsymbol{A}$ where $\boldsymbol{A} \triangleq E\{\mathcal{C}(n)^{\dagger}(\mathcal{C}(n)^{\dagger})^H\} = E\{(\mathcal{C}(n)^H\mathcal{C}(n))^{-1}\}$ into $J \times J$ submatrices. Since the (j, j)th submatrix is $\bar{\boldsymbol{R}}_i = \boldsymbol{g}_i\boldsymbol{g}_i^H + \sigma_v^2 \boldsymbol{A}_i$, further whitening it yields

$$\boldsymbol{R}_j \triangleq \boldsymbol{A}_j^{-\frac{1}{2}}\bar{\boldsymbol{R}}_j\boldsymbol{A}_j^{-\frac{1}{2}} = \boldsymbol{A}_j^{-\frac{1}{2}}\boldsymbol{g}_j\boldsymbol{g}_j^H\boldsymbol{A}_j^{-\frac{1}{2}} + \sigma_v^2 \boldsymbol{I} \quad (3)$$

where $\boldsymbol{A}_j \triangleq \boldsymbol{S}_j^T \boldsymbol{A} \boldsymbol{S}_j$ and $\boldsymbol{S}_j$ is a selection matrix defined as $\boldsymbol{S}_j = [\boldsymbol{0};\ \ \boldsymbol{I};\ \ \boldsymbol{0}]^T$. Applying the subspace technique to $\boldsymbol{R}_j$ immediately yields the channel estimate for user j as $\chi_j = argmax_{||\boldsymbol{\beta}||=1} \boldsymbol{\beta}^H \boldsymbol{R}_j \boldsymbol{\beta}$, $\boldsymbol{g}_j = \frac{\boldsymbol{A}_j^{\frac{1}{2}} \chi_j}{||\boldsymbol{A}_j^{\frac{1}{2}} \chi_j||}$, and the noise power as $\sigma_v^2 = \frac{1}{q}\sum_{i=2}^{q+1} \lambda_i$ where λ_i's are q least eigenvalues of $\boldsymbol{R}_j$. It can be easily verified from (3) that $\sigma_v^2 = \frac{1}{q} tr\{(\boldsymbol{U}_n^j)^H \boldsymbol{R}_j \boldsymbol{U}_n^j\}$, where "$tr$" is a trace operator.

Once all users' channel vectors are estimated, different receivers can be constructed. Here we present three typical linear symbol receivers, known as ZF, MMSE and RAKE receivers. Without loss of generality, user 1 is assumed to be the desired user. Since the signature matrix can be estimated at each time instant n using all users' estimated channel vectors and spreading codes as (2), the ZF receiver at time instant n is constructed as

$$\boldsymbol{f}_{zf}(n) = \boldsymbol{H}(n)(\boldsymbol{H}(n)^H \boldsymbol{H}(n))^{-1} \boldsymbol{e} \tag{4}$$

where $\boldsymbol{e}$ is a unitary vector with the first element as 1. Then the desired symbol is estimated by $\widehat{w}_{1,zf}(n) = \boldsymbol{f}_{zf}^H(n)\ \boldsymbol{y}(n)$. Similarly, the MMSE receiver is defined as

$$\boldsymbol{f}_{mmse}(n) = \mathcal{R}(n)^{-1} \boldsymbol{C}_1(n) \boldsymbol{g}_1 \tag{5}$$

where $\mathcal{R}(n)$ is the correlation matrix of $\boldsymbol{y}(n)$ at time n. Noticing that $\mathcal{R}(n)$ can not be estimated by conventional sample average. However, it can be constructed by

$$\mathcal{R}(n) = \boldsymbol{H}(n)\boldsymbol{H}(n)^H + \sigma_v^2 \boldsymbol{I}, \tag{6}$$

once $H(n)$ is constructed as (2) from all estimated channel vectors and estimated σ_v^2. The detected symbol is given by $\widehat{w}_{1,mmse}(n) = \boldsymbol{f}_{mmse}^H(n)\boldsymbol{y}(n)$. The RAKE receiver at time n is constructed as the desired signature [2]

$$\boldsymbol{f}_{rake}(n) = \boldsymbol{h}_1(n) \triangleq \boldsymbol{C}_1(n)\boldsymbol{g}_1, \tag{7}$$

and the desired user's symbol is estimated as $\widehat{w}_{1,rake}(n) = \boldsymbol{f}_{rake}^H(n)\boldsymbol{y}(n)$. Performance of these receivers will be evaluated next.

IV. Analytical Results

When estimated from finite data samples, data covariance matrix gets perturbed. Its perturbation will be carried over to both channel estimate and receivers. Denoting the perturbation by preceding the corresponding quantity by δ, and the perturbed quantity with $\tilde{}$, then the jth user's channel perturbation and its covariance have been shown to be [8]

$$\begin{aligned}
\delta \boldsymbol{g}_j &\approx \frac{1}{\gamma_j} \boldsymbol{\Pi}_{\boldsymbol{g}_j}^{\perp} \boldsymbol{A}_j^{\frac{1}{2}} \boldsymbol{U}_n^j (\boldsymbol{U}_n^j)^H \delta \boldsymbol{R}_j \chi_j, \\
E\{\delta \boldsymbol{g}_j \delta \boldsymbol{g}_j^H\} &\approx \frac{\sigma_v^2}{N} \boldsymbol{\Pi}_{\boldsymbol{g}_j}^{\perp} \boldsymbol{A}_j^{\frac{1}{2}} \boldsymbol{U}_n^j (\boldsymbol{U}_n^j)^H \boldsymbol{A}_j^{\frac{1}{2}} \boldsymbol{\Pi}_{\boldsymbol{g}_j}^{\perp}
\end{aligned} \tag{8}$$

where $\gamma_j^2 = \boldsymbol{g}_j^H \boldsymbol{A}_j^{-1} \boldsymbol{g}_j$, $\boldsymbol{\Pi}_{\boldsymbol{g}_j}^{\perp} \triangleq (\boldsymbol{I} - \frac{\boldsymbol{g}_j \boldsymbol{g}_j^H}{\boldsymbol{g}_j^H \boldsymbol{g}_j})$, $\chi_j = \frac{\boldsymbol{A}_j^{-\frac{1}{2}} \boldsymbol{g}_j}{||\boldsymbol{A}_j^{-\frac{1}{2}} \boldsymbol{g}_j||}$, $\boldsymbol{U}_n^j$ denotes the noise subspace orthogonal to χ_j. In this section, we will study the statistical performance of receivers in terms of SINR and BER based on (8).

A. SINRs of Different Receivers

SINR is an important performance indicator for receivers. The average SINR can be defined as $\mathrm{SINR} = \frac{\Phi(\mathcal{R}_1(n))}{\Phi(\mathcal{R}_{int}(n))}$ where $\mathcal{R}_1(n) = \boldsymbol{h}_1(n)\boldsymbol{h}_1(n)^H$, $\mathcal{R}_{int}(n) = \mathcal{R}(n) - \mathcal{R}_1(n)$, $\Phi(\boldsymbol{X}) = E\{\boldsymbol{f}(n)^H \boldsymbol{X} \boldsymbol{f}(n)\}$ denotes unperturbed signal or noise power with $\boldsymbol{X}$ replaced by $\mathcal{R}_1(n)$ or $\mathcal{R}_{int}(n)$, $\boldsymbol{f}(n)$ is any symbol receiver. Although fluctuation of SINR can been analyzed under perfect conditions, perturbation in channel estimation induced by finite data samples inevitably causes the receiver perturbed as $\widetilde{\boldsymbol{f}}(n) = \boldsymbol{f}(n) + \delta \boldsymbol{f}(n)$, where the first order perturbation $\delta \boldsymbol{f}(n)$ is assumed to have zero mean. Therefore perturbed SINR becomes

$$\widetilde{\mathrm{SINR}} = \frac{\Phi(\mathcal{R}_1(n)) + \Psi(\mathcal{R}_1(n))}{\Phi(\mathcal{R}_{int}(n)) + \Psi(\mathcal{R}_{int}(n))} \tag{9}$$

where $\Psi(\boldsymbol{X}) = E\{\delta \boldsymbol{f}(n)^H \boldsymbol{X} \delta \boldsymbol{f}(n)\}$ denotes perturbed power. Since different receivers take different forms, $\widetilde{\mathrm{SINR}}$ will be evaluated for each receiver respectively. For shorter notations, all receivers' subscripts are dropped later.

1) Perturbed SINR of the ZF Receiver: First, replacing the ZF receiver with (4), it can be shown that $\Phi(\mathcal{R}_1(n)) = 1$ and $\Phi(\mathcal{R}_{int}(n)) = \sigma_v^2 \boldsymbol{e}^H E\{[\boldsymbol{H}(n)^H \boldsymbol{H}(n)\ ]^{-1}\} \boldsymbol{e}$. It is much involved to further simplify $\Phi(\mathcal{R}_{int}(n))$, Therefore, approximation is obtained by time average over codes as $\Phi(\mathcal{R}_{int}(n)) \approx \sigma_v^2 \boldsymbol{e}^H \frac{1}{N} \sum_{n=1}^{N} [\boldsymbol{H}(n)^H \boldsymbol{H}(n)]^{-1} \boldsymbol{e}$. To evaluate the perturbation term $\Psi(\boldsymbol{X})$, $\delta \boldsymbol{f}(n)$ is first obtained according to (4). Since $\delta \boldsymbol{H}(n)$ is given by

$$\delta \boldsymbol{H}(n) = [\boldsymbol{C}_1(n)\delta \boldsymbol{g}_1, \ldots, \boldsymbol{C}_J(n)\delta \boldsymbol{g}_J], \tag{10}$$

noticing $\widetilde{\boldsymbol{H}}(n) = \boldsymbol{H}(n) + \delta \boldsymbol{H}(n)$, expanding $(\widetilde{\boldsymbol{H}}(n)^H \widetilde{\boldsymbol{H}}(n))^{-1}$ using Taylor series and keeping only the first-order terms, we have

$$\begin{aligned}
\delta \boldsymbol{f}(n) \approx\ & \boldsymbol{\Gamma}(n) \delta \boldsymbol{H}(n) (\boldsymbol{H}(n)^H \boldsymbol{H}(n))^{-1} \boldsymbol{e} \\
& - (\boldsymbol{H}(n)^{\dagger})^H \delta \boldsymbol{H}(n)^H (\boldsymbol{H}(n)^{\dagger})^H \boldsymbol{e}
\end{aligned} \tag{11}$$

where $\boldsymbol{\Gamma}(n) = \boldsymbol{I} - \boldsymbol{H}(n)\boldsymbol{H}(n)^{\dagger}$, and $\boldsymbol{H}(n)^{\dagger} = (\boldsymbol{H}(n)^H \boldsymbol{H}(n))^{-1} \boldsymbol{H}(n)^H$. Based on (11), $\Psi(\boldsymbol{X})$ can be immediately derived, which involves joint expectation over codes and $\delta \boldsymbol{g}_j$, and imposes incredible complexity for direct computation. Considering $\delta \boldsymbol{g}_j$ can be regarded as independent of $\boldsymbol{C}_j(n)$ for $j = 1, \ldots, J$ when the number of samples used to estimate $\boldsymbol{A}_j$ is sufficiently large, we thus simplify the computation of

$\Psi(\boldsymbol{X})$ as follows,

$$\begin{aligned}\Psi(\boldsymbol{X}) &\approx \frac{1}{N}\sum_{n=1}^{N}\Big\{\boldsymbol{e}^H\boldsymbol{H}(n)^\dagger\boldsymbol{D}_4(\boldsymbol{H}(n)^\dagger)^H\boldsymbol{e}\\ &+ \boldsymbol{e}^H[\boldsymbol{H}(n)^H\boldsymbol{H}(n)]^{-1}\boldsymbol{D}_1[\boldsymbol{H}(n)^H\boldsymbol{H}(n)]^{-1}\boldsymbol{e}\\ &- \boldsymbol{e}^H[\boldsymbol{H}(n)^H\boldsymbol{H}(n)]^{-1}\boldsymbol{D}_2[\boldsymbol{H}(n)^\dagger]^H\boldsymbol{e}\\ &- \boldsymbol{e}^H\boldsymbol{H}(n)^\dagger\boldsymbol{D}_3[\boldsymbol{H}(n)^H\boldsymbol{H}(n)]^{-1}\boldsymbol{e}\Big\} \qquad (12)\end{aligned}$$

where $\boldsymbol{D}_1 = E\{\delta\boldsymbol{H}(n)^H\boldsymbol{\Gamma}(n)\boldsymbol{X}\boldsymbol{\Gamma}(n)\delta\boldsymbol{H}(n)\}$, $\boldsymbol{D}_2 = E\{\delta\boldsymbol{H}(n)^H\boldsymbol{\Gamma}(n)\boldsymbol{X}(\boldsymbol{H}(n)^\dagger)^H\delta\boldsymbol{H}(n)^H\}$, $\boldsymbol{D}_3 = E\{\delta\boldsymbol{H}(n)\boldsymbol{H}(n)^\dagger\boldsymbol{X}\boldsymbol{\Gamma}(n)\delta\boldsymbol{H}(n)\}$, $\boldsymbol{D}_4 = E\{\delta\boldsymbol{H}(n)\boldsymbol{H}(n)^\dagger\boldsymbol{X}(\boldsymbol{H}(n)^\dagger)^H\delta\boldsymbol{H}(n)^H\}$. Clearly, $\boldsymbol{D}_1 \sim \boldsymbol{D}_4$ follow a general form of either $E\{\delta\boldsymbol{H}(n)^H\boldsymbol{Z}\delta\boldsymbol{H}(n)\}$, or $E\{\delta\boldsymbol{H}(n)\boldsymbol{Z}\delta\boldsymbol{H}(n)\}$ or $E\{\delta\boldsymbol{H}(n)\boldsymbol{Z}\delta\boldsymbol{H}(n)^H\}$ with $\boldsymbol{Z}$ replaced by corresponding quantities, we thus focus on computation of those general terms.

By (10), the (i,j)th element of $E\{\delta\boldsymbol{H}(n)^H\boldsymbol{Z}\delta\boldsymbol{H}(n)\}$ is given by $E\{\delta\boldsymbol{g}_i^H\boldsymbol{C}_i(n)^H\boldsymbol{Z}\boldsymbol{C}_j(n)\delta\boldsymbol{g}_j\}$. The diagonal term can be readily obtained after using tr (trace) operation and replacing $E\{\delta\boldsymbol{g}_j\delta\boldsymbol{g}_j^H\}$ by (8)

$$E\{\delta\boldsymbol{g}_j^H\boldsymbol{C}_j(n)^H\boldsymbol{Z}\boldsymbol{C}_j(n)\delta\boldsymbol{g}_j\} \approx \frac{\sigma_v^2}{N}tr\{\boldsymbol{C}_j(n)^H\boldsymbol{Z}\,\boldsymbol{C}_j(n)\boldsymbol{\Pi}_{\boldsymbol{g}_j}^\perp\boldsymbol{A}_j^{\frac{1}{2}}\boldsymbol{U}_n^j(\boldsymbol{U}_n^j)^H\boldsymbol{A}_j^{\frac{1}{2}}\boldsymbol{\Pi}_{\boldsymbol{g}_j}^\perp\}. \qquad (13)$$

On the other hand, using tr operation, off-diagonal terms can be shown to depend on $E\{\delta\boldsymbol{g}_j\delta\boldsymbol{g}_i^H\}$, which further depends on the cross correlation between $\delta\boldsymbol{R}_j$ and $\delta\boldsymbol{R}_i$ according to (8). Following similar steps in [11], it can be shown that $E\{\delta\boldsymbol{R}_j\boldsymbol{B}\delta\boldsymbol{R}_i\}$ is at the order of $O(\frac{\sigma_v^4}{N})$ for any deterministic $\boldsymbol{B}$ and thus negligible. $E\{\delta\boldsymbol{H}(n)^H\boldsymbol{Z}\delta\boldsymbol{H}(n)\}$ is then approximated by a diagonal matrix with its (i,i)th element given by (13).

Replacing $\delta\boldsymbol{H}(n)$ by (10), using (8) and ignoring higher order terms, $E\{\delta\boldsymbol{H}(n)\boldsymbol{Z}\delta\boldsymbol{H}(n)^H\}$ is obtained as

$$E\{\delta\boldsymbol{H}(n)\boldsymbol{Z}\delta\boldsymbol{H}(n)^H\} = \frac{\sigma_v^2}{N}\sum_{j=1}^{J}z_{j,j}\boldsymbol{C}_j(n)\boldsymbol{\Pi}_{\boldsymbol{g}_j}^\perp\boldsymbol{A}_j^{\frac{1}{2}}\boldsymbol{U}_n^j(\boldsymbol{U}_n^j)^H\boldsymbol{A}_j^{\frac{1}{2}}\boldsymbol{\Pi}_{\boldsymbol{g}_j}^\perp\boldsymbol{C}_j(n)^H \qquad (14)$$

where $z_{j,j}$ is the (j,j)th element of $\boldsymbol{Z}$.

To derive $E\{\delta\boldsymbol{H}(n)\boldsymbol{Z}\delta\boldsymbol{H}(n)\}$, we first rewrite $\delta\boldsymbol{H}(n)$ as $\delta\boldsymbol{H} = \mathcal{C}(n)diag\{\delta\boldsymbol{g}_1,\ldots,\delta\boldsymbol{g}_J\}$. Then the expectation can be readily shown to be zero for a complex system. For a real system, if we ignore the higher order terms induced by cross covariance between $\delta\boldsymbol{g}_i$ and $\delta\boldsymbol{g}_j$ for $i\neq j$, then

$$\begin{aligned}&E\{\delta\boldsymbol{H}(n)\boldsymbol{Z}\delta\boldsymbol{H}(n)\}\\ =\ &\frac{\sigma_v^2}{N}\Big[\boldsymbol{C}_1(n)\boldsymbol{\Pi}_{\boldsymbol{g}_1}^\perp\boldsymbol{A}_1^{\frac{1}{2}}\boldsymbol{U}_n^1(\boldsymbol{U}_n^1)^H\boldsymbol{A}_1^{\frac{1}{2}}\boldsymbol{\Pi}_{\boldsymbol{g}_1}^\perp\boldsymbol{z}_{11}^*,\ \ldots,\\ &\boldsymbol{C}_J(n)\boldsymbol{\Pi}_{\boldsymbol{g}_J}^\perp\boldsymbol{A}_J^{\frac{1}{2}}\boldsymbol{U}_n^j(\boldsymbol{U}_n^j)^H\boldsymbol{A}_J^{\frac{1}{2}}\boldsymbol{\Pi}_{\boldsymbol{g}_J}^\perp\boldsymbol{z}_{JJ}^*\Big] \qquad (15)\end{aligned}$$

where $\boldsymbol{z}_{ij}^H = [\boldsymbol{Z}]_{i,(q+1)(j-1)+1:j(q+1)}$, "*" represents complex conjugate.

Based on the above analysis, it can be shown that both $\Psi(\mathcal{R}_1(n))$ and $\Psi(\mathcal{R}_{int}(n))$ are at the order of $O(\frac{\sigma_v^2}{N})$, which is inversely proportional to N and proportional to σ_v^2.

2) Perturbed SINR of the MMSE Receiver: Substituting (5) for the receiver, the unperturbed term can be approximated as $\Phi(\boldsymbol{X}) = \boldsymbol{g}_1^H[\frac{1}{N}\sum_{n=1}^N\boldsymbol{C}_1(n)^H\mathcal{R}(n)^{-1}\ \boldsymbol{X}\mathcal{R}(n)^{-1}\boldsymbol{C}_1(n)]\boldsymbol{g}_1$. To evaluate the perturbed term, perturbation of the receiver is necessary, which is

$$\begin{aligned}\delta\boldsymbol{f}(n) &= \mathcal{R}(n)^{-1}\delta\boldsymbol{H}\boldsymbol{e} - \mathcal{R}(n)^{-1}[\delta\boldsymbol{H}(n)\boldsymbol{H}(n)^H\\ &+ \boldsymbol{H}(n)\delta\boldsymbol{H}(n)^H + \delta\sigma_v^2\boldsymbol{I}]\boldsymbol{f}(n). \qquad (16)\end{aligned}$$

Correspondingly, $\Psi(\boldsymbol{X})$ can be approximated by two-step expectations in the same way as for the ZF receiver

$$\begin{aligned}\Psi(\boldsymbol{X}) &= \frac{1}{N}\sum_{n=1}^{N}\Big\{\boldsymbol{e}^H\boldsymbol{\Delta}_1\boldsymbol{e} + \boldsymbol{f}(n)^H\boldsymbol{\Delta}_2\boldsymbol{f}(n)\\ &+ \boldsymbol{f}(n)^H\boldsymbol{H}(n)\boldsymbol{\Delta}_1\boldsymbol{H}(n)^H\boldsymbol{f}(n)\}\\ &+ \boldsymbol{f}(n)^H\mathcal{R}(n)^{-1}\boldsymbol{X}\mathcal{R}(n)^{-1}\boldsymbol{f}(n)\Delta_4\\ &+ \underbrace{\boldsymbol{f}(n)^H\boldsymbol{\Delta}_3\boldsymbol{H}(n)\boldsymbol{f}(n)}_{a_1} + a_1^*\\ &+ \underbrace{\boldsymbol{f}(n)^H\mathcal{R}(n)^{-1}\boldsymbol{X}\mathcal{R}(n)^{-1}\boldsymbol{\Delta}_5\boldsymbol{H}(n)^H\boldsymbol{f}(n)}_{a_2} + a_2^*\\ &+ \underbrace{\boldsymbol{f}(n)^H\boldsymbol{\Delta}_5\boldsymbol{H}(n)^H\mathcal{R}(n)^{-1}\boldsymbol{X}\mathcal{R}(n)^{-1}\boldsymbol{f}(n)}_{a_3} + a_3^*\\ &- \underbrace{\boldsymbol{e}^H\boldsymbol{\Delta}_1\boldsymbol{H}(n)^H\boldsymbol{f}(n)}_{a_4} - a_4^* - \underbrace{\boldsymbol{e}^H\boldsymbol{\Delta}_3^H\boldsymbol{f}(n)}_{a_5} - a_5^*\\ &- \underbrace{\boldsymbol{e}^H\boldsymbol{\Delta}_5^H\mathcal{R}(n)^{-1}\boldsymbol{X}\mathcal{R}(n)^{-1}\boldsymbol{f}(n)}_{a_6} - a_6^*\Big\} \qquad (17)\end{aligned}$$

where $\boldsymbol{\Delta}_1 = E\{\delta\boldsymbol{H}(n)^H\mathcal{R}(n)^{-1}\boldsymbol{X}\mathcal{R}(n)^{-1}\delta\boldsymbol{H}(n)\}$, $\boldsymbol{\Delta}_2 = E\{\delta\boldsymbol{H}(n)\boldsymbol{H}(n)^H\mathcal{R}(n)^{-1}\boldsymbol{X}\mathcal{R}(n)^{-1}\boldsymbol{H}(n)\delta\boldsymbol{H}(n)^H\}$, $\boldsymbol{\Delta}_3 = E\{\delta\boldsymbol{H}(n)\boldsymbol{H}(n)^H\mathcal{R}(n)^{-1}\boldsymbol{X}\mathcal{R}(n)^{-1}\delta\boldsymbol{H}(n)\}$, $\Delta_4 = E\{\delta\sigma_v^2\delta\sigma_v^2\}$, $\boldsymbol{\Delta}_5 = E\{\delta\boldsymbol{H}(n)\delta\sigma_v^2\}$. For shorter expression a_1 up to a_6 are also defined. Clearly, $\boldsymbol{\Delta}_1 \sim \boldsymbol{\Delta}_3$ can be directly computed as those general terms for the ZF receiver by replacing $\boldsymbol{Z}$ with corresponding quantities. We now turn to the computation of Δ_4 and $\boldsymbol{\Delta}_5$. Noticing $\delta\sigma_v^2 = \frac{1}{q}tr\{(\boldsymbol{U}_n^1)^H\delta\boldsymbol{R}_1\boldsymbol{U}_n^1\}$, using vec and tr operations, it follows that $\Delta_4 = \frac{1}{q^2}vec^H(\boldsymbol{U}_n^1(\boldsymbol{U}_n^1)^H)\ E\{vec(\delta\boldsymbol{R}_1)vec^H(\delta\boldsymbol{R}_1)\}\ vec\ (\boldsymbol{U}_n^1(\boldsymbol{U}_n^1)^H)$, which depends on $E\{vec(\delta\boldsymbol{R}_1)vec^H(\delta\boldsymbol{R}_1)\}$. If we denote the ith column of $\delta\boldsymbol{R}_1$ as $\delta\boldsymbol{r}_i$, i.e. $\delta\boldsymbol{r}_i = \delta\boldsymbol{R}_1\boldsymbol{\theta}_i$, where $\boldsymbol{\theta}_i$ is a unit vector with its ith element as 1, then the (i,j)th submatrix of $E\{vec(\delta\boldsymbol{R}_1)vec^H(\delta\boldsymbol{R}_1)\}$ becomes $E\{\delta\boldsymbol{R}_1\boldsymbol{\theta}_i\boldsymbol{\theta}_j^H\delta\boldsymbol{R}_1\}$, which can be readily computed as (43) or (50) in [11] without repetition here. Applying the above results, noticing that $vec(\boldsymbol{U}_n^1(\boldsymbol{U}_n^1)^H) = [\boldsymbol{\theta}_1^T\boldsymbol{U}_n^1\ (\boldsymbol{U}_n^1)^H,\ldots,$

$\boldsymbol{\theta}_{q+1}^T \boldsymbol{U}_n^1 (\boldsymbol{U}_n^1)^H]^H$, $(\boldsymbol{U}_n^1)^H \chi_1 = 0$, $(\boldsymbol{U}_n^1)^H \boldsymbol{R}_1 = \sigma_v^2 (\boldsymbol{U}_n^1)^H$, and ignoring higher order terms, we have

$$\Delta_4 = \frac{\sigma_v^2}{Nq^2} \sum_{i,j=1}^{q+1} (\boldsymbol{\theta}_j^T \boldsymbol{R}_1 \boldsymbol{\theta}_i) \boldsymbol{\theta}_i^T \boldsymbol{U}_n^1 (\boldsymbol{U}_n^1)^H \boldsymbol{\theta}_j.$$

To compute $\boldsymbol{\Delta}_5$, we express it in columns. Replacing $\delta\sigma_v^2$, the jth column is found to be $E\{\boldsymbol{C}_j(n) \delta \boldsymbol{g}_j tr\{(\boldsymbol{U}_n^1)^H \delta \boldsymbol{R}_1 \, \boldsymbol{U}_n^1\}\}$. Since $\delta \boldsymbol{g}_j$ depends on $\delta \boldsymbol{R}_j$, and the cross covariance between $\delta \boldsymbol{R}_j$ and $\delta \boldsymbol{R}_1$ are negligible for $j > 1$, we immediately have that except the frist column, all other ones can be approximated by $\boldsymbol{0}$. Replacing $\delta \boldsymbol{g}_1$ by (8) and following similar steps in deriving Δ_4, the first column of $\boldsymbol{\Delta}_5$ becomes

$$\begin{aligned} \boldsymbol{\Delta}_5 \approx & \frac{\sigma_v^2}{Nq\gamma_1} \boldsymbol{C}_1(n) \boldsymbol{\Pi}_{\boldsymbol{g}_1}^{\perp} \boldsymbol{A}_1^{\frac{1}{2}} \Big[\sum_{j=1}^{q+1} (\boldsymbol{\theta}_j^H \boldsymbol{R}_1 \boldsymbol{\theta}_1) \boldsymbol{U}_n^1 (\boldsymbol{U}_n^1)^H \\ & \boldsymbol{\theta}_j, \ldots, \sum_{j=1}^{q+1} (\boldsymbol{\theta}_j^H \boldsymbol{R}_1 \boldsymbol{\theta}_{q+1}) \boldsymbol{U}_n^1 (\boldsymbol{U}_n^1)^H \boldsymbol{\theta}_j \Big] \chi_1. \quad (18) \end{aligned}$$

To summarize, each expectation term ($\boldsymbol{\Delta}_1 \sim \boldsymbol{\Delta}_5$) in (17) is at the order of $O(\frac{\sigma_v^2}{N})$, the final perturbations for both the desired signal and interference plus noise are thus all at the order of $O(\frac{\sigma_v^2}{N})$.

3) Perturbed SINR of the RAKE Receiver: Closed form SINR for the RAKE receiver will be derived here. The signature matrix $\boldsymbol{H}(n)$ in (2) is first rewritten as $\boldsymbol{H}(n) = [\boldsymbol{G}_1 \boldsymbol{c}_{1,n}, \ldots, \boldsymbol{G}_J \boldsymbol{c}_{J,n}]$, where $\boldsymbol{c}_{j,n} = [c_{j,n}(0), \ \ldots, \ c_{j,n}(P-1)]^T$, $\boldsymbol{G}_j = [\bar{\boldsymbol{G}}_j]_{\mu+1-d_j:P-d_j,1:P} = \mathcal{S}_j \bar{\boldsymbol{G}}_j$, $\bar{\boldsymbol{G}}_j$ is Toeplitz matrix of channel vector $\boldsymbol{g}_j$ and $\mathcal{S}_j = [\boldsymbol{0}_{(p-\mu)\times(\mu-d_j)}; \boldsymbol{I}_{(P-\mu)\times(P-\mu)}; \boldsymbol{0}_{(P-\mu)\times(q+d_j)}]$. Based on the new format of $\boldsymbol{H}(n)$, the unperturbed desired power can be derived after replacing the RAKE receiver with (7) and $\boldsymbol{h}_1(n)$ with $\boldsymbol{G}_1 \boldsymbol{c}_{1,n}$, and applying tr, vec and kronecker product operations

$$\begin{aligned} \Phi(\mathcal{R}_1(n)) &= E\{\boldsymbol{c}_{1,n}^H \boldsymbol{G}_1^H \boldsymbol{G}_1 \boldsymbol{c}_{1,n} \boldsymbol{c}_{1,n}^H \boldsymbol{G}_1^H \boldsymbol{G}_1 \boldsymbol{c}_{1,n}\} \\ &= vec^H\{\boldsymbol{G}_1^H \boldsymbol{G}_1\} E\{(\boldsymbol{c}_{1,n}^* \otimes \boldsymbol{c}_{1,n}) \\ &\quad (\boldsymbol{c}_{1,n}^T \otimes \boldsymbol{c}_{1,n}^H)\} vec(\boldsymbol{G}_1^H \boldsymbol{G}_1). \quad (19) \end{aligned}$$

The expectation term is given by (36) or (46) in [11]. Similarly, $\Phi(\mathcal{R}_{int}(n))$ is derived as the following after noticing the independence between $\boldsymbol{c}_{1,n}$ and $\boldsymbol{c}_{j,n}$, and $E\{\boldsymbol{c}_{j,n} \boldsymbol{c}_{j,n}^H\} = \sigma_c^2 \boldsymbol{I}$ for $j = 1, \ldots, J$,

$$\Phi(\mathcal{R}_{int}(n)) = \sigma_c^4 \sum_{j=2}^{J} tr\{\boldsymbol{G}_1^H \boldsymbol{G}_j \boldsymbol{G}_j^H \boldsymbol{G}_1\} + \sigma_c^2 \sigma_v^2 tr\{\boldsymbol{G}_1^H \boldsymbol{G}_1\}. \quad (20)$$

We now proceed to evaluate $\Psi(\boldsymbol{X})$. Noticing $\boldsymbol{h}_1 = \boldsymbol{G}_1 \boldsymbol{c}_{1,n}$, we have $\delta \boldsymbol{f}(n) = \delta \boldsymbol{G}_1 \boldsymbol{c}_{1,n}$, where $\delta \boldsymbol{G}_1 = [\boldsymbol{B}_0 \delta \boldsymbol{g}_1, \ \ldots, \ \boldsymbol{B}_{P-1} \delta \boldsymbol{g}_1]$, $\boldsymbol{B}_i \triangleq \mathcal{S}_1 \boldsymbol{J}^i \boldsymbol{B}$, $\boldsymbol{B} = [\boldsymbol{I}_{M(q+1)}; \ \boldsymbol{0}]^T$, $\boldsymbol{J}$ is a shifting matrix with all 1s in the first sub-diagonal, and $\boldsymbol{J}^0$ is defined as an identity matrix for convenience. Based on the above results, following the same steps for deriving (19), and noticing that $\delta \boldsymbol{G}_1$ is independent of codes, then the perturbation of signal power is obtained as

$$\begin{aligned} \Psi(\mathcal{R}_1(n)) = tr\Big\{ & E\{(\boldsymbol{c}_{1,n}^* \otimes \boldsymbol{c}_{1,n})(\boldsymbol{c}_{1,n}^T \otimes \boldsymbol{c}_{1,n}^H)\} \\ & E\{vec\{\boldsymbol{G}_1^H \delta \boldsymbol{G}_1) vec^H(\boldsymbol{G}_1^H \delta \boldsymbol{G}_1)\}\Big\} \quad (21) \end{aligned}$$

where the first expectation term is given by [11] as explained before. Rewriting $vec(\boldsymbol{G}_1^H \delta \boldsymbol{G}_1)$ as $vec(\boldsymbol{G}_1^H \delta \boldsymbol{G}_1 \boldsymbol{I})$ and using vec operations, it is straight forward to simplify the second expectation as

$$\begin{aligned} & E\{vec(\boldsymbol{G}_1^H \delta \boldsymbol{G}_1) vec^H(\boldsymbol{G}_1^H \delta \boldsymbol{G}_1)\} \\ = & \frac{\sigma_v^2}{N} (\boldsymbol{I} \otimes \boldsymbol{G}_1^H) \mathcal{B} \boldsymbol{\Pi}_{\boldsymbol{g}_1}^{\perp} \boldsymbol{A}_1^{\frac{1}{2}} \boldsymbol{U}_n^1 (\boldsymbol{U}_n^1)^H \boldsymbol{A}_1^{\frac{1}{2}} \boldsymbol{\Pi}_{\boldsymbol{g}_1}^{\perp} \\ & \mathcal{B}^H (\boldsymbol{I} \otimes \boldsymbol{G}_1) \quad (22) \end{aligned}$$

where $\mathcal{B} = [\boldsymbol{B}_0^T, \ \ldots, \boldsymbol{B}_{P-1}^T]^T$. In a much similar way for deriving (20), the perturbation of interference plus noise power is computed as

$$\begin{aligned} \Psi(\mathcal{R}_{int}(n)) &= \sigma_c^4 \sum_{j=2}^{J} tr\Big\{ \boldsymbol{G}_j \boldsymbol{G}_j^H E\{\delta \boldsymbol{G}_1 \delta \boldsymbol{G}_1^H\}\Big\} \\ &+ \sigma_c^2 \sigma_v^2 tr\Big\{E\{\delta \boldsymbol{G}_1 \delta \boldsymbol{G}_1^H\}\Big\} \quad (23) \end{aligned}$$

where using $\delta \boldsymbol{G}_1 = [\boldsymbol{B}_0 \delta \boldsymbol{g}_1, \ldots, \boldsymbol{B}_{P-1} \delta \boldsymbol{g}_1]$ and ignoring $O(\frac{\sigma_v^4}{N})$ terms, we have

$$\begin{aligned} E\{\delta \boldsymbol{G}_1 \delta \boldsymbol{G}_1^H\} &= \sum_{i=0}^{P-1} \boldsymbol{B}_i E\{\delta \boldsymbol{g}_1 \delta \boldsymbol{g}_1^H\} \boldsymbol{B}_i^H \\ &= \frac{\sigma_v^2}{N} \sum_{i=0}^{P-1} \boldsymbol{B}_i \boldsymbol{\Pi}_{\boldsymbol{g}_1}^{\perp} \boldsymbol{A}_1^{\frac{1}{2}} \boldsymbol{U}_n^1 (\boldsymbol{U}_n^1)^H \boldsymbol{A}_1^{\frac{1}{2}} \boldsymbol{\Pi}_{\boldsymbol{g}_1}^{\perp} \\ & \quad \boldsymbol{B}_i^H. \quad (24) \end{aligned}$$

To summarize, the unperturbed signal power of the RAKE receiver, given by (19), depends on the desired user's channel conditions as well as the second and fourth order statistics of its own code sequence, while the unperturbed interference plus noise power given by (20) depends on the second order moment of the codes, and cross channel conditions between each interfering user and the desired user. Finally, according to (21) $\sim$ (24), both the desired user's power and interference plus noise are perturbed at the order of $O(\frac{\sigma_v^2}{N})$.

B. BER Performance of Different Receivers

For each receiver, once its output SINR is evaluated, BER can be obtained by assuming the interference is Gaussian distributed. This may not be necessarily correct, but this approximation has been shown to be relatively good [2], especially when number of interfering users is large. The BER for BPSK information symbol is

$$\text{BER} = Q\Big(\sqrt{\widetilde{\text{SINR}}}\ \Big) \quad (25)$$

where $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{\frac{-t^2}{2}} dt$.

V. Simulation Examples

Performance of the proposed receivers is numerically studied and compared with our analytical results as well as theoretical limits. Those theoretical values are based on ideal receivers constructed from all users' perfect channel vectors as well as noise power. Simulation parameters are set as follows: $P = 32$, $q = 2$, and $J = 8$. Totally 100 Monte Carlo simulations are performed to obtain the average results. Fig. 1 shows the SINRs of receivers over different N. It is observed that experimental, analytical and theoretical SINRs start to overlap from $N = 50$, indicating that the proposed receivers require very small data size to achieve their theoretical limit at $20dB$ SNR environment. On the other hand, the ZF and MMSE receivers show much better performance than the RAKE receiver. Fig. 2 compares experimental BERs of all receivers with their corresponding analytical ones respectively over various SNRs. The MMSE receiver shows slightly better performance than the ZF receiver. Both of them are significantly superior to the RAKE receiver. The convergence between the experimental and analytical values can be observed, indicating that our analytical SINR and BER can serve as good performance predictors. Moreover, the experimental values are found to be very close to their theoretical ones, showing that each proposed receiver constructed from the proposed channel estimator and estimated noise behaves as well as its ideal counterpart.

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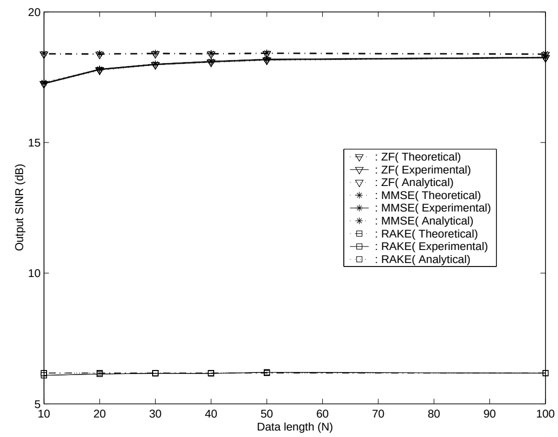

Fig. 1. Output SINR vs. N.

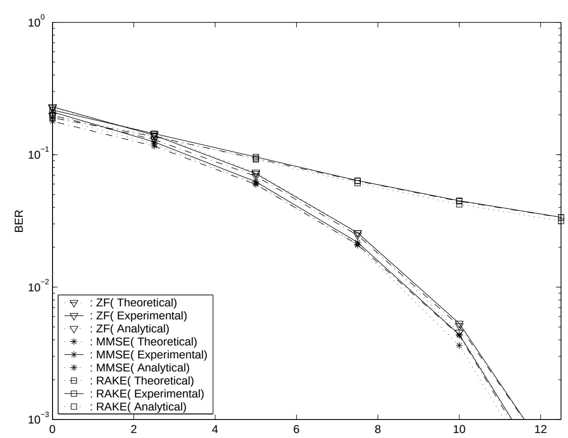

Fig. 2. BER vs. SNR.