# LINEAR MULTIUSER DETECTION FOR UPLINK LONG-CODE CDMA SYSTEMS

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**ABSTRACT**

*In the uplink of a long-code CDMA system, base station knows spreading codes of all serviced users. Given propagation delays, code-decorrelation can be performed to separate users' inputs in each symbol interval even in a multipath environment. To detect information symbols, we first apply the subspace technique to estimate multipath parameters for the user of interest based on the correlation matrix of whitened data. Then we construct different linear receivers. The statistics of estimated channel vectors are analyzed and verified by our numerical examples. Performance of receivers is also studied in simulations.*

## 1. INTRODUCTION

Direct sequence (DS) code division multiple access (CDMA) technology has become an appealing solution to support emerging multirate multiuser communications. Despite various advantages, adopted long spreading codes inevitably destroy cyclostationarity of CDMA signals, making many of the existing channel estimation and detection approaches for short code CDMA systems not directly applicable.

Tremendous efforts have been focused on developing solutions for downlink communications [3], [4], [7]. Uplink communications incur new problems due to synchronization and different code assignment strategies. Given pilot symbols of all users, least squares (LS) fitting or iterative maximum likelihood (ML) approaches have been reported [1], [2]. Blind methods have also appeared using correlation matching techniques [9], [12], or employing a space-time 2D RAKE receiver structure to maximize the output signal to interference plus noise ratio (SINR) [5, 6], or LS [8].

In this paper, we investigate channel estimation and detection methods in uplink long-code CDMA systems. Since the base station knows spreading codes of all serviced users, code-decorrelation can be performed to separate users' inputs in each symbol interval even in a multipath environment. Then a small data vector is extracted corresponding to the desired user. Considering that this data vector is corrupted by colored noise after pre-processing, data whitening is performed based on the ensemble average of a code-dependent matrix. Applying the subspace technique, the second order statistics of the whitened data vector are then used to estimate the multipath parameters. Then different linear detection techniques such as zero-forcing (ZF) and minimum mean-square-error (MMSE) are applied.

Different from [8] which processes a block of data, our method processes data symbol by symbol in much lower complexity. From channel estimation perspective, the identifiability condition can be easily satisfied by disregarding processing of some data vectors that give rise to rank-deficient code matrices. On the other hand, correlation of pre-processed data is able to provide an estimate of noise power to be used in the MMSE detection, which is impossible to be obtained in correlation of directly received data.

## 2. CDMA UPLINK WITH LONG CODES

Consider a quasi-synchronous uplink CDMA system [9], where $J$ mobile stations are communicating with a base station. The $i$th user's bit $w_i(n)$ is first spread by aperiodic codes $c_{i,n}(k)$ ($k = 0, \dots, P-1$), and then transmitted through a multipath channel $g_i(m)$. All channels are assumed to have maximum order $q$ ($q << P$). Then the chip-rate signal arriving at the base station is a superposition of signals from $J$ users corrupted by noise

$$y(n) = \sum_{i=1}^{J} \sum_{m=0}^{q} g_i(m) s_i(n - m - d_i) + v(n) \qquad (1)$$

where $s_i(n) = \sum_{k=-\infty}^{\infty} w_i(k) c_{i,k}(n - kP)$, $v(n)$ is zero-mean AWGN with variance $\sigma_v^2 = E\{|v(n)|^2\}$ and $d_i$ ($0 \leq d_i << P$) is the delay of user $i$. With quasi-synchronization, the intersymbol interference could be eliminated if we collect only $L = P - \mu$ samples in the $n$th bit interval into a vector $\boldsymbol{y}(n) = [y(nP + \mu), \dots, y(nP + P - 1)]^T$ with $\mu = max\{q + d_i\}$. Let $\boldsymbol{C}_i(n)$ be the corresponding code filtering matrix of user $i$. Then according to (1), a simple matrix form follows

$$\boldsymbol{y}(n) = \mathcal{C}(n)\boldsymbol{G}\boldsymbol{w}(n) + \boldsymbol{v}(n) = \boldsymbol{H}(n)\boldsymbol{w}(n) + \boldsymbol{v}(n) \qquad (2)$$

where $\mathcal{C}(n) = [\boldsymbol{C}_1(n), \dots, \boldsymbol{C}_J(n)]$, $\boldsymbol{G} = diag\{\boldsymbol{g}_1, \dots, \boldsymbol{g}_J\}$, $\boldsymbol{g}_i$ is the channel vector, $\boldsymbol{w}(n) = [w_1(n), \dots, w_J(n)]^T$ and $\boldsymbol{v}(n) = [v(nP + \mu), \dots, v(nP + P - 1)]^T$.

Through this paper, we make the following assumptions: **AS1)** All users' information sequences are mutually independent and temporally i.i.d. with unit power. **AS2)** Each user's codes and delay are known. **AS3)** The number of active users in the system satisfies $J < (P-q)/(q+1)$. **AS4)** The matrix $\boldsymbol{H}(n)$ has full column rank.

## 3. CHANNEL ESTIMATION AND MULTIUSER DETECTION IN CDMA UPLINK

### 3.1. Subspace Based Channel Estimation

It is clear from (2) that due to time varying codes $\mathcal{C}(n)$, the signature matrix $\boldsymbol{H}(n)$ at different time spans the whole operational space of the correlation matrix $E\{\boldsymbol{y}(n)\boldsymbol{y}(n)^H\}$. Consequently, the subspace method is not directly applicable to $E\{\boldsymbol{y}(n)\boldsymbol{y}(n)^H\}$. In this paper, we propose to decorrelate the data vector $\boldsymbol{y}(n)$ using the pseudo-inverse of code matrix $\mathcal{C}(n)$ at each symbol to obtain an approximately stationary sequence

$$\bar{\boldsymbol{u}}(n) = \mathcal{C}(n)^\dagger \boldsymbol{y}(n) = \boldsymbol{G}\boldsymbol{w}(n) + \mathcal{C}(n)^\dagger \boldsymbol{v}(n). \qquad (3)$$

The covariance of the decorrelated sequence becomes

$$\bar{\boldsymbol{R}} = \boldsymbol{G}\boldsymbol{G}^H + \sigma_v^2 \boldsymbol{A} \qquad (4)$$

where $\boldsymbol{A} \triangleq E\{\mathcal{C}(n)^\dagger(\mathcal{C}(n)^\dagger)^H\} = E\{(\mathcal{C}(n)^H\mathcal{C}(n))^{-1}\}$. If we partition $\bar{\boldsymbol{R}}$ and $\boldsymbol{A}$ diagonally into $J$ matrices and denote the $i$th diagonal matrices as $\bar{\boldsymbol{R}}_i$ and $\boldsymbol{A}_i$ respectively, then by (2) and (4), it follows that

$$\bar{\boldsymbol{R}}_i = \boldsymbol{g}_i\boldsymbol{g}_i^H + \sigma_v^2\boldsymbol{A}_i, \ \text{ where } \ i = 1, \ldots, J. \qquad (5)$$

The correlation matrix $\bar{\boldsymbol{R}}_i$ after decorrelation now contains the desired space spanned by the $i$th user's channel vector. However it is corrupted by colored noise. Therefore, whitening is necessary, which yields

$$\boldsymbol{R}_i \triangleq \boldsymbol{A}_i^{-\frac{1}{2}}\bar{\boldsymbol{R}}_i\boldsymbol{A}_i^{-\frac{1}{2}} = \boldsymbol{A}_i^{-\frac{1}{2}}\boldsymbol{g}_i\boldsymbol{g}_i^H\boldsymbol{A}_i^{-\frac{1}{2}} + \sigma_v^2\boldsymbol{I}. \qquad (6)$$

Since the matrix $\boldsymbol{A}_i$ is a constant matrix, its combination with the $j$th user's channel vector $\boldsymbol{A}_i^{-\frac{1}{2}}\boldsymbol{g}_i$ constitutes the unique signal space of $\boldsymbol{R}_i$. Applying the subspace technique immediately yields the following channel estimation method for user $i$

$$\boldsymbol{\chi}_i = arg \max_{||\boldsymbol{h}||=1} \boldsymbol{h}^H\boldsymbol{R}_i\boldsymbol{h}, \ \ \widehat{\boldsymbol{g}}_i = \frac{\boldsymbol{A}_i^{\frac{1}{2}}\boldsymbol{\chi}_i}{||\boldsymbol{A}_i^{\frac{1}{2}}\boldsymbol{\chi}_i||}. \qquad (7)$$

### 3.2. Multiuser Detection

Once the channel vectors of all users are estimated by (7), symbol level ZF and MMSE receivers can be constructed respectively to detect each user's symbols. The ZF receiver for user $i$ at time instant $n$ is defined as

$$\boldsymbol{f}_{zf,i}(n) = \widehat{\boldsymbol{H}}(n)^\dagger\boldsymbol{e}_i$$

where $\widehat{\boldsymbol{H}}(n) = [\boldsymbol{C}_1(n)\widehat{\boldsymbol{g}}_1, \ldots, \boldsymbol{C}_J(n)\widehat{\boldsymbol{g}}_J]$ and $\boldsymbol{e}_i$ is a unitary vector with the $i$th element as 1. Correspondingly, the estimated symbol is given by $\widehat{w}_{zf,i}(n) = \boldsymbol{f}_{zf,i}^H(n)\boldsymbol{y}(n)$. The ZF receiver is not an optimal one in the presence of noise, since it might enhance noise effect if $\boldsymbol{H}(n)$ is ill conditioned. In that case, an MMSE receiver is desirable,

$$\boldsymbol{f}_{mmse,i}(n) = [\widehat{\boldsymbol{H}}(n)\widehat{\boldsymbol{H}}(n)^H + \widehat{\sigma_v}^2\boldsymbol{I}]^{-1}\boldsymbol{C}_i\widehat{\boldsymbol{g}}_i.$$

The noise power $\widehat{\sigma}_v^2$ can be estimated from the minimum eigenvalue of $\boldsymbol{R}_i$ in (6). Similarly, the detected $i$th user's symbol is $\widehat{w}_{mmse,i}(n) = \boldsymbol{f}_{mmse,i}^H(n)\boldsymbol{y}(n)$.

## 4. CHANNEL ESTIMATION PERFORMANCE

In this section, we will study both channel identifiability and channel estimation mean-square-error (MSE).

### 4.1. Identifiability

*Proposition*: Let only those data vectors $\boldsymbol{y}(n)$ with corresponding full-rank code matrices $\mathcal{C}(n)$ are processed as in (3). If the number of such vectors is sufficiently large, then the subspace based channel estimation method in (7) guarantees identification of each user's channel vector within a scalar ambiguity.

*Proof:* The full column rank condition on the matrix $\mathcal{C}(n)$ implies the existence of $\mathcal{C}(n)^\dagger$ and $\boldsymbol{A}^{-1}$ for each symbol interval of interest. This condition, together with the sufficiently large number of data vectors, in turn guarantee all equations from (3) to (7) hold. As a result, identification of channel vector for each user within a scalar ambiguity is ensured. □

In practice, the rank condition holds for most symbol intervals under (**AS3**). Moreover, under (**AS2**) those unsatisfied data vectors can be easily identified and discarded in estimating $\bar{\boldsymbol{R}}$ for channel estimation. Therefore, the identification of each user's channel is ensured for the proposed approach. On the other hand, under (**AS4**) symbol detection is also guaranteed for the proposed ZF and MMSE receivers. It is worth to note that (**AS4**) is much weaker than the full column rank condition of the matrix $\mathcal{C}(n)$ as required by [8] for symbol detection, since the latter might not be satisfied for some symbol intervals, while the former is almost surely ensured due to the diversity of users' channels in the uplink. Although rank deficiency of $\mathcal{C}(n)$ for some symbol intervals does not affect channel estimation, it inevitably impairs the detection of associated information symbols in [8].

### 4.2. Mean-Square-Error

The MSE depends on finite data size $N$ and will be derived using perturbation techniques. It is observed from (7) that channel estimation depends on $\boldsymbol{R}_i$ and finally $\bar{\boldsymbol{R}}$. It is perturbed when $\bar{\boldsymbol{R}}$ is estimated from $N$ decorrelated data samples as $\tilde{\bar{\boldsymbol{R}}} = \frac{1}{N}\sum_{n=1}^{N}\bar{\boldsymbol{u}}(n)\bar{\boldsymbol{u}}(n)^H$. After extracting its $i$th diagonal block and whitening the block using $\widehat{\boldsymbol{A}}_i$ which is the $i$th diagonal block of $\widehat{\boldsymbol{A}} \triangleq \frac{1}{N}\sum_{n=1}^{N}[\mathcal{C}(n)^H\mathcal{C}(n)]^{-1}$, we have $\widetilde{\boldsymbol{R}}_i = \frac{1}{N}\sum_{n=1}^{N}[\boldsymbol{x}(n)+\widehat{\boldsymbol{A}}_i^{-\frac{1}{2}}\boldsymbol{\zeta}_i(n)][\boldsymbol{x}(n)+\widehat{\boldsymbol{A}}_i^{-\frac{1}{2}}\boldsymbol{\zeta}_i(n)]^H$, where $\boldsymbol{x}(n) = \widehat{\boldsymbol{A}}_i^{-\frac{1}{2}}\boldsymbol{g}_i w_i(n)$ and $\boldsymbol{\zeta}_i(n)$ is the $i$th subvector of $\mathcal{C}(n)^\dagger\boldsymbol{v}(n)$ with $q+1$ elements. Direct perturbation analysis based on $\widetilde{\boldsymbol{R}}_i$ is formidable due to the time varying product term of $\mathcal{C}(n)^\dagger\boldsymbol{v}(n)$. Noticing $\mathcal{C}(n)^\dagger$ is independent of white noise $\boldsymbol{v}(n)$, the time-averaging covariance of $\widehat{\boldsymbol{A}}_i^{-\frac{1}{2}}\boldsymbol{\zeta}_i(n)$ is $\sigma_v^2\boldsymbol{I}$ when $N$ is sufficiently large. Therefore, we approximate $\widehat{\boldsymbol{A}}_i^{-\frac{1}{2}}\boldsymbol{\zeta}_i(n)$ by a white noise $\boldsymbol{\nu}(n)$, and $\widetilde{\boldsymbol{R}}_i$ reduces to

$$\widetilde{\boldsymbol{R}}_i = \frac{1}{N}\sum_{i=1}^{N}[\boldsymbol{x}(n)+\boldsymbol{\nu}(n)][\boldsymbol{x}(n)+\boldsymbol{\nu}(n)]^H \qquad (8)$$

where $E\{||\boldsymbol{\nu}(n)||^2\} = \sigma_v^2\boldsymbol{I}$. Based on (8) the perturbation analysis can be readily conducted by directly applying some results in [10]. In the sequel, let's denote the perturbation by preceding the corresponding quantity by $\delta$, and the perturbed quantity with $\widetilde{\ }$, i.e., $\delta\boldsymbol{R}_i = \widetilde{\boldsymbol{R}}_i - \boldsymbol{R}_i$. By (7), $\boldsymbol{\chi}_i$ is the signal space of $\boldsymbol{R}_i$, thus its perturbation, when estimated from (8), is given by [10]

$$\delta\boldsymbol{\chi}_i = (1/\gamma_i^2)\boldsymbol{U}_n\boldsymbol{U}_n^H\delta\boldsymbol{R}_i\boldsymbol{\chi}_i \qquad (9)$$

where $\gamma_i^2 = \boldsymbol{g}_i^H\widehat{\boldsymbol{A}}_i^{-1}\boldsymbol{g}_i$ and $\boldsymbol{U}_n$ is the null space of $\boldsymbol{R}_i$. The perturbation of $\boldsymbol{\chi}_i$ will cause that of $\boldsymbol{g}_i$. By (7), the perturbed channel estimation is $\widetilde{\boldsymbol{g}}_i = (\widetilde{\boldsymbol{\chi}}_i^H\widehat{\boldsymbol{A}}_i\widetilde{\boldsymbol{\chi}}_i)^{-\frac{1}{2}}\widehat{\boldsymbol{A}}_i^{\frac{1}{2}}\widetilde{\boldsymbol{\chi}}_i$. Substituting $\widetilde{\boldsymbol{\chi}}_i$, expanding the power term using Taylor series and keeping only the first order terms, we have

$$\begin{aligned}\delta\boldsymbol{g}_i \;\approx\;& (\boldsymbol{\chi}_i^H\widehat{\boldsymbol{A}}_i\boldsymbol{\chi}_i)^{-\frac{1}{2}}\widehat{\boldsymbol{A}}_i^{\frac{1}{2}}\delta\boldsymbol{\chi}_i \\ -\;& \frac{1}{2}(\boldsymbol{\chi}_i^H\widehat{\boldsymbol{A}}_i\boldsymbol{\chi}_i)^{-\frac{3}{2}}(\delta\boldsymbol{\chi}_i^H\widehat{\boldsymbol{A}}_i\boldsymbol{\chi}_i + \boldsymbol{\chi}_i^H\widehat{\boldsymbol{A}}_i\delta\boldsymbol{\chi}_i)\widehat{\boldsymbol{A}}_i^{\frac{1}{2}}\boldsymbol{\chi}_i.\end{aligned} \qquad (10)$$

Since in-space error is approximately the square of orthogonal space error by [10], and noticing $\boldsymbol{g}_i = (\boldsymbol{\chi}_i^H\widehat{\boldsymbol{A}}_i\boldsymbol{\chi}_i)^{\frac{1}{2}}\widehat{\boldsymbol{A}}_i^{\frac{1}{2}}\boldsymbol{\chi}_i$, we further simply (10) to the following by keeping only the orthogonal space error

$$\delta\boldsymbol{g}_i \approx \boldsymbol{\Pi}^\perp(\boldsymbol{\chi}_i^H\widehat{\boldsymbol{A}}_i\boldsymbol{\chi}_i)^{-\frac{1}{2}}\widehat{\boldsymbol{A}}_i^{\frac{1}{2}}\delta\boldsymbol{\chi}_i \qquad (11)$$

where $\boldsymbol{\Pi}^\perp \triangleq (\boldsymbol{I} - \frac{\boldsymbol{g}_i\boldsymbol{g}_i^H}{\boldsymbol{g}_i^H\boldsymbol{g}_i})$. Since both $\widehat{\boldsymbol{A}}_i^{-\frac{1}{2}}\boldsymbol{g}_i$ and $\boldsymbol{\chi}_i$ are the unique eigenvectors of $\boldsymbol{R}_i$, we have $(\boldsymbol{\chi}_i^H\widehat{\boldsymbol{A}}_i\boldsymbol{\chi}_i) = \frac{1}{\gamma_i^2}$. Then applying the above results and (9), the covariance of $\delta\boldsymbol{g}_i$ becomes

$$\begin{aligned}E\{\delta\boldsymbol{g}_i\delta\boldsymbol{g}_i^H\} \;\approx\;& \frac{1}{\gamma_i^2}\boldsymbol{\Pi}^\perp\widehat{\boldsymbol{A}}_i^{\frac{1}{2}}\boldsymbol{U}_n\boldsymbol{U}_n^H E\{\delta\boldsymbol{R}_i\boldsymbol{\chi}_i\boldsymbol{\chi}_i^H\delta\boldsymbol{R}_i\} \\ & \boldsymbol{U}_n\boldsymbol{U}_n^H\widehat{\boldsymbol{A}}_i^{\frac{1}{2}}\boldsymbol{\Pi}^\perp.\end{aligned} \qquad (12)$$

It is shown [11] that for a given data model, statistical properties of the inputs and additive noise, $E\{\delta\boldsymbol{R}_i\boldsymbol{\chi}_i\boldsymbol{\chi}_i^H\delta\boldsymbol{R}_i\}$ can always be evaluated. Applying results in [11] and noticing $\boldsymbol{\chi}_i^H\boldsymbol{R}_i\boldsymbol{\chi}_i = \gamma_i^2 + \sigma_v^2$, $\boldsymbol{\chi}_i^H\boldsymbol{U}_n = 0$, (12) reduces to

$$E\{\delta\boldsymbol{g}_i\delta\boldsymbol{g}_i^H\} \approx \frac{\sigma_v^2(\gamma_i^2+\sigma_v^2)}{N\gamma_i^2}\boldsymbol{\Pi}^\perp\widehat{\boldsymbol{A}}_i^{\frac{1}{2}}\boldsymbol{U}_n\boldsymbol{U}_n^H\widehat{\boldsymbol{A}}_i^{\frac{1}{2}}\boldsymbol{\Pi}^\perp. \qquad (13)$$

The mean squared channel estimation error is then given by the trace of (13).

## 5. MULTIUSER DETECTION PERFORMANCE

The proposed ZF and MMSE receivers are built upon estimated channel vectors. Their performance depends on the channel estimation performance. The output signal to interference plus noise ratio (SINR) and bit error rate (BER) can be investigated. Due to lack of space, the corresponding results will be presented in the future. Instead, we turn our attention to numerical studies next.

## 6. SIMULATION EXAMPLES

We consider an uplink CDMA system, where each user transmits BPSK signals with equal power through a respective multipath channel of length 3. In the first experiment, we set $P = 16$, $J = 4$ and randomly generate 15 channels. For each channel, we obtain the normalized mean-square-errors (NMSEs) of channel estimation over 100 independent runs. Then all NMSEs are averaged. The experimental results over different $N$ are compared with their analytical values in Fig. 1. As expected, the experimental and analytical curves are highly consistent for all examined $N$s. Then we compare our method with the least square method [8] and PC-MMSE method [5]. We adopt $P = 32$, $J = 5$ as [8]. The channel is fixed for totally 100 runs. 100 data symbols are used for channel estimation and symbol detection for all three approaches. The NMSEs over various input signal to noise ratios (SNRs) are plotted in Fig. 2. The proposed method shows a very similar MSE level to the LS method, and is much better than the PC-MMSE method at moderate to high SNRs. The BERs of different receivers are plotted in Fig. 3. Both proposed receivers exhibit lowest BER levels when SNR is over $8dB$, while the PC-MMSE receiver has the worst performance due to lack of spatial diversity.

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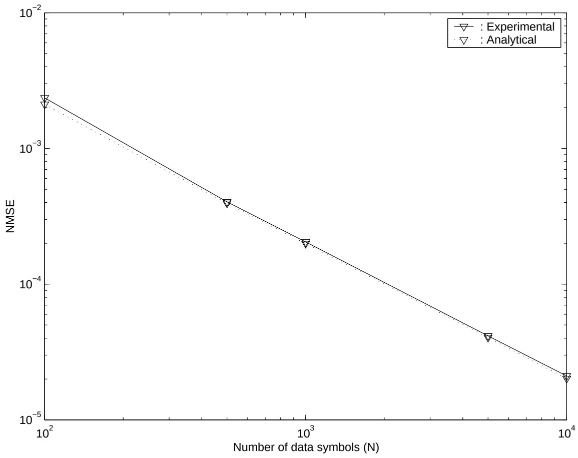


**Fig. 1**. NMSE v.s. N, $P$=16, $J$=4, $SNR$=20$dB$.

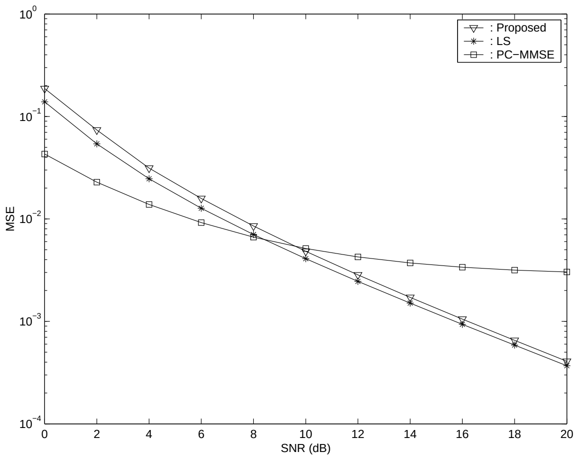


**Fig. 2**. NMSE v.s. SNR, $P$=32, $J$=5, $N$=100.

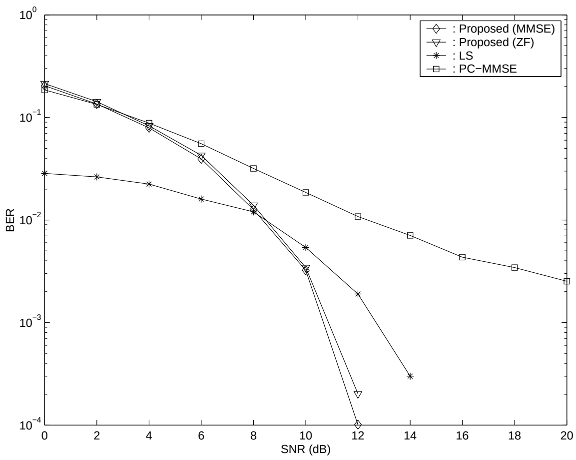


**Fig. 3**. BER v.s. SNR, $P$=32, $J$=5, $N$=100.