Performance of Channel Estimation Assisted Linear Multiuser Receivers for Downlink CDMA Systems with Long Spreading Codes

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***Abstract*— With availability of spatial/temporal diversity in the downlink of long code CDMA systems, subspace technique has been shown to be able to estimate channel parameters. Based on the estimated channel, we construct three typical linear receivers as ZF, MMSE and RAKE receivers. Due to channel estimation error, those receivers show perturbed performance compared with their optimal counterparts. Their statistical performance is thus analyzed by applying perturbation theory. Those analyses are well supported by numerical results.**

I. Introduction

Long code CDMA systems have received considerable attention in recent years due to adoption of long codes in the third generation communication standards. Study on channel estimation and detection techniques is still on-going. Given transmitted pilot symbols of all users, least squares (LS) fitting or iterative maximum likelihood (ML) approaches have been reported [1], [2]. A semi-blind channel estimation solution via subspace based data projection for downlink is derived [3]. Blind methods have also appeared for either downlink [4] or uplink [5], [6]. Various symbol-level and chip-level channel equalization and interference suppression schemes have been presented [7], [8].

In this paper, we first provide three typical linear symbol receivers constructed from estimated channel parameters by the method [4], where both the ZF and MMSE receivers utilize characteristics of spreading codes in CDMA downlink and are composed of corresponding equalizers followed by code despreaders, while the RAKE receiver directly applies the desired symbol's time varying signature. Due to channel estimation error, perfect equalization and detection becomes impossible. We then analyze performance degradation of those receivers in terms of output signal to interference plus noise ratio (SINR) and bit error rate (BER) based on perturbation theory. All analytical results are verified by numerical examples.

II. Downlink CDMA System Model

Consider a base station communicating with J mobile stations in a CDMA system. The jth user's aperiodic spreading codes $c_{j,n}(k)$ $(k = 0, \ldots, P-1)$, which is the combination of its Hadamard codes and base station's long codes, is used to spread bit $w_j(n)$. Let the chip sequence be transmitted through M subchannels with unknown coefficients $g_m(n)$ for the mth subchannel. Each subchannel is assumed to be FIR and has order q ($q < P$). If we collect P chip rate samples $[y_m(nP),$ $\cdots, y_m(nP{+}P{-}1)]$ from each subchannel and collect samples from all M subchannels in a big vector, then the received data becomes [4]

$$\boldsymbol{y}(n) = \mathcal{G}\boldsymbol{b}(n) + \boldsymbol{v}(n), \quad \boldsymbol{b}(n) = \sum_{j=1}^{J} \boldsymbol{b}_j(n) \tag{1}$$

where $\mathcal{G}$ is a $MP \times (P+q)$ block Toeplitz matrix whose first block row is $[\boldsymbol{G}, \boldsymbol{0}]$ with $\boldsymbol{G}$ given by

$$\boldsymbol{G} = \left[\begin{array}{ccc} g_1(q) & \cdots & g_1(0) \\ \vdots & & \vdots \\ g_M(q) & \cdots & g_M(0) \end{array}\right],$$
$$\boldsymbol{b}_j(n) = [w_j(n-1)\bar{\boldsymbol{c}}_{j,n-1}^T, w_j(n)\boldsymbol{c}_{j,n}^T]^T, \tag{2}$$

$\boldsymbol{c}_{j,n} = [c_{j,n}(0), \cdots, c_{j,n}(P-1)]^T$, $\bar{\boldsymbol{c}}_{j,n-1} = \boldsymbol{c}_{j,n-1}(P-q : P-1)$, and $\boldsymbol{v}(n)$ is an AWGN vector.

III. Symbol Detection

According to [4], $\mathcal{G}$ is a tall matrix when $M \geq 2$ and $q < P$. If all subchannels have no common zeros, then subspace technique is directly applicable to estimate all subchannels up to a scalar ambiguity. According to (1), the data covariance matrix is derived as $\boldsymbol{R} = E\{\boldsymbol{y}(n)\boldsymbol{y}(n)^H\} = \rho\mathcal{G}\mathcal{G}^H + \sigma_v^2\boldsymbol{I}$, where $\rho = J\sigma_w^2\sigma_c^2$, σ_w^2 is the transmission power of each user's symbol and σ_c^2 is the covariance of the base station's random codes. Let $\boldsymbol{h} = [g_1(0),\cdots, g_M(0), g_1(1),\cdots,g_M(1),\cdots,g_1(q), \cdots,g_M(q)]^T$. then $\boldsymbol{h}$ can be estimated by [4] $\boldsymbol{h} = \arg\min_{||\boldsymbol{\alpha}||=1} \boldsymbol{\alpha}^H \boldsymbol{X}\boldsymbol{\alpha}$, where $\boldsymbol{X} = \sum_{i=-q}^{P-1} \boldsymbol{A}_i^H \boldsymbol{U}_n \boldsymbol{U}_n^H \boldsymbol{A}_i$, $\boldsymbol{A}_i \triangleq \boldsymbol{J}^{iM}\boldsymbol{A}$, $\boldsymbol{A} = [\boldsymbol{I}_{M(q+1)}, \boldsymbol{0}]^T$, $\boldsymbol{J}$ is a Jordan matrix with all $1's$ in the first sub-diagonal, and $\boldsymbol{U}_n$ is the null space of the noise-free data covariance matrix. Once $\boldsymbol{h}$ is estimated, the ith column of $\mathcal{G}$ can be formed by $\boldsymbol{A}_i\boldsymbol{h}$ $(i = -q, \ldots, P)$. Applying $\mathcal{G}$ and $\boldsymbol{h}$, we next propose different symbol detection schemes. Without loss of generality, user 1 is assumed to be the desired user.

In ZF detection, a ZF equalizer, defined as

$$\boldsymbol{f}_{zf} = \mathcal{G}(\mathcal{G}^H\mathcal{G})^{-1}\boldsymbol{e} \tag{3}$$

where $\boldsymbol{e}$ is a unitary vector with its $(q+1)$th element as 1, is first applied to P consecutive data vectors $\boldsymbol{Y}_n$=$[\boldsymbol{y}(n),\ldots, \boldsymbol{y}(n+P-1)]$ to yield an approximate sum signal. Then the desired user's codes are used to despread the equalized data and yield the estimated symbol

$$\widehat{w}_{1,zf} = \boldsymbol{f}_{zf}^H \boldsymbol{Y}_n \boldsymbol{c}_{1,n}^* \tag{4}$$

where "*" denotes complex conjugate. An MMSE receiver consists of an MMSE equalizer and code despreader. The MMSE equalizer might work on partial data vectors for fast convergence under small sample size N, as shown in the simulation section. Suppose the length of the MMSE equalizer is taken as νM, $\nu \leq P$. then the MMSE equalizer obeys the following form

$$\boldsymbol{f}_{mmse} = \boldsymbol{R}_\nu{}^{-1}\boldsymbol{\Gamma}_\nu \boldsymbol{A}\boldsymbol{h} \tag{5}$$

where $\boldsymbol{R}_\nu = E\{[\boldsymbol{y}(n)]_{1:\nu M}([\boldsymbol{y}(n)]_{1:\nu M})^H\}$, and $\boldsymbol{\Gamma}_\nu = [\boldsymbol{I}_{\nu M},\ \boldsymbol{0}_{\nu M\times(P-\nu)M}]$ is a selection matrix. To guarantee $\boldsymbol{\Gamma}_\nu\boldsymbol{A}\boldsymbol{h}$ contains full $\boldsymbol{h}$ for signal detection, ν should satisfy $\nu \geq q+1$. Applying the MMSE equalizer to P consecutive partial data vectors which are collected in a matrix $\boldsymbol{Y}_\nu(n) = \boldsymbol{\Gamma}_\nu\boldsymbol{Y}(n)$, and decorrelating the equalized data with the desired user's codes result in the following symbol estimate

$$\widehat{w}_{1,mmse}(n) = \boldsymbol{f}_{mmse}^H \boldsymbol{Y}_\nu(n)\boldsymbol{c}_{1,n}^*. \tag{6}$$

To construct the RAKE receiver, we partition $\mathcal{G}$ as $\mathcal{G} = [(\mathcal{G}_1)_{PM\times q},\ \ (\mathcal{G}_2)_{PM\times P}]$. Clearly current input is separated from previous input on the right hand side of (1)

$$\begin{aligned}\boldsymbol{y}(n) &= \mathcal{G}_2\boldsymbol{c}_{1,n}w_1(n) + \sum_{j=2}^{J}\mathcal{G}_2\boldsymbol{c}_{j,n}w_j(n)\\ &+ \sum_{j=1}^{J}\mathcal{G}_1\bar{\boldsymbol{c}}_{j,n-1}w_j(n-1) + \boldsymbol{v}_(n).\end{aligned} \tag{7}$$

The signature of the desired symbol $w_1(n)$ is $\mathcal{G}_2\boldsymbol{c}_{1,n}$. The symbol-level RAKE receiver is then constructed as

$$\boldsymbol{f}_{rake}(n) = \mathcal{G}_2\boldsymbol{c}_{1,n}, \tag{8}$$

and the estimated symbol is

$$\widehat{w}_{1,rake}(n) = \boldsymbol{f}_{rake}^H(n)\boldsymbol{y}(n). \tag{9}$$

Performance of these three linear receivers will be evaluated next.

IV. Analytical Results

Both channel estimator and equalizers depend on $\boldsymbol{R}$. They will be perturbed if $\boldsymbol{R}$ is estimated from N vectors as $\widetilde{\boldsymbol{R}} = \frac{1}{N}\sum_{n=1}^{N}\boldsymbol{y}(n)\boldsymbol{y}^H(n)$. Denoting the perturbation by preceding the corresponding quantity by δ, and the perturbed quantity with $\widetilde{\ }$, the channel perturbation and its covariance are give by [10]

$$\begin{aligned}\delta\boldsymbol{h} &\approx \frac{1}{\rho}\sum_{i=-q}^{P-1}\boldsymbol{T}_i\boldsymbol{U}_n^H\delta\boldsymbol{R}\boldsymbol{t}_i\\ \mathrm{Cov}_h &\approx \frac{\sigma_v^2}{N\rho^2}\sum_{i,j}(\boldsymbol{t}_j^H\boldsymbol{R}\boldsymbol{t}_i)\boldsymbol{T}_i\boldsymbol{T}_j^H\end{aligned} \tag{10}$$

where $\boldsymbol{T}_i$ and $\boldsymbol{t}_i$ are deterministic quantities

$$\boldsymbol{T}_i = \boldsymbol{X}^\dagger\boldsymbol{A}_i^H\boldsymbol{U}_n, \quad \boldsymbol{t}_i = (\mathcal{G}\mathcal{G}^H)^\dagger\boldsymbol{A}_i\boldsymbol{h},$$

"$\dagger$" represents pseudo-inverse. Next, we will study the statistical performance of three receivers in terms of SINR and BER based on (10).

A. SINR

SINR is an important performance indicator for receivers. Although ideal SINRs of different receivers can be obtained under perfect conditions, perturbation in channel estimation induced by finite data samples inevitably causes SINRs of those receivers to deviate from their optimal values. We will derive perturbed SINRs for the receivers constructed from estimated channel parameters. Moreover, analytical results obtained in this subsection can be further used to predict the BER performance of those receivers, as shown in the next subsection. For shorter notations, all receivers' subscripts are dropped later.

1) ZF receiver: Since ZF equalizer filters the data vector $\boldsymbol{Y}_n\boldsymbol{c}_{1,n}$ (see eq. (4)), it is essential to express the data vector explicitly in terms of desired symbol, interfering symbols and noise in order to evaluate SINR. If we define $\boldsymbol{B}_n = [\boldsymbol{b}(n),\ldots,\boldsymbol{b}(n+P-1)]$ and $\boldsymbol{V}_n = [\boldsymbol{v}(n),\ldots,\boldsymbol{v}(n+P-1)]$, then $\boldsymbol{Y}_n\boldsymbol{c}_{1,n}^* = \mathcal{G}\boldsymbol{B}_n\boldsymbol{c}_{1,n}^* + \boldsymbol{V}_n\boldsymbol{c}_{1,n}^*$. After substituting $\boldsymbol{b}(n),\ldots,\boldsymbol{b}(n+p-1)$ with (1) and (2), $\boldsymbol{B}_n\boldsymbol{c}_{1,n}^*$ can be split as $\boldsymbol{B}_n\boldsymbol{c}_{1,n}^* = \boldsymbol{s}_1 w_1(n) + \boldsymbol{H}_{int}\boldsymbol{w}_{int}(n)$, where $\boldsymbol{s}_1$ and $\boldsymbol{H}_{int}$ have particular structures of users' codes. Applying the random property of both the base station's codes and noise, and invoking the orthogonality of Hadamard codes, one can verify that

$$E\{\boldsymbol{V}_n\boldsymbol{c}_{1,n}^*\boldsymbol{c}_{1,n}^T\boldsymbol{V}_n^H\} = \gamma\boldsymbol{I},$$

$$E\{\boldsymbol{s}_1\boldsymbol{s}_1^H\} = \sigma_c^4 diag\{(P-q):(P-1), P^2, (P-1):1\},$$

$$\begin{aligned} E\{\boldsymbol{H}_{int}\boldsymbol{H}_{int}^H)\} &= (J-1)diag\{P,\ldots,P,0,P,\ldots,P\}) \\ &+ \sigma_c^4(diag\{q:1,0:P-1\}, \end{aligned}$$

where $\gamma = P\sigma_c^2\sigma_v^2$, ":" represents succession of integers, 0 is in the $(q+1)$th position of each of the last two diagonal matrices. Based on the above results, SINR is then computed as SINR$= \boldsymbol{f}^H\boldsymbol{R}_1\boldsymbol{f}/\boldsymbol{f}^H\boldsymbol{R}_{int}\boldsymbol{f}$ where

$$\boldsymbol{R}_1 = \sigma_w^2\mathcal{G}E\{\boldsymbol{s}_1\boldsymbol{s}_1^H\}\mathcal{G}^H, \tag{11}$$

$$\boldsymbol{R}_{int} = \sigma_w^2\mathcal{G}E\{\boldsymbol{H}_{int}\boldsymbol{H}_{int}^H\}\mathcal{G}^H + \gamma\boldsymbol{I}. \tag{12}$$

Obviously, a perturbation of $\boldsymbol{R}$ causes a perturbation of the equalizer and finally the SINR

$$\widetilde{\text{SINR}} \approx \frac{\boldsymbol{f}^H\boldsymbol{R}_1\boldsymbol{f} + E\{\delta\boldsymbol{f}^H\boldsymbol{R}_1\delta\boldsymbol{f}\}}{\boldsymbol{f}^H\boldsymbol{R}_{int}\boldsymbol{f} + E\{\delta\boldsymbol{f}^H\boldsymbol{R}_{int}\delta\boldsymbol{f}\}}. \tag{13}$$

To evaluate (13), we first obtain $\delta\boldsymbol{f}$. Using (3) and noticing that the perturbed term $(\widetilde{\mathcal{G}}^H\widetilde{\mathcal{G}})^{-1}$ can be obtained by applying Taylor expansion, the perturbation of the ZF equalizer is derived as

$$\delta\boldsymbol{f} \approx \boldsymbol{\Pi}^\perp\delta\mathcal{G}(\mathcal{G}^H\mathcal{G})^{-1}\boldsymbol{e} - (\mathcal{G}^\dagger)^H\delta\mathcal{G}^H(\mathcal{G}^\dagger)^H\boldsymbol{e} \tag{14}$$

where $\boldsymbol{\Pi}^\perp \triangleq \boldsymbol{I} - \mathcal{G}\mathcal{G}^\dagger$, $\mathcal{G}^\dagger \triangleq (\mathcal{G}^H\mathcal{G})^{-1}\mathcal{G}^H$. Then we proceed to evaluate $E\{\delta\boldsymbol{f}^H\boldsymbol{R}_1\delta\boldsymbol{f}\}$ and $E\{\delta\boldsymbol{f}^H\boldsymbol{R}_{int}\delta\boldsymbol{f}\}$ to obtain $\widetilde{SINR}$. Because $\boldsymbol{\Pi}^\perp\mathcal{G} = \boldsymbol{0}$ and $\boldsymbol{R}_1$ in (11) has a particular structure, it can be found that

$$E\{\delta\boldsymbol{f}^H\boldsymbol{R}_1\delta\boldsymbol{f}\} \approx \boldsymbol{e}^H\mathcal{G}^\dagger E\{\delta\mathcal{G}\mathcal{G}^\dagger\boldsymbol{R}_1(\mathcal{G}^\dagger)^H\delta\mathcal{G}^H\}(\mathcal{G}^\dagger)^H\boldsymbol{e}. \tag{15}$$

Thus, to obtain this quantity, it suffices to evaluate a typical form $E\{\delta\mathcal{G}\boldsymbol{\Phi}_1\delta\mathcal{G}^H\}$ and then replace $\boldsymbol{\Phi}_1$ by $\mathcal{G}^\dagger\boldsymbol{R}_1(\mathcal{G}^\dagger)^H$. Noticing the definition of $\mathcal{G}$, $\delta\mathcal{G}$ is related to $\delta\boldsymbol{h}$ as

$$\delta\mathcal{G} = [\boldsymbol{A}_{-q}\delta\boldsymbol{h},\ldots,\boldsymbol{A}_{P-1}\delta\boldsymbol{h}]. \tag{16}$$

Then

$$E\{\delta\mathcal{G}\boldsymbol{\Phi}_1\delta\mathcal{G}^H\} = \sum_{i,j=1}^{P+q}\boldsymbol{\Phi}_1^{(i,j)}\boldsymbol{A}_{i-1-q}\text{Cov}_\text{h}\boldsymbol{A}_{j-1-q}^H \tag{17}$$

where $\boldsymbol{\Phi}_1^{(i,j)}$ is the (i,j)th element of $\boldsymbol{\Phi}_1$, and Cov_h is given by (10). To simplify $E\{\delta\boldsymbol{f}^H\boldsymbol{R}_{int}\delta\boldsymbol{f}\}$, let us define the first term of $\boldsymbol{R}_{int}$ in (12) as $\boldsymbol{\Sigma} = \sigma_w^2\mathcal{G}E\{\boldsymbol{H}_{int}\boldsymbol{H}_{int}^H\}\mathcal{G}^H$. It can be verified that

$$\begin{aligned} E\{\delta\boldsymbol{f}^H\boldsymbol{R}_{int}\delta\boldsymbol{f}\} &\approx \boldsymbol{e}^H\mathcal{G}^\dagger E\{\delta\mathcal{G}\mathcal{G}^\dagger\boldsymbol{\Sigma}(\mathcal{G}^\dagger)^H\delta\mathcal{G}^H\}(\mathcal{G}^\dagger)^H\boldsymbol{e} \\ &+ \gamma\boldsymbol{e}^H\mathcal{G}^\dagger E\{\delta\mathcal{G}(\mathcal{G}^H\mathcal{G})^{-1}\delta\mathcal{G}^H\}(\mathcal{G}^\dagger)^H\boldsymbol{e} \\ &+ \gamma\boldsymbol{e}^H(\mathcal{G}^H\mathcal{G})^{-1}E\{\delta\mathcal{G}^H\boldsymbol{\Pi}^\perp\delta\mathcal{G}\}(\mathcal{G}^H\mathcal{G})^{-1}\boldsymbol{e}. \end{aligned} \tag{18}$$

The first and second terms can be derived by following (17). To obtain the third term, we need $E\{\delta\mathcal{G}^H\boldsymbol{\Pi}^\perp\delta\mathcal{G}\}$. Considering the ith column of $\delta\mathcal{G}$ is $\boldsymbol{A}_{i-1-q}\delta\boldsymbol{h}$, the (i,j)th element of this matrix is $E\{\delta\boldsymbol{h}^H\boldsymbol{\Theta}_{i,j}\delta\boldsymbol{h}\}$ where $\boldsymbol{\Theta}_{i,j} = \boldsymbol{A}_{i-1-q}^H\boldsymbol{\Pi}^\perp\boldsymbol{A}_{j-1-q}$. Substituting (10) for $\delta\boldsymbol{h}$, we have

$$\begin{aligned} E\{\delta\boldsymbol{h}^H\boldsymbol{\Theta}_{i,j}\delta\boldsymbol{h}\} &\approx \frac{1}{\rho^2}\sum_{k_1,k_2=-q}^{P-1}\boldsymbol{t}_{k_1}^H E\{\delta\boldsymbol{R}\boldsymbol{U}_n\boldsymbol{T}_{k_1}^H \\ &\qquad \boldsymbol{\Theta}_{i,j}\boldsymbol{T}_{k_2}\boldsymbol{U}_n^H\delta\boldsymbol{R}\}\boldsymbol{t}_{k2}. \end{aligned} \tag{19}$$

The expectation on the right hand side follows general-form matrix $\boldsymbol{\Psi} = E\{\delta\boldsymbol{R}\boldsymbol{D}\delta\boldsymbol{R}\}$, where $\boldsymbol{D}$ can be replaced by corresponding deterministic quantities respectively later. The computation of $\boldsymbol{\Psi}$ is given by (43) or (50) in [9] and will not be repeated here due to limited space. Using (15) and (18), $\widetilde{SINR}$ follows.

2) MMSE receiver: According to (6), the MMSE equalizer filters the data vector $\boldsymbol{Y}_\nu(n)\boldsymbol{c}_{1,n}^*$. Since $\boldsymbol{Y}_\nu(n) = \boldsymbol{\Gamma}_\nu\boldsymbol{Y}(n)$, (11)-(13) can be directly applied after replacing $\boldsymbol{R}_1$ with $\boldsymbol{\Gamma}_\nu\boldsymbol{R}_1\boldsymbol{\Gamma}_\nu^H$ and $\boldsymbol{R}_{int}$ with $\boldsymbol{\Gamma}_\nu\boldsymbol{R}_{int}\boldsymbol{\Gamma}_\nu^H$. To compute (13), we first obtain the perturbation of the MMSE equalizer. According to (5), noticing that $\widetilde{\boldsymbol{R}_\nu} = \boldsymbol{R}_\nu + \delta\boldsymbol{R}_\nu$ and keeping only the first order terms, $\delta\boldsymbol{f}$ is found to be

$$\delta\boldsymbol{f} \approx \boldsymbol{R}_\nu^{-1}\boldsymbol{\Gamma}_\nu\boldsymbol{A}\delta\boldsymbol{h} - \boldsymbol{R}_\nu^{-1}\delta\boldsymbol{R}_\nu\boldsymbol{R}_\nu^{-1}\boldsymbol{\Gamma}_\nu\boldsymbol{A}\boldsymbol{h}. \tag{20}$$

Since $[\boldsymbol{y}(n)]_{1:\nu M} = \boldsymbol{\Gamma}_\nu\boldsymbol{y}(n)$, then $\boldsymbol{R}_\nu = \boldsymbol{\Gamma}_\nu\boldsymbol{R}\boldsymbol{\Gamma}_\nu^H$. Its perturbation is related to $\delta\boldsymbol{R}$ by $\delta\boldsymbol{R}_\nu = \boldsymbol{\Gamma}_\nu\delta\boldsymbol{R}\boldsymbol{\Gamma}_\nu^H$. Using this result and substituting (10), $\delta\boldsymbol{f}$ is related to $\delta\boldsymbol{R}$ by

$$\delta\boldsymbol{f} \approx \frac{1}{\rho}\sum_{i=-q}^{P-1}\boldsymbol{Q}_i\delta\boldsymbol{R}\boldsymbol{t}_i - \boldsymbol{R}_\nu^{-1}\boldsymbol{\Gamma}_\nu\delta\boldsymbol{R}\boldsymbol{\Gamma}_\nu^H\boldsymbol{f} \tag{21}$$

where $\boldsymbol{Q}_i = \boldsymbol{R}_\nu^{-1}\boldsymbol{\Gamma}_\nu\boldsymbol{A}\boldsymbol{T}_i\boldsymbol{U}_n^H$. Two perturbation quantities in evaluating $\widetilde{SINR}$ in (13) follow the same form $E\{\delta\boldsymbol{f}^H\boldsymbol{\Phi}\delta\boldsymbol{f}\}$ with corresponding substitution of a deterministic matrix $\boldsymbol{\Phi}$. Using (21), we have

$$\begin{aligned} E\{\delta\boldsymbol{f}^H\boldsymbol{\Phi}\delta\boldsymbol{f}\} &\approx \frac{1}{\rho^2}\sum_{i,j}\boldsymbol{t}_i^H E\{\delta\boldsymbol{R}\boldsymbol{Q}_i^H\boldsymbol{\Phi}\boldsymbol{Q}_j\delta\boldsymbol{R}\}\boldsymbol{t}_j \\ &+ \boldsymbol{f}^H\boldsymbol{\Gamma}_\nu E\{\delta\boldsymbol{R}\boldsymbol{\Gamma}_\nu^H\boldsymbol{R}_\nu^{-1}\boldsymbol{\Phi}\boldsymbol{R}_\nu^{-1}\boldsymbol{\Gamma}_\nu\delta\boldsymbol{R}\}\boldsymbol{\Gamma}_\nu^H\boldsymbol{f} \\ &- \frac{1}{\rho}\sum_i\boldsymbol{t}_i^H E\{\delta\boldsymbol{R}\boldsymbol{Q}_i^H\boldsymbol{\Phi}\boldsymbol{R}_\nu^{-1}\boldsymbol{\Gamma}_\nu\delta\boldsymbol{R}\}\boldsymbol{\Gamma}_\nu^H\boldsymbol{f} \\ &- \frac{1}{\rho}\boldsymbol{f}^H\boldsymbol{\Gamma}_\nu\sum_i E\{\delta\boldsymbol{R}\boldsymbol{\Gamma}_\nu^H\boldsymbol{R}_\nu^{-1}\boldsymbol{\Phi}\boldsymbol{Q}_i\delta\boldsymbol{R}\}\boldsymbol{t}_i. \end{aligned} \tag{22}$$

Evaluation of each term on the right hand side involves a typical matrix $E\{\delta\boldsymbol{R}\boldsymbol{D}\delta\boldsymbol{R}\}$ which has been provided by [9].

3) SINR of the RAKE receiver: To derive SINR for the RAKE receiver, we start from (7), where $\boldsymbol{y}(n)$ shows clear contributions from the desired symbol, interfering symbols and noise. Therefore, the output SINR of the rake receiver is computed as

$$\text{SINR} = \frac{\sigma_w^2 E\{|\boldsymbol{f}(n)^H \mathcal{G}_2 \boldsymbol{c}_{1,n}|^2\}}{\sigma_w^2 E\{||\boldsymbol{f}(n)^H \boldsymbol{Z}_{int}(n)||^2\} + \sigma_v^2 E\{||\boldsymbol{f}(n)||^2\}}$$

where $\boldsymbol{Z}_{int}(n)$ contains all interfering signatures

$$\boldsymbol{Z}_{int}(n) = [\mathcal{G}_2\boldsymbol{c}_{2,n}, \ldots, \mathcal{G}_2\boldsymbol{c}_{J,n}, \mathcal{G}_1\bar{\boldsymbol{c}}_{1,n-1}, \ldots, \mathcal{G}_1\bar{\boldsymbol{c}}_{J,n-1}]. \tag{23}$$

The perturbation of $\boldsymbol{R}$ causes the perturbation of $\mathcal{G}$ and then the receiver, which finally makes SINR perturbed. It can be shown that the perturbed signal power is $\sigma_w^2\Big(E\{|\boldsymbol{f}(n)^H \mathcal{G}_2\boldsymbol{c}_{1,n}|^2\} + E\{|\delta\boldsymbol{f}(n)^H\mathcal{G}_2\boldsymbol{c}_{1,n}|^2\}\Big)$, and the perturbed interference plus noise power is $\Big(\sigma_w^2\ E\{\ ||\boldsymbol{f}(n)^H\ \boldsymbol{Z}_{int}(n)||^2\} + \sigma_v^2\ E\{||\boldsymbol{f}(n)||^2\} + \sigma_w^2\ E\{||\delta\ \boldsymbol{f}(n)^H\ \boldsymbol{Z}_{int}(n)||^2\} + \sigma_v^2\ E\{||\delta\ \boldsymbol{f}(n)\ ||^2\ \}\Big)$. We now turn to evaluation of each non-perturbed or perturbed term by first obtaining the perturbation of the RAKE receiver as $\delta\boldsymbol{f}(n) = \delta\mathcal{G}_2\boldsymbol{c}_{1,n}$, where after considering (16), $\delta\mathcal{G}_2$ can be shown to be

$$\delta\mathcal{G}_2 = [\boldsymbol{A}_0\delta\boldsymbol{h}, \ldots, \boldsymbol{A}_{P-1}\delta\boldsymbol{h}]. \tag{24}$$

First, using trace and vec operations, the power of the desired symbol is derived as

$$\begin{aligned} & E\{|\boldsymbol{f}(n)^H\mathcal{G}_2\boldsymbol{c}_{1,n}|^2\} \\ = \ & E\{tr\{\mathcal{G}_2^H\mathcal{G}_2\boldsymbol{c}_{1,n}\boldsymbol{c}_{1,n}^H\mathcal{G}_2^H\mathcal{G}_2^H\boldsymbol{c}_{1,n}\boldsymbol{c}_{1,n}^H\}\} \\ = \ & vec^H(\mathcal{G}_2^H\mathcal{G}_\in)E\{(\boldsymbol{c}_{1,n}^*\otimes\boldsymbol{c}_{1,n})(\boldsymbol{c}_{1,n}^T\otimes\boldsymbol{c}_{1,n}^H)\} \\ & vec(\mathcal{G}_2^H\mathcal{G}_\in) \end{aligned} \tag{25}$$

where $E\{(\boldsymbol{c}_{1,n}^*\otimes\boldsymbol{c}_{1,n})(\boldsymbol{c}_{1,n}^T\otimes\boldsymbol{c}_{1,n}^H)\}$ can be directly obtained as (36) or (46) in [9] without repetition here. Following similar steps, $E\{|\delta\boldsymbol{f}(n)^H\mathcal{G}_2\boldsymbol{c}_{1,n}|^2\}$ can be shown to be similar to (25) with $\mathcal{G}_2^H\mathcal{G}_\in$ replaced by $\mathcal{G}_2^H\delta\mathcal{G}_\in$. If we further use trace operation, and replace $\mathcal{G}_2^H\delta\mathcal{G}_\in$ by $\mathcal{G}_2^H\delta\mathcal{G}_\in\boldsymbol{I}$, and then use vec operations, the perturbation of the power of the desired symbol can be shown to be

$$\begin{aligned} & E\{|\delta\boldsymbol{f}(n)^H\mathcal{G}_2\boldsymbol{c}_{1,n}|^2\} \\ = \ & tr\Big\{E\{(\boldsymbol{c}_{1,n}^*\otimes\boldsymbol{c}_{1,n})(\boldsymbol{c}_{1,n}^T\otimes\boldsymbol{c}_{1,n}^H)\}(\boldsymbol{I}\otimes\mathcal{G}_2^H) \\ & E\{vec(\delta\mathcal{G}_2)vec(\delta\mathcal{G}_2)^H\}(\boldsymbol{I}\otimes\mathcal{G}_2^H)\Big\} \end{aligned} \tag{26}$$

where we invoke a mild assumption on the independence of $\boldsymbol{c}_{1,n}$ and $\delta\mathcal{G}_2$. Although this assumption is inaccurate for moderate N, it yields satisfactory prediction of receiver's performance for large N, as verified by simulations. The expectation term $E\{vec(\delta\mathcal{G}_2)vec(\delta\mathcal{G}_2)^H\}$ can be obtained as

$$E\{vec(\delta\mathcal{G}_2)vec(\delta\mathcal{G}_2)^H\} = \boldsymbol{\Lambda}\text{Cov}_h\boldsymbol{\Lambda}^H. \tag{27}$$

where $\boldsymbol{\Lambda} = [\boldsymbol{A}_0^T,\ \ldots, \boldsymbol{A}_{P-1}^T]^T$.

Now, we proceed with interfering symbols' power and its perturbation. Using (23), following similar steps in (25), and considering $\boldsymbol{c}_{1,n}$ is independent of $\boldsymbol{c}_{j,n-1}$ which implies $E\{(\boldsymbol{c}_{1,n}^*\boldsymbol{c}_{1,n}^T)\otimes(\bar{\boldsymbol{c}}_{j,n-1}\ \bar{\boldsymbol{c}}_{j,n-1}^H)\} = \sigma_c^4\boldsymbol{I}$, the interfering symbols' power is immediately given by

$$\begin{aligned} & E\{||\boldsymbol{f}(n)^H\boldsymbol{Z}_{int}(n)||^2\} \\ = \ & \sum_{j=2}^{J} vec^H(\mathcal{G}_2^H\mathcal{G}_\in)E\{(\boldsymbol{c}_{1,n}^*\boldsymbol{c}_{1,n}^T)\otimes(\boldsymbol{c}_{j,n}\boldsymbol{c}_{j,n}^H)\} \\ & vec(\mathcal{G}_2^H\mathcal{G}_\in) + J\sigma_c^4 tr(\mathcal{G}_2^H\mathcal{G}_\infty\mathcal{G}_\infty{}^H\mathcal{G}_\in) \end{aligned} \tag{28}$$

$\boldsymbol{c}_{1,n}$ and $\boldsymbol{c}_{j,n}$ are dependent with each other, since they share the same base station codes. If we denote the jth user's Hadamard codes as $\boldsymbol{\beta}_j$ and the base station codes at time instant n as $\boldsymbol{\xi}_n$, then $\boldsymbol{c}_{j,n} = \boldsymbol{\Omega}_j\boldsymbol{\xi}_n$, where $\boldsymbol{\Omega}_j$ is a $P\times P$ diagonal matrix with its (k,k)th element as $\boldsymbol{\beta}_j(k)$. Based on this notation of $\boldsymbol{c}_{j,n}$ and using the property of Kronecker product, we immediately obtain

$$\begin{aligned} & E\{(\boldsymbol{c}_{1,n}^*\boldsymbol{c}_{1,n}^T)\otimes(\boldsymbol{c}_{j,n}\boldsymbol{c}_{j,n}^H)\} \\ = \ & (\boldsymbol{\Omega}_1^*\otimes\boldsymbol{\Omega}_j)E\{(\boldsymbol{\xi}_n^*\otimes\boldsymbol{\xi}_n)(\boldsymbol{\xi}_n^T\otimes\boldsymbol{\xi}_n^H)\}(\boldsymbol{\Omega}_1^T\otimes\boldsymbol{\Omega}_j^H) \end{aligned}$$

where $E\{(\boldsymbol{\xi}_n^*\otimes\boldsymbol{\xi}_n)(\boldsymbol{\xi}_n^T\otimes\boldsymbol{\xi}_n^H)\}$ is given by (36) or (46) in [9]. In a much similar way, the perturbation of interfering symbols' power can be shown to be

$$\begin{aligned} & E\{||\delta\boldsymbol{f}(n)^H\boldsymbol{Z}_{int}(n)||^2\} \\ = \ & \sum_{j=2}^{J} tr\Big\{E\{(\boldsymbol{c}_{1,n}^*\otimes\boldsymbol{c}_{j,n})(\boldsymbol{c}_{1,n}^T\otimes\boldsymbol{c}_{j,n}^H)\}(\boldsymbol{I}\otimes\mathcal{G}_2^H) \\ & \boldsymbol{\Lambda}\text{Cov}_h\boldsymbol{\Lambda}^H(\boldsymbol{I}\otimes\mathcal{G}_2^H))\Big\} + J\sigma_c^4 tr\Big\{E\{\delta\mathcal{G}_2^H\mathcal{G}_1 \\ & \mathcal{G}_1^H\delta\mathcal{G}_2\}\Big\} \end{aligned} \tag{29}$$

where $E\{\delta\mathcal{G}_2^H\ \mathcal{G}_1\ \mathcal{G}_1^H\ \delta\mathcal{G}_2\}$ has a similar form as $E\{\delta\mathcal{G}^H\ \boldsymbol{\Pi}^\perp\ \delta\mathcal{G}\}$. After considering (24), its (i,j)th element can be similarly evaluated as (19) with $k_1,\ k_2$ taking values from 0 to $P-1$, and $\boldsymbol{\Theta}_{i,j} = \boldsymbol{A}_{i-1}^H\mathcal{G}_1\mathcal{G}_1^H\boldsymbol{A}_{j-1}$. Finally, replacing $\boldsymbol{f}(n)$ by (8) and noticing that $\boldsymbol{c}_{1,n}$ is independent of noise and $E\{\boldsymbol{c}_{1,n}\boldsymbol{c}_{1,n}^H\} = \sigma_c^2\boldsymbol{I}$, noise power and its perturbation are readily derived as

$$\begin{aligned} \sigma_v^2 E\{||\boldsymbol{f}(n)||^2\} &= \sigma_v^2\sigma_c^2 tr\{\mathcal{G}_2^H\mathcal{G}_2\}, \\ \sigma_v^2 E\{||\delta\boldsymbol{f}(n)||^2\} &= \sigma_v^2\sigma_c^2 tr\{E\{\delta\mathcal{G}_2^H\delta\mathcal{G}_2\}\} \end{aligned} \tag{30}$$

where $E\{\delta\mathcal{G}_2^H\delta\mathcal{G}_2\}$ in (30) can be obtained similarly as above by setting $\boldsymbol{\Theta}_{i,j} = \boldsymbol{A}_{i-1}^H\boldsymbol{A}_{j-1}$.

To conclude, the perturbed signal power can be obtained by (25) and (26), and the perturbed interference plus noise power by (28), (29), and (30). Then the perturbed SINR immediately follows from the division of the perturbed signal power over the perturbed interference plus noise power.

B. BER Performance

For each receiver, once its output SINR is evaluated, BER can now be obtained by assuming the interference is Gaussian distributed. This may not be necessarily correct, but this approximation has been shown to be relatively good [8], especially when number of interfering users is large. The BER for BPSK information symbol is

$$\mathrm{BER} = Q(\sqrt{\mathrm{SINR}}) \tag{31}$$

where $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{\frac{-t^2}{2}} dt$.

V. Simulation Examples

Performance of the proposed receivers is numerically studied and compared with our analytical results. The transmitted sequences are drawn from a binary constellation $\{\pm 1\}$. Simulation parameters are set as follows: $P = 16$, $q = 3$, $J = 10$ and $M = 2$. Totally 100 Monte Carlo simulations are performed to obtain the average results. Fig. 1 shows the SINRs of receivers over different N when signal to noise ratio is 15dB. Clearly, each receiver's experimental SINRs approach their corresponding analytical results as N increases, verifying our SINR analysis. It is interesting to note that MMSE2 ($\nu = P$) shows worse performance than MMSE1 ($\nu = q+1$) when N is small, while the former gradually outperforms the latter when N becomes large. The cross point is around $N = 700$. This observation gives us a guideline of choosing the MMSE receiver's length. We thus set $\nu = q+1$ in the next experiment. Fig. 2 compares experimental BERs of all receivers with their corresponding analytical ones respectively over various SNRs. We first use a data record of length $N = 500$ for channel estimation and obtain different receivers. Experimental BER is then computed over an independent data of length 5000. Experimental results are very close to their analytical ones especially at low SNRs. The slight differences at high SNRs are caused by the tenuous assumption of Gaussian distribution of interference [8].

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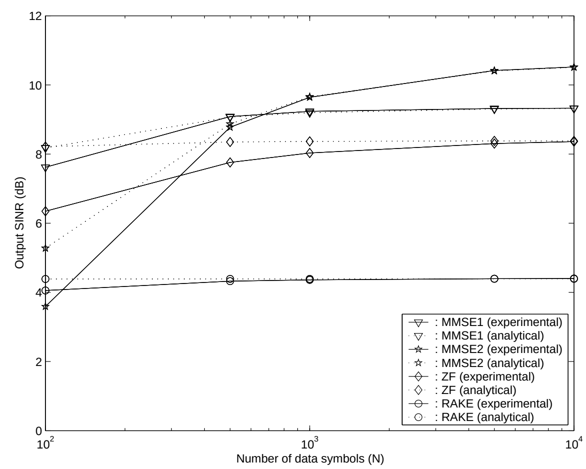

Fig. 1. Output SINR vs. N.

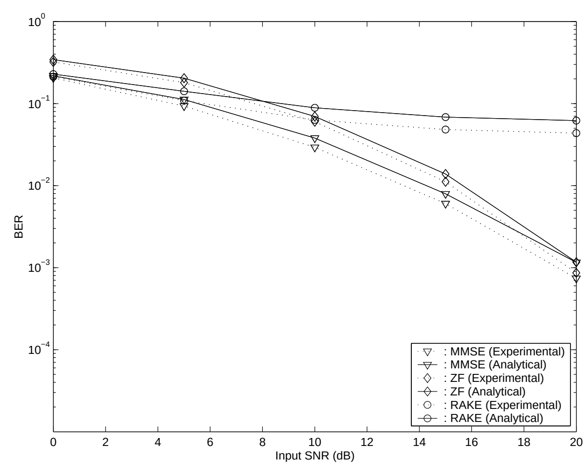

Fig. 2. BER vs. SNR.