

On the Second-Order Statistics of the Sample Covariance

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Abstract— The second-order statistics of the sample covariance are encountered in many covariance based processing algorithms such as subspace based and covariance-matching based channel estimation methods. This paper provides closed-form expressions for the covariance of either the weighted sample covariance matrix or vectored sample covariance. Given a system model, the results explicitly rely on the second-order and fourth-order statistics of the channel noise and inputs. They can be directly applied to study the statistical properties of covariance based channel estimators and detectors. Various numerical examples are provided to further verify derived results.

I. INTRODUCTION

In a wireless communication system, digital signals are transmitted through multipath channels which usually cause severe signal distortion. In order to reliably recover the input sequence, channel impulse responses can be first estimated based on the channel output. Then receivers are designed. If training symbols are not available, blind channel estimation methods are required. Typical second-order statistics (SOS) based methods rely on the SOS of the received signal, among which subspace based [9], [10] and covariance matching based methods are widely studied [2], [4], [5], [11].

It is observed that the covariance of the received data plays an important role in determining the performance of each channel estimator/detector. In practice, estimation of the covariance is performed easily from sample average based on received data. Performance of this covariance estimation method characterizes each individual method. If Gaussian sources are assumed, the second-order statistics of the estimated covariance can be found in [1]. It has been applied to analyze the asymptotic property of eigenvectors of a covariance matrix [3]. A statistical analysis of the subspace method [9] is carried out in [6] under some asymptotic approximations for involved quantities. For a CDMA system, performance of a subspace based channel estimator is also studied in [12].

However, lack of a closed form expression results in partially redundant efforts on performance evaluation of associated methods.

In this paper, we consider both a weighted covariance of a sample covariance matrix and a covariance of vectored data covariance when the channel inputs and noise are independent random processes and data covariance is estimated from independent observations. Since correlations of real random variables and complex random variables show different statistical properties [1], two cases are differentiated: the system is real and the system is complex. By appropriate manipulations in tedious but straightforward procedures and employing attractive properties of the Kronecker product (“ \otimes ”) [7], closed-form expressions are derived respectively. It turns out that they depend on the second- as well as fourth-order statistics of both the inputs and channel noise. Numerical examples are provided to further validate the analytical results. Those results can be directly applied to obtain the channel mean-square-errors of the subspace-based channel estimation methods.

II. DATA MODEL AND ASSUMPTIONS

Consider a vector form channel input/output data model

$$\mathbf{y}_n = \mathbf{H}\mathbf{b}_n + \mathbf{v}_n \quad (1)$$

where \mathbf{y}_n is a ν -dimensional received data vector at time n with samples stacked in an ascending order of time, \mathbf{H} is a $\nu \times L$ channel matrix (could be structured or not structured), \mathbf{b}_n is a L -dimensional input vector and \mathbf{v}_n represents the additive noise. This model can describe a variety of communication systems. For example, in a single-input multiple-output (SIMO) system, \mathbf{b}_n includes inputs from time n to $n + L - 1$ emitted from the same source. When it is used to represent a multiple-input multiple-output (MIMO) system, \mathbf{y}_n has samples from either multiple antennas or upsamplers. Accordingly, \mathbf{b}_n contains symbols from different sources. The

received signal in a DS/CDMA system can also fit into this model where \mathbf{H} becomes a signature waveform matrix.

Without loss of generality, we assume all entries in \mathbf{b}_n are mutually independent sequences. They are drawn from the same constellation with zero odd moments, equal variance σ_b^2 , fourth-order absolute moment m_{4b} and fourth-order cumulant κ_{4b} . In the case when multiple sources have different transmission powers, it is an easy task to cast amplitudes into the columns of channel matrix \mathbf{H} . Additionally, we assume \mathbf{v}_n is an additive white Gaussian noise (AWGN) vector whose entries have zero odd moments, variance σ_v^2 . For shorter notation, denote Hermitian - complex conjugate $(\cdot)^*$ transpose $(\cdot)^T$ by $(\cdot)^H$, trace of a matrix by $tr(\cdot)$, expectation of a random variable by $E\{\cdot\}$, the a th ($a = 1, 2, \dots$) column of a matrix \mathbf{U} as \mathbf{u}_a , $\mathbf{1}_a$ as a column vector of length a with all elements equal to one, \mathbf{I}_a as an identity matrix of degree a whose b th column is denoted as $\mathbf{e}_{a,b}$, the Kronecker product as “ \otimes ” [7], a “ vec ” operation to successively stack columns of a matrix into a big column vector [7], the matrix Hadamard product “ \odot ” to represent element wise multiplication, the Khatri-Rao product “ \square ” to represent column-wise Kronecker product of two matrices: $\mathbf{U}\square\mathbf{W} = [\mathbf{u}_1 \otimes \mathbf{w}_1, \mathbf{u}_2 \otimes \mathbf{w}_2, \dots]$. If inputs and noise are complex quantities, their real parts and imaginary parts are assumed to be i.i.d. as well. Cumulant is related to moment and variance [8]. For a real input sequence, $\kappa_{4b} = m_{4b} - 3\sigma_b^4$, while for a complex input sequence, $\kappa_{4b} = m_{4b} - 2\sigma_b^4$. If a sequence is a Gaussian random process, then it has zero cumulant. This applies to the noise sequence as well.

III. STATISTICS OF ESTIMATED COVARIANCE

According to (1), the covariance of \mathbf{y}_n has a form

$$\mathbf{R} = E\{\mathbf{y}_n \mathbf{y}_n^H\} = \sigma_b^2 \mathbf{H} \mathbf{H}^H + \sigma_v^2 \mathbf{I}_\nu \quad (2)$$

where H represents Hermitian - complex conjugate (*) transpose (T). In practice, \mathbf{R} is usually estimated from N independent data vectors by sample average [12]

$$\hat{\mathbf{R}} = \frac{1}{N} \sum_{n=1}^N \mathbf{y}_n \mathbf{y}_n^H \quad (3)$$

to approximate the expectation. The approximation error is denoted by $\delta \mathbf{R} = \hat{\mathbf{R}} - \mathbf{R}$. It is noticed that $\delta \mathbf{R}$ is a Hermitian symmetric matrix $\delta \mathbf{R}^H = \delta \mathbf{R}$ because both $\hat{\mathbf{R}}$ and \mathbf{R} are Hermitian symmetric.

Here, we are interested in the performance of the covariance estimator $\hat{\mathbf{R}}$. The performance measure may appear in two

different forms. One is directly related to $\delta \mathbf{R}$ in terms of a weighted covariance

$$\Phi(\hat{\mathbf{R}}) = E\{\delta \mathbf{R} \Psi \delta \mathbf{R}\} \quad (4)$$

where Ψ is a deterministic weighting matrix which may vary in different applications. This expression is involved in algorithms when operations are directly performed on \mathbf{R} such as the subspace decomposition in subspace based algorithms. The other one is a vectored version

$$\Gamma(\hat{\mathbf{r}}) = E\{\delta \mathbf{r} \delta \mathbf{r}^H\} \quad (5)$$

where $\hat{\mathbf{r}} = vec(\hat{\mathbf{R}})$, $\delta \mathbf{r} = vec(\delta \mathbf{R})$. Although each entry of (5) can be obtained directly from (4) by properly defining Ψ as a selection matrix whose elements are all zeros except only one “1” in an appropriate position, we still consider them separately for compact and easy-to-use forms. We provide our results after simplification and skip tedious derivations due to lack of space. Some of them have appeared in [13] while all others will be reported in detail elsewhere. Instead, we will verify them by simulations. Since real and complex systems result in different expressions for the quantities to be derived, a communication system is differentiated by either a real system whose quantities are real-valued or complex system whose quantities are complex-valued with independent real and imaginary parts. Our major results are then provided by following propositions.

Proposition 1 (weighted covariance of $\hat{\mathbf{R}}$) [13]: If channel model follows (1) with specified dimensions L and ν , and covariance is estimated by (3) from N independent data vectors, then for a real system, the weighted covariance of $\hat{\mathbf{R}}$ with any constant weight Ψ satisfies

$$\begin{aligned} N\Phi(\hat{\mathbf{R}}) &= \kappa_{4b} \mathbf{H} [\mathbf{I}_L \odot (\mathbf{H}^T \Psi \mathbf{H})] \mathbf{H}^T \\ &+ tr(\mathbf{R} \Psi) \mathbf{R} + \mathbf{R} \Psi^T \mathbf{R}, \end{aligned} \quad (6)$$

while for a complex system, it follows

$$\begin{aligned} N\Phi(\hat{\mathbf{R}}) &= \kappa_{4b} \mathbf{H} [\mathbf{I}_L \odot (\mathbf{H}^H \Psi \mathbf{H})] \mathbf{H}^H \\ &+ tr(\mathbf{R} \Psi) \mathbf{R} \end{aligned} \quad (7)$$

□

Proposition 2 (covariance of $\hat{\mathbf{r}}$): If channel model follows (1) with specified dimensions L and ν , and covariance is estimated by (3) from N independent data vectors, then for a

real system, the covariance of $\hat{\mathbf{r}}$ satisfies

$$N\mathbf{\Gamma}(\hat{\mathbf{r}}) = \kappa_{4b}(\mathbf{H}\square\mathbf{H})(\mathbf{H}\square\mathbf{H})^T + \mathbf{R} \otimes \mathbf{R} + [(\mathbf{I}_\nu \otimes \mathbf{1}_\nu)\mathbf{R}(\mathbf{1}_\nu^T \otimes \mathbf{I}_\nu)] \odot [(\mathbf{1}_\nu \otimes \mathbf{I}_\nu)\mathbf{R}(\mathbf{I}_\nu \otimes \mathbf{1}_\nu^T)], \quad (8)$$

while for a complex system it satisfies

$$N\mathbf{\Gamma}(\hat{\mathbf{r}}) = \kappa_{4b}(\mathbf{H}^*\square\mathbf{H})(\mathbf{H}^*\square\mathbf{H})^H + \mathbf{R}^* \otimes \mathbf{R}. \quad (9)$$

□

Discussion

Our model is based on the assumption that all inputs have the same power. If their powers are different, then proper scaling of the corresponding column vector in the channel matrix can be performed. *Proposition 1* provides statistics of the estimated covariance matrix with an arbitrary deterministic weighting matrix, while *Proposition 2* presents statistics of the vectored form of the covariance matrix. They are directly applicable in analyzing covariance-based algorithms. The former fits well into a situation where covariance matrix \mathbf{R} instead of its vectored form \mathbf{r} is directly involved in the corresponding methods. It may be interesting to seek connections of *Proposition 1* with *Proposition 2*. *Proposition 2* pertains to covariance of all possible pairs of elements in $\hat{\mathbf{r}}$. If we set Ψ in *Proposition 1* to be a matrix with all zero elements except “1” in one position, then the resulting $\Phi(\hat{\mathbf{R}})$ will contain only one non-zero entry which becomes a particular entry of $\mathbf{\Gamma}(\hat{\mathbf{r}})$. Repeating this procedure by changing Ψ accordingly, all entries of $\mathbf{\Gamma}(\hat{\mathbf{r}})$ can be obtained. However, it requires exhaustive operations (there are totally ν^2 elements in $\mathbf{\Gamma}(\hat{\mathbf{r}})$) to obtain final results. Meanwhile, no close-form expression can be easily derived in such a way. Thus, *Proposition 2* is still necessary for easy application. It is also worth to mention that these results are accurate. No approximations are made. They hold for an arbitrary N which is not necessarily large, although large N is required to apply them in some applications.

These two propositions provide direct methods to compute the covariances for given system parameters. It can be observed that either $\Phi(\hat{\mathbf{R}})$ or $\mathbf{\Gamma}(\hat{\mathbf{r}})$ is inversely proportional to N . Before we verify these results by computer simulations, we will first show how to apply *Proposition 1* to analyze a subspace-based channel estimator. Application of *Proposition 2* is discussed once more after *Proposition 2* is taken into account although some analytical results in complicated forms have appeared in [11].

IV. APPLICATIONS

Our previous results can be applied to analyze covariance based detectors or estimators. Here we provide two examples to evaluate performance of different channel estimators in a CDMA system.

A. Performance of a subspace based channel estimator

Consider a K -user synchronous CDMA system. User k is assigned spreading codes $c_k(0), \dots, c_k(P-1)$. Its channel coefficients are stacked in a vector \mathbf{g}_k . All channels are assumed to have maximum length q . Then in one symbol interval, the received data vector has a form [11], [12]

$$\mathbf{y}_n = \sum_{k=1}^K \mathbf{C}_k \mathbf{g}_k b_k(n) + \mathbf{v}_n \quad (10)$$

where \mathbf{C}_k is a code filtering matrix obtained from the spreading codes of user k , \mathbf{g}_k is the channel vector. Without loss of generality, assume user 1 is the desired user. We denote the noise-free data covariance matrix by \mathbf{Z} and the noise subspace of \mathbf{R} by \mathbf{U}_n . Since $\mathbf{U}_n^H \mathbf{C}_1 \mathbf{g}_1 = \mathbf{0}$, under some identifiability conditions, \mathbf{h}_1 can be estimated from

$$\hat{\mathbf{g}}_1 = \arg \min_{\|\alpha\|=1} \alpha^H \mathbf{C}_1^H \mathbf{U}_n \mathbf{U}_n^H \mathbf{C}_1 \alpha \quad (11)$$

up to a multiplicative scalar. $\hat{\mathbf{g}}_1$ is then a unique eigenvector of the following objective matrix $\mathbf{O}_1 = \mathbf{C}_1^H \mathbf{U}_n \mathbf{U}_n^H \mathbf{C}_1$ corresponding to its null eigenvalue. If $\delta\mathbf{R}$ exists for large N , the perturbation of the noise subspace of \mathbf{R} has the following form [3]

$$\delta\mathbf{U}_n \approx -\mathbf{Z}^\dagger \delta\mathbf{R} \mathbf{U}_n \quad (12)$$

where $(\cdot)^\dagger$ denotes the pseudo-inverse, and the perturbation in the estimated channel vector becomes [3]

$$\delta\mathbf{g}_1 \approx -\mathbf{O}_1^\dagger \mathbf{C}_1^H \mathbf{U}_n \delta\mathbf{U}_n^H \mathbf{C}_1 \mathbf{g}_1. \quad (13)$$

Combining (12), (13), and applying our results in *Proposition 1*, one can easily find that

$$E\{\delta\mathbf{g}_1 \delta\mathbf{g}_1^H\} \approx \frac{\gamma_1 \sigma_v^2}{N} \mathbf{O}_1^\dagger \mathbf{C}_1^H \mathbf{U}_n \mathbf{U}_n^H \mathbf{C}_1 \mathbf{O}_1^\dagger \quad (14)$$

where

$$\gamma_1 = \mathbf{g}_1^H \mathbf{C}_1^H \mathbf{Z}^\dagger \mathbf{R} \mathbf{Z}^\dagger \mathbf{C}_1 \mathbf{g}_1. \quad (15)$$

The mean square error (MSE) of the estimated channel vector is then the trace of (14). It is proportional to $\frac{\sigma_v^2}{N}$ and is approximately inversely proportional to the power of the transmitted signal due to the term $\mathbf{Z}^\dagger \mathbf{R} \mathbf{Z}^\dagger$. It also depends on projections of the code matrix onto the signal subspace and noise subspace. However, it does not depend on the fourth-order statistics of the input sequence although statistical performance of the estimated covariance matrix does.

B. Performance of a correlation-matching based channel estimator

Consider a correlation-matching based channel estimator proposed in [11]. The correlation of (10) becomes

$$\mathbf{R} = \sum_{k=1}^K \mathbf{C}_k \mathbf{G}_k \mathbf{C}_k^H + \sigma_v^2 \mathbf{I}_\nu \quad (16)$$

where $\mathbf{G}_k = \sigma_b^2 \mathbf{g}_k \mathbf{g}_k^H$. For convenience, define $\mathbf{x}_k = \text{vec}(\mathbf{G}_k)$. Collect all \mathbf{x}_k and σ_v^2 in an unknown vector \mathbf{x} . Then after *vec* operation on \mathbf{R} : $\mathbf{r} = \text{vec}(\mathbf{R})$, it can be easily found that \mathbf{x} satisfies the following [11]

$$\mathbf{x} = (\mathbf{S}^H \mathbf{S})^{-1} \mathbf{S}^H \mathbf{r} \quad (17)$$

with \mathbf{S} being defined as

$$\mathbf{S} = [\mathbf{C}_1 \otimes \mathbf{C}_1, \dots, \mathbf{C}_K \otimes \mathbf{C}_K, \text{vec}(\mathbf{I}_\nu)].$$

If \mathbf{r} is estimated from received data vectors, then \mathbf{x} can be estimated according to (17). For the desired user, \mathbf{x}_1 can be extracted from \mathbf{x} . Then \mathbf{G}_1 is reconstructed by the reverse *vec* operation. These operations can be described by

$$\widehat{\mathbf{G}}_1 = [\mathbf{A}_1 \widehat{\mathbf{r}}, \dots, \mathbf{A}_q \widehat{\mathbf{r}}] \quad (18)$$

where q is the length of \mathbf{g}_1 ,

$$\mathbf{A}_i = (\mathbf{e}_{q,i}^T \otimes \mathbf{I}_q) [\mathbf{e}_{K,1}^T \otimes \mathbf{I}_{q^2}, \mathbf{0}_{q^2 \times 1}]_{q^2 \times (q^2 K + 1)} (\mathbf{S}^H \mathbf{S})^{-1} \mathbf{S}^H$$

for $i = 1, \dots, q$. From our definition of \mathbf{G}_1 , \mathbf{g}_1 is an eigenvector corresponding to its unique non-zero eigenvalue. If \mathbf{G}_1 is perturbed by $\delta \mathbf{G}_1$ as

$$\delta \mathbf{G}_1 = [\mathbf{A}_1 \delta \mathbf{r}, \dots, \mathbf{A}_q \delta \mathbf{r}], \quad (19)$$

then the first-order perturbation in this eigenvector becomes [14]

$$\delta \mathbf{g}_1 \approx \sigma_b^{-2} \mathbf{\Pi} \delta \mathbf{G}_1 \mathbf{g}_1, \quad \mathbf{\Pi} = \mathbf{\Sigma}_n \mathbf{\Sigma}_n^H \quad (20)$$

where $\mathbf{\Sigma}_n$ spans a $(q-1)$ -dimensional subspace orthogonal to \mathbf{g}_1 . Using (20) and (19), we obtain the covariance $\text{Cov}(\delta \mathbf{g}_1)$

$$\text{Cov}(\delta \mathbf{g}_1) \approx \sigma_b^{-4} \mathbf{\Pi} \left(\sum_{i=1}^q g_1(i) \mathbf{A}_i \right) \mathbf{\Gamma}(\widehat{\mathbf{r}}) \left(\sum_{i=1}^q g_1(i) \mathbf{A}_i \right)^H \mathbf{\Pi} \quad (21)$$

where $\mathbf{\Gamma}(\widehat{\mathbf{r}})$ is the covariance of $\widehat{\mathbf{r}}$ given by *Proposition 2*. Then the channel MSE is the trace of $\text{Cov}(\delta \mathbf{g}_1)$. Together with (14), *Proposition 1* and *Proposition 2*, it will be verified by simulations next.

V. SIMULATION

We test the results derived in the previous sections. We only consider real systems while their counter parts for the complex systems are omitted. To test *Proposition 1*, we adopt the normalized squared Frobenius norm of the difference between $\mathbf{\Phi}(\widehat{\mathbf{R}})$ and its estimate as the performance measure

$$\epsilon = \frac{\|\mathbf{\Phi}(\widehat{\mathbf{R}}) - \widehat{\mathbf{\Phi}}\|_F^2}{\|\mathbf{\Phi}(\widehat{\mathbf{R}})\|_F^2}$$

and test the asymptotic performance with respect to the number of independent snapshots (N). Parameters are set as follows: $\nu = 10$, $L = 4$, $\mathbf{\Psi} = \mathbf{I}$. Channel coefficients are randomly selected from a zero-mean white Gaussian process with unit variance and fixed in all realizations. Channel noise is Gaussian distributed giving a 10dB signal to noise ratio (SNR). Inputs are drawn from real constellations such as BPSK, PAM and Gaussian. The average estimation errors from 100 independent Monte Carlo realizations are plotted in Fig. 1. The solid line is for the BPSK source, the dashed-dotted line for the 8PAM source and the dashed line for the Gaussian source. Differences are observed with different inputs which implies that the sample covariance estimation method may perform differently. We also apply the derived results to channel estimation for a synchronous CDMA system with BPSK inputs. We assume 10 equally-powered users in a system with 15dB AWGN. Each user is assigned a Gold sequence of length 31 to spread its data streams [11]. Channel order is set to be 3 for each user. Again, our experimental result (by a solid line) is compared with the analytical result (by a dashed line) for different N in Fig. 2. High consistency between two lines can be observed for longer observation time.

To test *Proposition 2*, we adopt the MSE to quantify the covariance estimation error. It is the trace of $\mathbf{\Gamma}(\widehat{\mathbf{r}})$. Experimental results are compared with analytical results. For a real system using BPSK sources, the MSE versus N is plotted in Fig. 3 which shows that the experimental result agrees with the dashed line from our analysis. The channel MSEs based on (21) are also compared with the experimental results. Same CDMA system as before is simulated. The desired user's channel vector is estimated by the correlation matching method with an identity weighting matrix. Fig. 4 plots the experimental result (solid line) and analytical result. High consistency between two curves can be observed.

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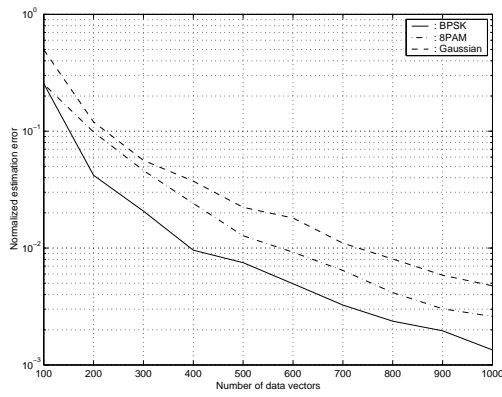


Fig. 1. Covariance estimation error for a real system with real inputs.

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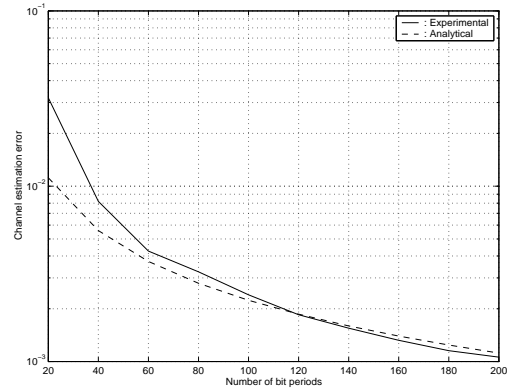


Fig. 2. Data length effect on subspace-based channel estimation error for a synchronous CDMA system.

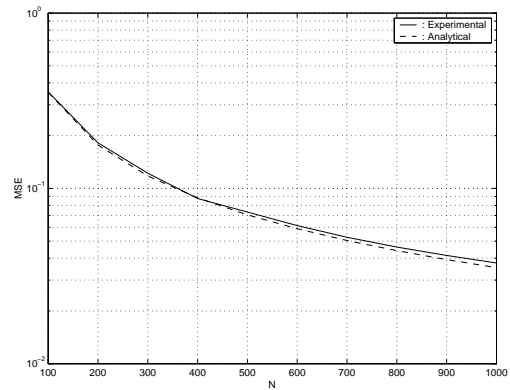


Fig. 3. Mean square error of vectored covariance estimation versus N for real systems.

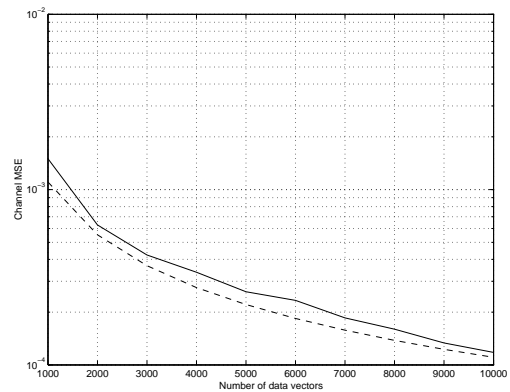


Fig. 4. Data length effect on correlation-matching based channel estimation error for a synchronous CDMA system.