PERFORMANCE OF CDMA RECEIVERS UNDER IMPERFECT CHANNEL ESTIMATION

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ABSTRACT

In a CDMA system, imperfect channel estimation causes performance degradation of a minimum mean-square-error (MMSE) receiver. This paper studies effects of channel estimation error on the performance of direct matrix inversion (DMI) MMSE receiver and subspace MMSE receiver when channel parameters are estimated by a covariance-matching technique based on finite data samples. Receivers' output signal to interference plus noise ratios (SINRs) and bit-error-rates (BERs) are adopted for performance measures. Those performance indicators under such an imperfect condition are derived from a perturbation perspective and verified by simulation examples.

1. INTRODUCTION

In a CDMA system, multiuser interference (MUI) is a typical obstacle to be obviated. Minimum mean-square-error (MMSE) receivers can be designed to detect input signals while suppressing MUI. However, in a wireless communication environment, channel parameters are not known *a priori* and need to be estimated. Blind methods offer an increased effective data rate. Typical second-order statistics based methods include subspace approaches [1], [2], minimum variance or minimum output energy (MOE) methods [3], [4], power of R (POR) technique [5], etc.

Most multiuser detection and channel estimation techniques are developed under perfect conditions first and then applied to practical scenarios to test performance degradation and robustness. Since various imperfectness may stem from many sources such as background noise, finite sample size, unknown channel order, synchronization error, user variation, etc., performance prediction under those conditions is necessary to better evaluate individual method. For example, sensitivity of multiuser detectors' performance to channel mismatch is analyzed in [6] when transmitted signals suffer from flat fading. Subspace based multipath channel estimation errors for a multirate CDMA system are derived for given finite number of observations [7]. In [8], perturbation to subspace decomposition of a matrix is studied when errors are introduced to the matrix by either noise or finite sample size.

In this paper, we analyze performance of direct matrix inversion (DMI) MMSE and subspace MMSE receivers [2] when channel is imperfectly estimated by a covariance-matching technique [9]. Each receiver's output signal to interference plus noise ratio (SINR) and bit-error-rate (BER) are adopted for performance measures. Imperfectness from finite sample size is treated as perturbation whose effect is particularly studied. To achieve our goal, statistics of sample covariance are provided in the light of [10]. Similar to [11], they can be regarded as general results and possibly applied to analysis of other methods. Then SINR and BER under such an imperfect condition are analyzed from a perturbation perspective and verified by extensive simulations.

Notations throughout the paper are defined as follows. Denote Hermitian - complex conjugate $(\cdot)^*$ transpose $(\cdot)^T$ by $(\cdot)^H$, integer ceiling by $\lceil\cdot\rceil$, vector 2-norm by $||\cdot||$ and matrix Frobenius norm by $||\cdot||_F$ [12], trace by $tr(\cdot)$, expectation by $E\{\cdot\}$, the ath column of a matrix $\boldsymbol{U}$ as $\boldsymbol{u}_a$, $\mathbf{1}_a$ as a column vector of length a with all elements equal to one, $\boldsymbol{I}_a$ as an identity matrix of degree a whose bth column is denoted as $\boldsymbol{e}_{a,b}$, the Kronecker product as "$\otimes$" [12], "vec" as a vectorized operation, matrix Hadamard product "$\odot$" to represent element wise multiplication, the Khatri-Rao product "$\square$" to represent column-wise Kronecker product [12]: $\boldsymbol{U}\square\boldsymbol{W} = [\boldsymbol{u}_1\otimes\boldsymbol{w}_1, \boldsymbol{u}_2\otimes\boldsymbol{w}_2, \cdots]$. A diagonal or block diagonal matrix with main diagonal entries $\boldsymbol{x}_i$ is denoted as $diag\{\boldsymbol{x}_1, \boldsymbol{x}_2, \cdots\}$. We also denote a perturbation by preceding the corresponding quantity by δ, and the perturbed quantity with $\widehat{\cdot}$. For example, $\delta\boldsymbol{R} = \widehat{\boldsymbol{R}} - \boldsymbol{R}$.

2. DATA MODEL

Consider an uplink CDMA system with J users. User j is assigned periodic spreading codes $c_j(k)$ of length P to spread its information symbol $w_j(n)$ of zero-mean and unit variance. Let its chip sequence be transmitted through a discrete-time chip-rate channel $g_j(l)$. Then the received signal $y_j(n)$ at the chip-synchronized receiver has a form [3]

$$y_j(n) = \sum_{l=-\infty}^{\infty} w_j(l)h_j(n-d_j-lP), h_j(n) = \sum_{i=-\infty}^{\infty} g_j(i)c_j(n-i), \quad (1)$$

where d_j is the propagation delay of user j in chip periods. After considering all J users and zero-mean additive white Gaussian noise (AWGN) $v(n)$ whose variance is denoted as σ_v^2, the received signal becomes $y(n) = \sum_{j=1}^{J} y_j(n) + v(n)$.

The discrete-time model can be easily formulated into a matrix/vector representation. For convenience, we assume a quasi-synchronous system with $d_j \ll P$ and absorb propagation delay into the channel for each user. The maximum delay spread of all multipath channels is q chips. If we collect $\nu = MP$ chip-rate samples in a vector $\boldsymbol{y}_n$ at the receiver, then it is given by [3]

$$\boldsymbol{y}_n = \sum_{j=1}^{J}\sum_{m=-K}^{M-1} \boldsymbol{C}_{j,m}\boldsymbol{g}_j w_j(n+m) + \boldsymbol{v}_n = \boldsymbol{H}\boldsymbol{w}_n + \boldsymbol{v}_n, \quad (2)$$

where $K = \lceil(q-1)/P\rceil$, $\boldsymbol{g}_j$ is the channel vector of length q, $\boldsymbol{C}_{j,m}$ is the code filtering matrix of user j for symbol $w_j(n+$

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m) which can be obtained from $\boldsymbol{C}_{j,0}$ (corresponding to symbol $w_j(n)$) by shifting it up (if $m<0$) or down (if $m>0$) by $|m|P$ rows, $\boldsymbol{H}$ is called a signature waveform matrix

$$\boldsymbol{H}=[\boldsymbol{H}_1,\cdots,\boldsymbol{H}_J],\quad \boldsymbol{H}_j=[\boldsymbol{C}_{j,-K}\boldsymbol{g}_j,\cdots,\boldsymbol{C}_{j,M-1}\boldsymbol{g}_j], \quad (3)$$

$\boldsymbol{w}_n$ contains all $L=J(M+K)$ inputs, $\boldsymbol{v}_n$ is the noise component. Without loss of generality, assume $w_1(n)$ is to be detected at time n. Its signature vector is $\boldsymbol{h}_{K+1}=\boldsymbol{C}_{1,0}\boldsymbol{g}_1$. Structures of users' signature waveforms have been exploited to estimate all channel vectors blindly by a covariance-matching technique [9] under assumptions that all spreading codes and the maximum channel delay spread are known.

3. COVARIANCE-MATCHING CHANNEL ESTIMATION AND LINEAR DETECTION

Covariance-matching technique is based on the covariance of $\boldsymbol{y}_n$

$$\boldsymbol{R}=\sum_{j=1}^{J}\sum_{m=-K}^{M-1}\boldsymbol{C}_{j,m}\boldsymbol{G}_j\boldsymbol{C}_{j,m}^H+\sigma_v^2\boldsymbol{I}_\nu \quad (4)$$

where $\boldsymbol{G}_j=\boldsymbol{g}_j\boldsymbol{g}_j^H$. Define $\alpha_j=tr(\boldsymbol{G}_j)=||\boldsymbol{g}_j||^2$. It is observed that $\boldsymbol{R}$ is parameterized by $\boldsymbol{G}_j$. If it is matched with its estimate from N data vectors

$$\widehat{\boldsymbol{R}}=\frac{1}{N}\sum_{n=1}^{N}\boldsymbol{y}_n\boldsymbol{y}_n^H, \quad (5)$$

and the resulting error $||\boldsymbol{R}-\widehat{\boldsymbol{R}}||_F^2$ is minimized, $\boldsymbol{G}_j$ can be estimated. Define $\boldsymbol{r}=vec(\boldsymbol{R})$, $\widehat{\boldsymbol{r}}=vec(\widehat{\boldsymbol{R}})$, $\boldsymbol{x}_j=vec(\boldsymbol{G}_j)$, and $\boldsymbol{x}=[\boldsymbol{x}_1^T,\cdots,\boldsymbol{x}_J^T,\sigma_v^2]^T$. Noticing $||\boldsymbol{A}||_F^2=||vec(\boldsymbol{A})||^2$, the criterion can be described as follows [9]

$$\widehat{\boldsymbol{x}}=\arg\min||\boldsymbol{r}-\widehat{\boldsymbol{r}}||^2=\arg\min||\boldsymbol{S}\boldsymbol{x}-\widehat{\boldsymbol{r}}||^2 \quad (6)$$

where

$$\boldsymbol{S}=[\boldsymbol{S}_1,\cdots,\boldsymbol{S}_J,vec(\boldsymbol{I}_\nu)],\quad \boldsymbol{S}_j=\sum_{m=-K}^{M-1}\boldsymbol{C}_{j,m}^*\otimes\boldsymbol{C}_{j,m}. \quad (7)$$

Under some identifiability conditions [9], the solution to (6) becomes

$$\widehat{\boldsymbol{x}}=\boldsymbol{Q}\widehat{\boldsymbol{r}},\quad \boldsymbol{Q}=(\boldsymbol{S}^H\boldsymbol{S})^{-1}\boldsymbol{S}^H, \quad (8)$$

Once $\boldsymbol{x}$ is estimated, $\boldsymbol{x}_j$ can be extracted. Then $\boldsymbol{G}_j$ is reconstructed by the reverse vec operation. These operations can be described by

$$\widehat{\boldsymbol{G}}_j=[\boldsymbol{A}_{j,1}\widehat{\boldsymbol{r}},\cdots,\boldsymbol{A}_{j,q}\widehat{\boldsymbol{r}}] \quad (9)$$

where

$$\boldsymbol{A}_{j,i}=(\boldsymbol{e}_{q,i}^T\otimes\boldsymbol{I}_q)[\boldsymbol{e}_{J,j}^T\otimes\boldsymbol{I}_{q^2},\boldsymbol{0}_{q^2\times1}]_{q^2\times(Jq^2+1)}(\boldsymbol{S}^H\boldsymbol{S})^{-1}\boldsymbol{S}^H$$

for $i=1,\cdots,q$, $j=1,\cdots,J$. If singular value decomposition is performed on $\boldsymbol{G}_j$, then the maximum singular value is $\alpha_j=||\boldsymbol{g}_j||^2$ and corresponding singular vector becomes the channel vector up to a scalar ambiguity. Therefore, once $\widehat{\boldsymbol{G}}_j$ is obtained, channel vector $\boldsymbol{g}_j$ can be estimated by finding the maximum singular vector of $\widehat{\boldsymbol{G}}_j$ and scaling the vector by $\sqrt{\widehat{\alpha}_j}$ to adjust its norm.

The estimated channel vector $\widehat{\boldsymbol{g}}_1$ can be used for design of DMI MMSE and subspace MMSE receivers. The DMI MMSE receiver can be defined from direct inversion of $\boldsymbol{R}$ as

$$\boldsymbol{f}_{mmse,dmi}=\boldsymbol{R}^{-1}\boldsymbol{C}_{1,0}\boldsymbol{g}_1. \quad (10)$$

The MMSE receiver can also be expressed in terms of the subspace components of $\boldsymbol{R}$. Let the eigenvalue decomposition of $\boldsymbol{R}$ be

$$\boldsymbol{R}=\boldsymbol{U}_s\boldsymbol{\Lambda}_s\boldsymbol{U}_s^H+\boldsymbol{U}_n\boldsymbol{\Lambda}_n\boldsymbol{U}_n^H, \quad (11)$$

where $\boldsymbol{\Lambda}_s=diag\{\lambda_1^2,\ \ldots\ ,\lambda_\xi^2\}$, $\boldsymbol{\Lambda}_n=\sigma_v^2\boldsymbol{I}$, $\boldsymbol{U}_s$ and $\boldsymbol{U}_n$ represent the signal and noise subspaces respectively. Invoking the orthogonality between $\boldsymbol{U}_n$ and $\boldsymbol{C}_{1,0}\boldsymbol{g}_1$, the subspace MMSE receiver takes the following form [2]

$$\boldsymbol{f}_{mmse,sub}=\boldsymbol{U}_s\boldsymbol{\Lambda}_s^{-1}\boldsymbol{U}_s^H\boldsymbol{C}_{1,0}\boldsymbol{g}_1. \quad (12)$$

It can be observed that these receivers are coupled with channel vectors. Their performance will be investigated jointly with the channel estimator next.

4. PERFORMANCE STUDY

Performance loss is incurred to receivers when finite received data samples are processed as (5). Assume N is sufficiently large such that perturbation technique is applicable [8]. Performance of the channel estimator is required to evaluate each receiver's performance and is thus studied first. We will propose more compact results than those in [9] based on the property of cumulant.

4.1. Channel Estimation Performance

All perturbations are due to an estimation error for $\boldsymbol{R}$ or equivalently $\boldsymbol{r}$. If $\widehat{\boldsymbol{r}}$ has an estimation error $\delta\boldsymbol{r}=\widehat{\boldsymbol{r}}-\boldsymbol{r}$ due to finite N, then from (9), $\boldsymbol{G}_j$ is perturbed by $\delta\boldsymbol{G}_j$ as

$$\delta\boldsymbol{G}_j=[\boldsymbol{A}_{j,1}\delta\boldsymbol{r},\cdots,\boldsymbol{A}_{j,q}\delta\boldsymbol{r}]. \quad (13)$$

Then the first-order perturbation in its maximum singular vector becomes [8]

$$\delta\boldsymbol{g}_j\approx\frac{1}{\alpha_j}\boldsymbol{\Pi}_{\boldsymbol{g}_j}^{\perp}\delta\boldsymbol{G}_j\boldsymbol{g}_j,\ \ \boldsymbol{\Pi}_{\boldsymbol{g}_j}^{\perp}=\boldsymbol{\Sigma}_j\boldsymbol{\Sigma}_j^H \quad (14)$$

where $\boldsymbol{\Sigma}_j$ is in size of $q\times(q-1)$ and spans a $(q-1)$-dimensional subspace orthogonal to $\boldsymbol{g}_j$. Substituting (13) into (14), we obtain

$$\delta\boldsymbol{g}_j\approx\boldsymbol{\Gamma}_j\delta\boldsymbol{r},\quad \boldsymbol{\Gamma}_j=\frac{1}{\alpha_j}\boldsymbol{\Pi}_{\boldsymbol{g}_j}^{\perp}\sum_{i=1}^{q}g_j(i)\boldsymbol{A}_{j,i}. \quad (15)$$

The auto-covariance of channel estimate thus becomes

$$\mathrm{Cov}(\delta\boldsymbol{g}_j,\delta\boldsymbol{g}_j)=E\{\delta\boldsymbol{g}_j\delta\boldsymbol{g}_j^H\}\approx\boldsymbol{\Gamma}_j\boldsymbol{\Phi}(\widehat{\boldsymbol{r}})\boldsymbol{\Gamma}_j^H, \quad (16)$$

where $\boldsymbol{\Phi}(\widehat{\boldsymbol{r}})=E\{\delta\boldsymbol{r}\delta\boldsymbol{r}^H\}$ is the covariance of $\widehat{\boldsymbol{r}}$. It depends on data model (2), covariance estimation method (5) and up to the fourth-order statistics of channel inputs and noise. We will present general results in the light of [10] whose results for complex symmetric sources are extended to both a real system and a complex system here. The following properties of "vec", "$\otimes$" and "$\square$" have found to be useful [12]

$$vec(\boldsymbol{ABC})=(\boldsymbol{C}^T\otimes\boldsymbol{A})vec(\boldsymbol{B}), \quad (17)$$

$$(\boldsymbol{A}\otimes\boldsymbol{B})(\boldsymbol{C}\otimes\boldsymbol{D})=(\boldsymbol{AC})\otimes(\boldsymbol{BD}), \quad (18)$$

$$(\boldsymbol{A}\otimes\boldsymbol{B})(\boldsymbol{C}\square\boldsymbol{D})=(\boldsymbol{AC})\square(\boldsymbol{BD}). \quad (19)$$

Although (5) does not require independence of different data vectors, they are assumed independent for convenience of analysis.

Proposition: If channel model follows (2) with ν outputs and L inputs with same distribution and fourth-order cumulant κ_{4w}, and data covariance is estimated from N independent data vectors as (5), then for a real system, $\mathbf{\Phi}(\widehat{\boldsymbol{r}})$ is given by

$$N\mathbf{\Phi}(\widehat{\boldsymbol{r}}) = \kappa_{4w}(\boldsymbol{H}\square\boldsymbol{H})(\boldsymbol{H}\square\boldsymbol{H})^T + \boldsymbol{R}\otimes\boldsymbol{R} + \boldsymbol{A}\odot\boldsymbol{A}^T, \quad (20)$$

$$\boldsymbol{A} = [(\boldsymbol{I}_\nu \otimes \mathbf{1}_\nu)\boldsymbol{R}(\mathbf{1}_\nu^T \otimes \boldsymbol{I}_\nu)]$$

while for a complex system it has a form

$$N\mathbf{\Phi}(\widehat{\boldsymbol{r}}) = \kappa_{4w}(\boldsymbol{H}^*\square\boldsymbol{H})(\boldsymbol{H}^*\square\boldsymbol{H})^H + \boldsymbol{R}^*\otimes\boldsymbol{R}. \quad (21)$$

Proof: It is omitted due to limited space. See [13] for details. □

4.2. Performance of Receivers

Perturbation in channel estimation induced by finite data samples inevitably causes each receiver perturbed to be $\widehat{\boldsymbol{f}} = \boldsymbol{f} + \delta\boldsymbol{f}$, where the first-order perturbation in $\delta\boldsymbol{f}$ can be assumed to have zero mean due to zero-mean of $\delta\boldsymbol{R}$. According to (2), perturbed SINR has the following form

$$\widehat{\text{SINR}} = \frac{\boldsymbol{f}^H\boldsymbol{R}_1\boldsymbol{f} + E\{\delta\boldsymbol{f}^H\boldsymbol{R}_1\delta\boldsymbol{f}\}}{\boldsymbol{f}^H\boldsymbol{R}_{int}\boldsymbol{f} + E\{\delta\boldsymbol{f}^H\boldsymbol{R}_{int}\delta\boldsymbol{f}\}} \quad (22)$$

where $\boldsymbol{R}_1 = \boldsymbol{C}_{1,0}\boldsymbol{g}_1\boldsymbol{g}_1^H\boldsymbol{C}_{1,0}^H$ and $\boldsymbol{R}_{int} = \boldsymbol{R} - \boldsymbol{R}_1$. Then BER can be evaluated by assuming Gaussian interference [14]

$$\widehat{\text{BER}} \approx Q(\sqrt{\widehat{\text{SINR}}}\,) \quad (23)$$

where $Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^\infty e^{\frac{-t^2}{2}}dt$. According to (22), the SINR depends on both unperturbed terms (signal power, interference plus noise power) and corresponding perturbations. Perturbations follow a typical form of $\Psi(\boldsymbol{X}) = E\{\delta\boldsymbol{f}^H\boldsymbol{X}\delta\boldsymbol{f}\}$, where $\boldsymbol{X}$ can be replaced by $\boldsymbol{R}_1$ or $\boldsymbol{R}_{int}$. Since different receivers take different forms with correspondingly different $\delta\boldsymbol{f}$, evaluation of $\Psi(\boldsymbol{X})$ will be discussed for each receiver respectively. For shorter notations, receivers' subscripts are dropped later and simply denoted by $\boldsymbol{f}$. However, no confusion will be caused in the context.

4.2.1. DMI MMSE Receiver

Replacing $\boldsymbol{R}$ in (10) by $\boldsymbol{R} + \delta\boldsymbol{R}$ and considering channel estimation error, we obtain the first-order perturbation

$$\delta\boldsymbol{f} \approx \boldsymbol{R}^{-1}\boldsymbol{C}_{1,0}\delta\boldsymbol{g}_1 - \boldsymbol{R}^{-1}\delta\boldsymbol{R}\boldsymbol{R}^{-1}\boldsymbol{C}_{1,0}\boldsymbol{g}_1. \quad (24)$$

Substituting (15) into (24), $\delta\boldsymbol{f}$ is related to perturbation in covariance estimation by

$$\delta\boldsymbol{f} \approx \boldsymbol{R}^{-1}\boldsymbol{C}_{1,0}\mathbf{\Gamma}_1\delta\boldsymbol{r} - \boldsymbol{R}^{-1}\delta\boldsymbol{R}\boldsymbol{R}^{-1}\boldsymbol{C}_{1,0}\boldsymbol{g}_1. \quad (25)$$

Then perturbation of signal/noise power $\Psi(\boldsymbol{X})$ is given by

$$\begin{aligned}\Psi(\boldsymbol{X}) \approx\ & tr(\underline{E\{\delta\boldsymbol{r}\delta\boldsymbol{r}^H\}}\mathbf{\Gamma}_1^H\boldsymbol{C}_{1,0}^H\boldsymbol{R}^{-1}\boldsymbol{X}\boldsymbol{R}^{-1}\boldsymbol{C}_{1,0}\mathbf{\Gamma}_1)\\ +\ & \boldsymbol{g}_1^H\boldsymbol{C}_{1,0}^H\boldsymbol{R}^{-1}\underline{E\{\delta\boldsymbol{R}\boldsymbol{R}^{-1}\boldsymbol{X}\boldsymbol{R}^{-1}\delta\boldsymbol{R}\}}\boldsymbol{R}^{-1}\boldsymbol{C}_{1,0}\boldsymbol{g}_1\\ -\ & \boldsymbol{g}_1^H\boldsymbol{C}_{1,0}^H\boldsymbol{R}^{-1}\underline{E\{\delta\boldsymbol{R}\boldsymbol{R}^{-1}\boldsymbol{X}\boldsymbol{R}^{-1}\boldsymbol{C}_{1,0}\mathbf{\Gamma}_1\delta\boldsymbol{r}\}}\\ -\ & \underline{E\{\delta\boldsymbol{r}^H\mathbf{\Gamma}_1^H\boldsymbol{C}_{1,0}^H\boldsymbol{R}^{-1}\boldsymbol{X}\boldsymbol{R}^{-1}\delta\boldsymbol{R}\}}\boldsymbol{R}^{-1}\boldsymbol{C}_{1,0}\boldsymbol{g}_1 \quad (26)\end{aligned}$$

Those underlined terms are required for evaluation of $\Psi(\boldsymbol{X})$. The first term can be easily obtained from *Proposition*. The second term has been derived in [7] in a general form $E\{\delta\boldsymbol{R}\boldsymbol{Z}\delta\boldsymbol{R}\}$ where $\boldsymbol{Z}$ is an arbitrary deterministic matrix. The third underlined term $E\{\delta\boldsymbol{R}\boldsymbol{Z}\delta\boldsymbol{r}\}$ can be easily related to a form $E\{\delta\boldsymbol{R}\widetilde{\boldsymbol{Z}}\delta\boldsymbol{R}\}$ after noticing $\delta\boldsymbol{r} = vec(\delta\boldsymbol{R})$. Therefore, all of them can be evaluated from given system parameters.

4.2.2. Subspace MMSE Receiver

According to (12) and expanding $(\mathbf{\Lambda}_s + \delta\mathbf{\Lambda}_s)^{-1}$, we obtain

$$\begin{aligned}\delta\boldsymbol{f} \approx\ & \delta\boldsymbol{U}_s\mathbf{\Lambda}_s^{-1}\boldsymbol{U}_s^H\boldsymbol{C}_{1,0}\boldsymbol{g}_1 - \boldsymbol{U}_s\mathbf{\Lambda}_s^{-1}\delta\mathbf{\Lambda}_s\mathbf{\Lambda}_s^{-1}\boldsymbol{U}_s^H\boldsymbol{C}_{1,0}\boldsymbol{g}_1\\ +\ & \boldsymbol{U}_s\mathbf{\Lambda}_s^{-1}\delta\boldsymbol{U}_s^H\boldsymbol{C}_{1,0}\boldsymbol{g}_1 + \boldsymbol{U}_s\mathbf{\Lambda}_s^{-1}\boldsymbol{U}_s^H\boldsymbol{C}_{1,0}\delta\boldsymbol{g}_1. \quad (27)\end{aligned}$$

Perturbation $\delta\boldsymbol{R}$ or $\delta\boldsymbol{r}$ causes not only estimated channel vector perturbed, but also subspace components of $\boldsymbol{R}$ perturbed [8]

$$\delta\boldsymbol{U}_s \approx \boldsymbol{U}_n\boldsymbol{U}_n^H\delta\boldsymbol{R}\boldsymbol{U}_s\mathbf{\Omega}^{-1},\ \delta\boldsymbol{U}_n \approx -\boldsymbol{U}_s\mathbf{\Omega}^{-1}\boldsymbol{U}_s^H\delta\boldsymbol{R}\boldsymbol{U}_n,$$

$$\delta\mathbf{\Lambda}_s \approx \boldsymbol{U}_s^H\delta\boldsymbol{R}\boldsymbol{U}_s,\ \delta\mathbf{\Lambda}_n \approx \boldsymbol{U}_n^H\delta\boldsymbol{R}\boldsymbol{U}_n, \quad (28)$$

where $\mathbf{\Omega} = \mathbf{\Lambda}_s - \sigma_v^2\boldsymbol{I}$, and approximation is valid up to the first order of $\delta\boldsymbol{R}$. Since $\boldsymbol{U}_n^H\boldsymbol{C}_{1,0}\boldsymbol{g}_1 = \mathbf{0}$, substituting (28) in (27) and invoking (15), we obtain

$$\delta\boldsymbol{f} \approx \boldsymbol{B}_n\delta\boldsymbol{R}\boldsymbol{B}_\gamma\boldsymbol{C}_{1,0}\boldsymbol{g}_1 - \boldsymbol{B}_s\delta\boldsymbol{R}\boldsymbol{B}_s\boldsymbol{C}_{1,0}\boldsymbol{g}_1 + \boldsymbol{B}_s\boldsymbol{C}_{1,0}\mathbf{\Gamma}_1\delta\boldsymbol{r} \quad (29)$$

where for convenience we have defined

$$\boldsymbol{B}_n \triangleq \boldsymbol{U}_n\boldsymbol{U}_n^H,\ \boldsymbol{B}_s \triangleq \boldsymbol{U}_s\mathbf{\Lambda}_s^{-1}\boldsymbol{U}_s^H,\ \boldsymbol{B}_\gamma \triangleq \boldsymbol{U}_s(\mathbf{\Omega}\mathbf{\Lambda}_s)^{-1}\boldsymbol{U}_s^H.$$

Then $\Psi(\boldsymbol{X})$ can be expressed in terms of statistics of the covariance estimation error

$$\begin{aligned}\Psi(\boldsymbol{X}) \approx\ & \boldsymbol{g}_1^H\boldsymbol{C}_{1,0}^H\boldsymbol{B}_\gamma\underline{E\{\delta\boldsymbol{R}\boldsymbol{B}_n\boldsymbol{X}\boldsymbol{B}_n\delta\boldsymbol{R}\}}\boldsymbol{B}_\gamma\boldsymbol{C}_{1,0}\boldsymbol{g}_1\\ -\ & \boldsymbol{g}_1^H\boldsymbol{C}_{1,0}^H\boldsymbol{B}_\gamma\underline{E\{\delta\boldsymbol{R}\boldsymbol{B}_n\boldsymbol{X}\boldsymbol{B}_s\delta\boldsymbol{R}\}}\boldsymbol{B}_s\boldsymbol{C}_{1,0}\boldsymbol{g}_1\\ +\ & \boldsymbol{g}_1^H\boldsymbol{C}_{1,0}^H\boldsymbol{B}_\gamma\underline{E\{\delta\boldsymbol{R}\boldsymbol{B}_n\boldsymbol{X}\boldsymbol{B}_s\boldsymbol{C}_{1,0}\mathbf{\Gamma}_1\delta\boldsymbol{r}\}}\\ -\ & \boldsymbol{g}_1^H\boldsymbol{C}_{1,0}^H\boldsymbol{B}_s\underline{E\{\delta\boldsymbol{R}\boldsymbol{B}_s\boldsymbol{X}\boldsymbol{B}_n\delta\boldsymbol{R}\}}\boldsymbol{B}_\gamma\boldsymbol{C}_{1,0}\boldsymbol{g}_1\\ +\ & \boldsymbol{g}_1^H\boldsymbol{C}_{1,0}^H\boldsymbol{B}_s\underline{E\{\delta\boldsymbol{R}\boldsymbol{B}_s\boldsymbol{X}\boldsymbol{B}_s\delta\boldsymbol{R}\}}\boldsymbol{B}_s\boldsymbol{C}_{1,0}\boldsymbol{g}_1\\ -\ & \boldsymbol{g}_1^H\boldsymbol{C}_{1,0}^H\boldsymbol{B}_s\underline{E\{\delta\boldsymbol{R}\boldsymbol{B}_s\boldsymbol{X}\boldsymbol{B}_s\boldsymbol{C}_{1,0}\mathbf{\Gamma}_1\delta\boldsymbol{r}\}}\\ +\ & \underline{E\{\delta\boldsymbol{r}^H\mathbf{\Gamma}_1^H\boldsymbol{C}_{1,0}^H\boldsymbol{B}_s\boldsymbol{X}\boldsymbol{B}_n\delta\boldsymbol{R}\}}\boldsymbol{B}_\gamma\boldsymbol{C}_{1,0}\boldsymbol{g}_1\\ -\ & \underline{E\{\delta\boldsymbol{r}^H\mathbf{\Gamma}_1^H\boldsymbol{C}_{1,0}^H\boldsymbol{B}_s\boldsymbol{X}\boldsymbol{B}_s\delta\boldsymbol{R}\}}\boldsymbol{B}_s\boldsymbol{C}_{1,0}\boldsymbol{g}_1\\ +\ & tr(\underline{E\{\delta\boldsymbol{r}\delta\boldsymbol{r}^H\}}\mathbf{\Gamma}_1^H\boldsymbol{C}_{1,0}^H\boldsymbol{B}_s\boldsymbol{X}\boldsymbol{B}_s\boldsymbol{C}_{1,0}\mathbf{\Gamma}_1). \quad (30)\end{aligned}$$

Each underlined term in (30) can be evaluated similarly as before.

5. SIMULATION EXAMPLES

Since channel estimation MSEs have been well studied in [9], we only show SINRs and BERs of each receiver averaged over 100 independent realizations. We assume $J = 8$, Gold sequences of $P = 31$, and Gaussian random channels of length 5 chips. Fig. 1 presents effect of N when $SNR = 10dB$. Dashed-dotted lines are based on ideal receivers ($N = \infty$), dashed lines represent analytical results obtained according to (22) or (23) and solid lines stand for experimental results. Analytical results agree well with experimental ones for relatively large N, say 1000. However, the DMI MMSE receiver has slower convergence than the subspace MMSE receiver because it is more sensitive to the estimation error in the data covariance due to matrix inversion. Effect of noise is tested when $N = 500$ with results presented in Fig. 2. Performance changes differently for different receivers. The DMI MMSE receiver (with finite N) does not improve with increased SNR monotonically, due to the same reason as before. Also the error between the analytical and experimental results turns out to be larger. The subspace MMSE receiver performs satisfactorily. It is close to the ideal MMSE receiver. The sensitivity to noise and sample size can be further observed from Fig. 3. Dashed line in each subplot represents performance of the ideal MMSE receiver associated with $N = \infty$ while other six solid lines correspond to $N = 50, 150, 300, 500, 1000, 2000$ sequentially. Only marks $N = 50$ and $N = 150$ are made for clear representation. The SINR peaks for the DMI MMSE receiver are clearly observed for small to moderate N. They shift to high SNR regions as N increases. As $N \to \infty$, peak disappears and the solid line converges to the dashed one. However, the SINR of the subspace MMSE receiver almost shows no peak for all N. It converges to the ideal one much faster than that of the DMI MMSE receiver. Similar conclusions can be made for BER results.

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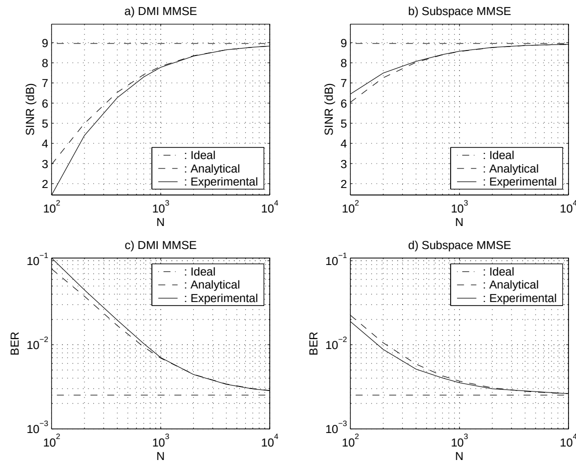

Fig. 1. Effect of sample size.

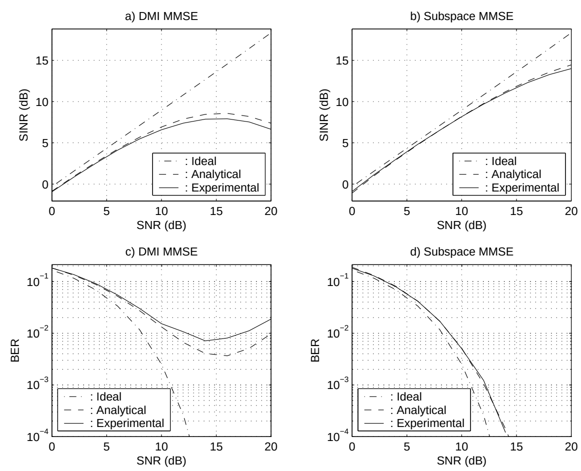

Fig. 2. Effect of noise.

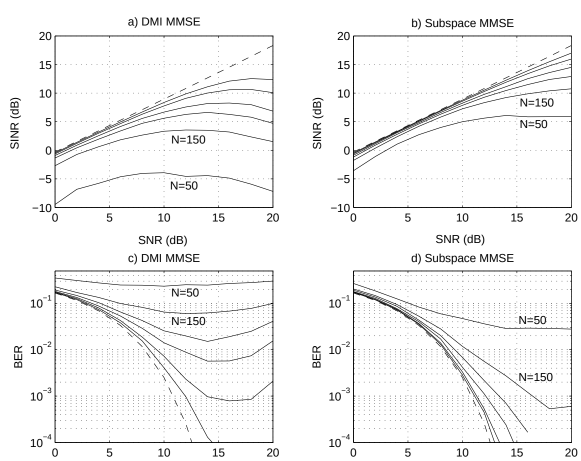

Fig. 3. Effects of sample size and noise.