CORRELATION MATCHING IN CHANNEL ESTIMATION FOR MULTIRATE DS/CDMA

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ABSTRACT

Channel estimation is studied for a multicode (MC) or multiple processing gain (MPG) multirate CDMA system based on correlation matching. All users are partitioned into groups in terms of their raw data rates. Since a communication channel may span several symbol intervals especially for a high rate user, severe intersymbol interference (ISI) exists in the received data. However, signature waveforms of various symbols from a user are convolutions of spreading codes with a common channel. This structure is beneficial to channel estimation due to multiple contributions to the output correlation. By properly selecting a weighting matrix in the matching cost function, significant improvement is observed. The optimal weighting matrix and covariance of the channel estimate are further derived in closed forms.

1. INTRODUCTION

The increasing demand for integrated wideband services has accelerated research activities in developing new wireless communication technologies. Due to its inherent merits, direct sequence (DS) CDMA technique becomes one of the best candidates to satisfy the multirate service requirement.

There exists two multirate access schemes potentially suitable for a multirate DS/CDMA system [1, 3, 5]: multicode (MC) access where each high rate user is assigned multiple codes to spread different bits and multiple processing gain (MPG) access where each user is assigned one periodic code sequence with period determined by its data rate. As explained in [5], the variable chip rate (VCR) access introduces extra difficulty to chip synchronization and frequency planning. We only focus on MC and MPG schemes.

These two access schemes use different strategy in code assignment. For a MC-based system, data stream from a high-rate (HR) user is converted into a series of parallel low-rate (LR) streams spread by different code sequences. Therefore, a MC multirate system is well translated into a single-rate system with additional virtual users entering the system. Many existing single-rate receivers are directly applicable to detect the user, such as conventional RAKE receiver, decorrelating receiver or maximum-likelihood (ML) receiver. However, channel capacity may decrease since each HR user occupies several code channels. It is thus preferable to adopt the MPG access scheme if the spectrum efficiency is of particular concern. Each user's data rate can be easily adjusted by changing the length of the code sequence [3, 5]. The major obstacle arising from such a MPG-based system is that interference characteristic changes from symbol to symbol for a HR user and thus creates challenges in multiuser detection.

In this paper, we study channel estimation by correlation matching. Since the channel of each user could span several symbol intervals, severe intersymbol interference (ISI) exists in the received data. However, exploitation of signatures of all interfering symbols is beneficial to channel estimation. Because those signatures share information about a common channel, they enhance contributions to the data correlation. It is well known that performance of a correlation matching estimator highly depends on the choice of the weighting matrix. An optimal matrix exists in principle, but is not a priori-known since it is parameterized by parameters to be estimated. Fortunately, the asymptotically optimal performance is still achievable if this matrix is directly estimated from samples. We will show by simulations that the performance of such a estimator is comparable to the optimal estimator even for a finite number of received samples.

2. MULTIRATE DS/CDMA MODEL

Consider a multirate synchronous DS/CDMA system [3]. We partition total K users into M groups based on their data rates. In group m, assume there are K_m users with data rate R_m. Then $K = \sum_{m=1}^{M} K_m$. For notational convenience, we denote the kth user in the mth group by (k, m). Multirate access can be achieved by either MC access or MPG access which will be discussed in detail next.

2.1. Multirate system with MPG access

For MPG access, code sequences have different lengths for users in different groups. Assume the lowest data rate is R_1 with correspondingly largest spreading length P_1. Then $R_m = mR_1$ and $P_1 = mP_m$. User (k, m) is assigned spreading codes $[c_{k,m}(0), \cdots, c_{k,m}(P_m - 1)]^T \triangleq \boldsymbol{c}_{k,m}$ of length P_m. Corresponding to one symbol interval of the rate-R_1 users, it has m symbols transmitted. For convenience, we partition its symbol stream $b_{k,m}(n)$ into blocks with each block m symbols. Denote the jth symbol in the

nth block by $b_{k,m,n}(j) \triangleq b_{k,m}(mn+j)$. After spreading, the spread signal passes through a multipath channel with finite impulse response. Assume the channel has order $q_{k,m}$ and impulse response is described by chip rate coefficients $g_{k,m}(n)$. Then the received signal due to user (k, m) is [8]

$$y_{k,m}(u) = \sum_{l=-\infty}^{\infty} \sum_{i=0}^{q_{k,m}} b_{k,m}(l) c_{k,m}(u - i - lP_m) g_{k,m}(i) \tag{1}$$

Total $\nu = QP_1$ chip samples of $y_{k,m}(u)$ can be collected in a vector $\boldsymbol{y}_{k,m}(n)$ from $u = nP_1$ to $u = nP_1 + \nu - 1$. Due to the channel spread, the number of symbols involved in this vector depends on $q_{k,m}$. We thus factorize it by P_m as follows

$$q_{k,m} = \alpha_{k,m} P_m + \beta_{k,m} \tag{2}$$

with $\beta_{k,m} < P_m$. Here channel order can be arbitrarily large for a general discussion, making it distinguished from conventional CDMA systems. If we define a code filtering matrix $\boldsymbol{C}_{k,m}$ and a Jordan matrix $\boldsymbol{J}$ with all $1's$ in the first diagonal below the main diagonal, then we obtain a data vector due to user (k, m) [8]

$$\boldsymbol{y}_{k,m}(n) = \sum_{j=-j_{k,m,0}}^{mQ-1} \boldsymbol{A}_{k,m,j} \boldsymbol{g}_{k,m} b_{k,m}(mn + j) \tag{3}$$

where

$$\boldsymbol{A}_{k,m,j} = \boldsymbol{J}^{jP_m} \boldsymbol{C}_{k,m},$$

$\boldsymbol{J}^{-1}$ is defined as $\boldsymbol{J}^T$ and $\boldsymbol{J}^0$ as an identity matrix. In (3), the total number of symbols depends on the channel order given in (2)

$$L_{k,m} = mQ + j_{k,m,0}, \;\; j_{k,m,0} = \alpha_{k,m} + 1 - \delta(\beta_{k,m}) \tag{4}$$

where $\delta(\cdot)$ is the Dirac delta function taking values either zero or one. Pre-multiplying the signature waveform $\boldsymbol{C}_{k,m}\boldsymbol{g}_{k,m}$ by a matrix $\boldsymbol{J}^{jP_m}$ will shift its elements up ($j < 0$) or down ($j > 0$) by multiples of P_m positions. After considering all K active users in the system and the additive white Gaussian noise $\boldsymbol{v}(n)$, the received data has the form

$$\boldsymbol{y}(n) = \sum_{m,k,j} \boldsymbol{A}_{k,m,j} \boldsymbol{g}_{k,m} b_{k,m}(mn + j) + \boldsymbol{v}(n) \tag{5}$$

where j, k, m take all integers in $[-j_{k,m,0}, mQ-1]$, $[1, K_m]$ and $[1, M]$ respectively. It can be easily observed that significant ISI comes from not only intra-block symbols, but also inter-block ones. The situation becomes worse if the channel has a long delay spread. However, signature waveforms of interfering symbols share common channel information $\boldsymbol{g}_{k,m}$. This structure will be helpful to performance improvement of the correlation based channel estimator. Before detailed discussion on the method, let us briefly describe the MC access scheme.

2.2. Multirate system with MC access

In the MC access scheme, corresponding to one symbol interval of a LR user, m symbols from user (k, m) are multiplexed by m spreading sequences which are periodic with period P_1. To differentiate this case with the previous one, we add a $\widetilde{\cdot}$ and an additional subscript to the corresponding quantities. Then the code vector for symbol $\widetilde{b}_{k,m}(mn + i)$ is $\widetilde{\boldsymbol{c}}_{k,m,i}$ where $i = 0, \cdots, m-1$ and n is still the block index. In particular, $\widetilde{\boldsymbol{c}}_{k,m,i} = [\widetilde{c}_{k,m,i}(0), \cdots, \widetilde{c}_{k,m,i}(P_1 - 1)]^T$. Now we factorize the channel order by P_1: $q_{k,m} = \widetilde{\alpha}_{k,m} P_1 + \widetilde{\beta}_{k,m}$ with $\widetilde{\beta}_{k,m} < P_1$. The received data vector of length $\nu = QP_1$ has a form [8]

$$\widetilde{\boldsymbol{y}}_{k,m}(n) = \sum_{i=0}^{m-1} \sum_{j=-\widetilde{j}_{k,m,0}}^{Q-1} \widetilde{\boldsymbol{A}}_{k,m,j,i} \boldsymbol{g}_{k,m} \widetilde{b}_{k,m}(mn + jn + i) \tag{6}$$

where

$$\widetilde{\boldsymbol{A}}_{k,m,j,i} = \boldsymbol{J}^{jP_1} \widetilde{\boldsymbol{C}}_{k,m,i}.$$

and $\widetilde{\boldsymbol{C}}_{k,m,i}$ is similarly constructed as $\boldsymbol{C}_{k,m}$ by replacing $c_{k,m}$ with $\widetilde{c}_{k,m,i}$. The total number of symbols from user (k, m) can be calculated as

$$\widetilde{L}_{k,m} = m(Q + \widetilde{j}_{k,m,0}), \;\; \widetilde{j}_{k,m,0} = \widetilde{\alpha}_{k,m} + 1 - \delta(\widetilde{\beta}_{k,m}) \tag{7}$$

Finally, the received data vector becomes

$$\widetilde{\boldsymbol{y}}(n) = \sum_{i,j,k,m} \widetilde{\boldsymbol{A}}_{k,m,j,i} \boldsymbol{g}_{k,m} \widetilde{b}_{k,m}(mn + jn + i) + \widetilde{\boldsymbol{v}}(n) \tag{8}$$

where i, j, k, m take all integers in $[1, m]$, $[-\widetilde{j}_{k,m,0}, Q-1]$, $[1, K_m]$ and $[1, M]$ respectively. Structured signature waveforms of various symbols from user (k, m) will be utilized in our channel estimation.

3. BLIND CHANNEL ESTIMATION

Our channel estimator will be derived based on the data correlation matrix. Corresponding to different multirate access schemes, these methods will be described in the next two subsections respectively.

3.1. Channel estimation for MPG CDMA

According to the data model (5), the correlation matrix is easily found to be

$$\boldsymbol{R} = \sum_{m,k,j} \sigma_b^2 \boldsymbol{A}_{k,m,j} \boldsymbol{g}_{k,m} \boldsymbol{g}_{k,m}^H \boldsymbol{A}_{k,m,j}^H + \sigma_v^2 \boldsymbol{I} \tag{9}$$

where σ_b^2 is the input power, σ_v^2 is the noise power. It is clear that $\boldsymbol{R}$ is linearly parameterized by $\boldsymbol{g}_{k,m}\boldsymbol{g}_{k,m}^H$ and σ_v^2. Assume $\widehat{\boldsymbol{R}}$ is an estimate of $\boldsymbol{R}$. By matching $\widehat{\boldsymbol{R}}$ with $\boldsymbol{R}$, one could minimize the matching error and obtain those unknowns. To achieve this goal, we take a vec operation on $\boldsymbol{R}$ to obtain a long vector. After defining

$$\boldsymbol{r} = vec(\boldsymbol{R}), \quad \boldsymbol{G}_{k,m} = \sigma_b^2 \boldsymbol{g}_{k,m} \boldsymbol{g}_{k,m}^H,$$

$$\boldsymbol{x}_{k,m} = vec(\boldsymbol{G}_{k,m}), \quad \boldsymbol{S}_{k,m} = \sum_j \boldsymbol{A}^*_{k,m,j} \otimes \boldsymbol{A}_{k,m,j},$$

we can find $\boldsymbol{r} = \boldsymbol{\Pi x}$ where $\otimes$ denotes the Kronecker product [2], $\boldsymbol{x}$ is a long vector including all $\boldsymbol{x}_{k,m}$ and σ_v^2 as its entries, and correspondingly $\boldsymbol{\Pi}$ has entries $\boldsymbol{S}_{k,m}$ and $vec(\boldsymbol{I})$ stacked row-wise. Since $\boldsymbol{R}$ is a Hermitian matrix, we introduce a selection matrix $\boldsymbol{M}$ to remove some redundant elements in $\boldsymbol{r}$ by picking up only elements of the lower triangular part of $\boldsymbol{R}$ (including the diagonal elements) [7]

$$\boldsymbol{M} = diag\{\boldsymbol{I}_\nu, diag\{\boldsymbol{M}_1, \cdots, \boldsymbol{M}_{\nu-1}\}\},$$

$$\boldsymbol{M}_i = [\boldsymbol{0}_{(\nu-i)\times i}, \boldsymbol{I}_{\nu-i}], \;\; i = 1, \cdots, \nu-1.$$

We thus can build the following matching cost function with a positive weighting matrix $\boldsymbol{W}$

$$\mathcal{J} = (\widehat{\boldsymbol{r}} - \boldsymbol{\Pi x})^H \boldsymbol{M}^T \boldsymbol{W} \boldsymbol{M} (\widehat{\boldsymbol{r}} - \boldsymbol{\Pi x}) \qquad (10)$$

It is observed that $\mathcal{J}$ is a quadratic function of $\boldsymbol{x}$. Minimizing $\mathcal{J}$ leads to a unique solution

$$\widehat{\boldsymbol{x}} = \boldsymbol{U}_1 \widehat{\boldsymbol{r}}, \;\; \boldsymbol{U}_1 = (\boldsymbol{\Pi}^H \boldsymbol{M}^T \boldsymbol{W} \boldsymbol{M} \boldsymbol{\Pi})^{-1} \boldsymbol{\Pi}^H \boldsymbol{M}^T \boldsymbol{W} \boldsymbol{M}. \qquad (11)$$

Since $\boldsymbol{G}_{k,m}$ can be reconstructed from $\boldsymbol{x}$ by extracting corresponding $q_{k,m}+1$ consecutive sub-vectors each of which is of length $q_{k,m}+1$, and stacking into a matrix

$$\boldsymbol{G}_{k,m} = \boldsymbol{T}_{k,m}(\boldsymbol{I} \otimes \boldsymbol{x}), \qquad (12)$$

an estimate of $\boldsymbol{G}_{k,m}$ can be similarly obtained from $\widehat{\boldsymbol{x}}$. After eigenvalue decomposition (EVD) on $\boldsymbol{G}_{k,m}$, the estimate of $\boldsymbol{g}_{k,m}$ is the eigenvector associated with its maximum eigenvalue $\lambda_{k,m,1}$ with a scalar ambiguity.

3.2. Channel estimation for MC CDMA

Following the same line and starting from (8), one can derive a vectorized correlation $\widetilde{\boldsymbol{r}} = \widetilde{\boldsymbol{\Pi}} \boldsymbol{x}$ where $\widetilde{\boldsymbol{\Pi}}$ has entries as $\widetilde{\boldsymbol{S}}_{k,m}$ and $vec(\boldsymbol{I})$, and

$$\widetilde{\boldsymbol{S}}_{k,m} = \sum_{j,i} \widetilde{\boldsymbol{A}}^*_{k,m,j,i} \otimes \widetilde{\boldsymbol{A}}_{k,m,j,i}.$$

The cost function to estimate the channel has a similar form as (10). Due to this similarity, it suffices to discuss the MPG access scheme in detail while obtaining results for the MC access scheme by slight modification. Next we will analyze the performance of the proposed estimator.

4. PERFORMANCE ANALYSIS

In this section, we will derive the covariance of the channel estimate. In our estimation, $\boldsymbol{R}$ is estimated from N data sample vectors $\widehat{\boldsymbol{R}} = \frac{1}{N}\sum_{n=1}^N \boldsymbol{y}(n)\boldsymbol{y}^H(n)$. This estimate is deviated from $\boldsymbol{R}$ due to insufficient (finite) data samples. Denote the perturbed quantity by preceding it with δ: $\delta\boldsymbol{R} = \widehat{\boldsymbol{R}} - \boldsymbol{R}$ and correspondingly $\delta\boldsymbol{r} = \widehat{\boldsymbol{r}} - \boldsymbol{r}$. According to (11), $\delta\boldsymbol{r}$ results in an error in $\boldsymbol{x}$: $\delta\boldsymbol{x} = \boldsymbol{U}_1\delta\boldsymbol{r}$. Consequently, $\delta\boldsymbol{G}_{k,m} = \boldsymbol{T}_{k,m}(\boldsymbol{I} \otimes \delta\boldsymbol{x})$. Denote the bias of channel estimate by $\delta\boldsymbol{g}_{k,m}$. Since $\boldsymbol{g}_{k,m}$ is an eigenvector of $\boldsymbol{G}_{k,m}$ corresponding to its maximum eigenvalue $\lambda_{k,m,1}$, then $\delta\boldsymbol{g}_{k,m}$ has the following form [6]

$$\delta\boldsymbol{g}_{k,m} \approx -(\boldsymbol{G}_{k,m} - \lambda_{k,m,1}\boldsymbol{I})^\dagger \delta\boldsymbol{G}_{k,m}\boldsymbol{g}_{k,m} \qquad (13)$$

where $\dagger$ represents pseudo-inverse. After substituting $\delta\boldsymbol{G}_{k,m}$ and using the property of $\otimes$ for any vectors $\boldsymbol{a}$ and $\boldsymbol{b}$: $(\boldsymbol{I} \otimes \boldsymbol{a})\boldsymbol{b} = \boldsymbol{b} \otimes \boldsymbol{a}$, we obtain $\delta\boldsymbol{g}_{k,m}$ from $\delta\boldsymbol{x}$

$$\delta\boldsymbol{g}_{k,m} \approx \boldsymbol{U}_2[\boldsymbol{g}_{k,m} \otimes (\boldsymbol{U}_1\delta\boldsymbol{r})] \qquad (14)$$

where

$$\boldsymbol{U}_2 = -(\boldsymbol{G}_{k,m} - \lambda_{k,m,1}\boldsymbol{I})^\dagger \boldsymbol{T}_{k,m}.$$

It shows that $\delta\boldsymbol{g}_{k,m}$ is affected by $\delta\boldsymbol{r}$ - an indicator of accuracy of correlation estimation. According to (14), the covariance of $\delta\boldsymbol{g}_{k,m}$ can be derived from the covariance of $\delta\boldsymbol{r}$

$$\boldsymbol{\Phi}_g \approx \boldsymbol{U}_2[\boldsymbol{G}_{k,m} \otimes (\boldsymbol{U}_1\boldsymbol{\Phi}_r\boldsymbol{U}_1^H)]\boldsymbol{U}_2^H \qquad (15)$$

where

$$\boldsymbol{\Phi}_g = E\{\delta\boldsymbol{g}_{k,m}\delta\boldsymbol{g}_{k,m}^H\}, \quad \boldsymbol{\Phi}_r = E\{\delta\boldsymbol{r}\delta\boldsymbol{r}^H\}.$$

It is worth to emphasize that $\boldsymbol{\Phi}_r$ is also essential for the optimality of the estimator. If $\boldsymbol{W} = (\boldsymbol{M}\boldsymbol{\Phi}_r\boldsymbol{M}^T)^{-1}$, then $\widehat{\boldsymbol{g}}_{k,m}$ is an asymptotically optimal estimator [4]. Therefore, we will derive a closed form for $\boldsymbol{\Phi}_r$ next.

It can be observed that $\boldsymbol{\Phi}_r$ depends on the fourth order statistics of the channel output. To simplify our notations, we rewrite (5) differently

$$\boldsymbol{y}_n = \boldsymbol{H}\boldsymbol{b}_n + \boldsymbol{v}_n \qquad (16)$$

where all inputs are collected in a vector $\boldsymbol{b}_n$ with total J elements, $\boldsymbol{H}$ includes signature vectors of those inputs. The following can be easily verified first

$$\boldsymbol{\Phi}_r = \frac{1}{N}(E\{\boldsymbol{r}_n\boldsymbol{r}_n^H\} - \boldsymbol{r}\boldsymbol{r}^H), \;\; \boldsymbol{r}_n = vec(\boldsymbol{y}_n\boldsymbol{y}_n^H). \quad (17)$$

Assume inputs are i.i.d. with $\sigma_b^2 \triangleq E\{|b_{m,k}(n)|^2\}$, $\eta_b \triangleq E\{b_{m,k}^2(n)\}$, $m_{4b} \triangleq E\{|b_{m,k}(n)|^4\}$. Their third order moments are assumed to be zero. Similarly we define $\sigma_v^2 \triangleq E\{|v(n)|^2\}$, $\eta_v \triangleq E\{v^2(n)\}$, $m_{4v} \triangleq E\{|v(n)|^4\}$. Then after some manipulations, it can be shown that [7]

$$\begin{aligned}
E\{\boldsymbol{r}_n\boldsymbol{r}_n^H\} &= (\boldsymbol{H}^* \otimes \boldsymbol{H})\boldsymbol{B}_b(\boldsymbol{H}^T \otimes \boldsymbol{H}^H) + \boldsymbol{B}_v \\
&+ \sigma_b^2\sigma_v^2[(\boldsymbol{H}\boldsymbol{H}^H)^* \otimes \boldsymbol{I}_\nu + \boldsymbol{I}_\nu \otimes (\boldsymbol{H}\boldsymbol{H}^H)] \\
&+ \sigma_b^2\sigma_v^2 vec(\boldsymbol{H}\boldsymbol{H}^H)vec^T(\boldsymbol{I}_\nu) \\
&+ \sigma_b^2\sigma_v^2 vec(\boldsymbol{I}_\nu)vec^H(\boldsymbol{H}\boldsymbol{H}^H) \\
&+ \eta_b^*\eta_v(\boldsymbol{H}^* \otimes \boldsymbol{I}_\nu)\boldsymbol{B}_1(\boldsymbol{I}_\nu \otimes \boldsymbol{H}^H) \\
&+ \eta_b\eta_v^*(\boldsymbol{I}_\nu \otimes \boldsymbol{H})\boldsymbol{B}_1^T(\boldsymbol{H}^T \otimes \boldsymbol{I}_\nu) \qquad (18)
\end{aligned}$$

where $\boldsymbol{B}_b$ and $\boldsymbol{B}_v$ depend on the high order statistics

$$\begin{aligned}
\boldsymbol{B}_b &= E\{(\boldsymbol{b}_n\boldsymbol{b}_n^H)^* \otimes (\boldsymbol{b}_n\boldsymbol{b}_n^H)\} \\
&= (m_{4b} - 2\sigma_b^4 - |\eta_b|^2)\boldsymbol{D}_J + \sigma_b^4\boldsymbol{I}_{J^2} \\
&+ \sigma_b^4 vec(\boldsymbol{I}_J)vec^T(\boldsymbol{I}_J) + |\eta_b|^2\boldsymbol{E}_J \qquad (19)
\end{aligned}$$

$$
\begin{aligned}
\boldsymbol{B}_v &= E\{(\boldsymbol{v}_n\boldsymbol{v}_n^H)^* \otimes (\boldsymbol{v}_n\boldsymbol{v}_n^H)\} \\
&= (m_{4v} - 2\sigma_v^4 - |\eta_v|^2)\boldsymbol{D}_\nu + \sigma_v^4 \boldsymbol{I}_{\nu^2} \\
&+ \sigma_v^4 vec(\boldsymbol{I}_\nu) vec^T(\boldsymbol{I}_\nu) + |\eta_v|^2 \boldsymbol{E}_\nu \qquad (20)
\end{aligned}
$$

$$
\boldsymbol{D}_i = diag\{\boldsymbol{e}_{1,i}\boldsymbol{e}_{1,i}^T, \cdots, \boldsymbol{e}_{i,i}\boldsymbol{e}_{i,i}^T\},\ \ i = J \text{ or } \nu \qquad (21)
$$

$$
\boldsymbol{E}_i = [\boldsymbol{I}_i \otimes \boldsymbol{e}_{1,i}, \cdots, \boldsymbol{I}_i \otimes \boldsymbol{e}_{i,i}]^T,\ \ i = J \text{ or } \nu \qquad (22)
$$

$$
\boldsymbol{B}_1 = [\boldsymbol{B}_{i,j}]_{J\times\nu},\ \ \boldsymbol{B}_{i,j} = \boldsymbol{e}_{j,\nu}\boldsymbol{e}_{i,J}^T \qquad (23)
$$

$\boldsymbol{e}_{m_1,m_2}$ is a unitary vector of length m_2 with only the m_1-th element unity, $\boldsymbol{I}_i$ is an identity matrix of dimension i.

Once $E\{\boldsymbol{r}_n\boldsymbol{r}_n^H\}$ is obtained, $\boldsymbol{\Phi}_r$ can be evaluated from (17) thus $\boldsymbol{\Phi}_g$ according to (15). Asymptotically optimal performance can be achieved by setting $\boldsymbol{W} = (\boldsymbol{M}\widehat{\boldsymbol{\Phi}}_r\boldsymbol{M}^T)^{-1}$ where $\widehat{\boldsymbol{\Phi}}_r$ can be directly estimated from data

$$
\widehat{\boldsymbol{\Phi}}_r = \frac{1}{N}\sum_{n=1}^{N}(\widehat{\boldsymbol{r}}_n\widehat{\boldsymbol{r}}_n^H) - \frac{1}{N^2}\sum_{n_1=1}^{N}\widehat{\boldsymbol{r}}_{n_1}\sum_{n_2=1}^{N}\widehat{\boldsymbol{r}}_{n_2}^H \qquad (24)
$$

Simulation examples will be provided for verification next.

5. SIMULATION

We simulate a dual rate MC CDMA system with random spreading codes and multipath effect. The mean square channel estimation error (MSE) estimated from 50 realizations is adopted as the performance measure. Parameters are set as follows: $\nu = P_1 = 16$, $K_1 = 2$, $K_2 = 2$, $R_2 = 2R_1$, $q = 3$. Fig. 1 shows results with respect to different N for a low-rate user. Stars, solid line, circles, dashed-dotted line and dashed line are obtained based on $\boldsymbol{W} = \boldsymbol{I}$, estimated $\boldsymbol{W}$ ($\boldsymbol{W} = (\boldsymbol{M}\widehat{\boldsymbol{\Phi}}_r\boldsymbol{M}^T)^{-1}$), optimal $\boldsymbol{W}$ ($\boldsymbol{W} = (\boldsymbol{M}\boldsymbol{\Phi}_r\boldsymbol{M}^T)^{-1}$), theoretical analysis with $\boldsymbol{W} = \boldsymbol{I}$ and theoretical analysis with optimal $\boldsymbol{W}$. It can be observed that the weighting matrix significantly affects the performance. The estimators based on the optimal weighting matrix performs better than those based on $\boldsymbol{W} = \boldsymbol{I}$. It is also seen that the solid line agrees with the circles after a few hundred symbol periods. This indicates that the data sample based estimator performs as well as the optimal estimator. Additionally, our simulation results are consistent with the corresponding analytical results. Similar conclusions can be made based on the results for a high-rate user in Fig. 2. However, the overall errors are smaller than those for the LR user.

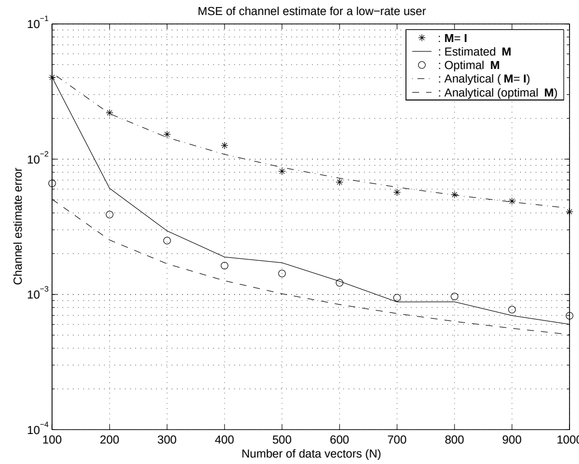

Fig. 1. Performance of the proposed channel estimator for a low-rate user.

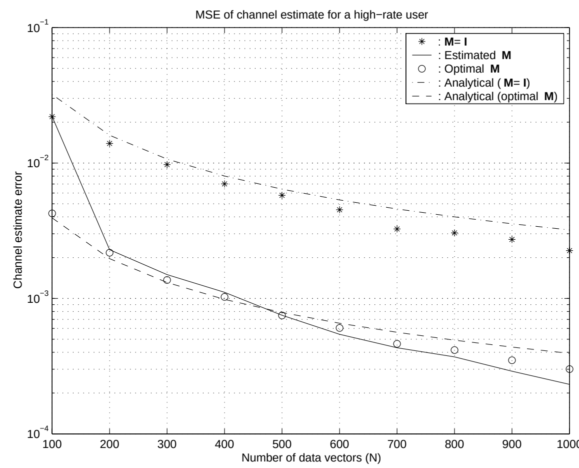

Fig. 2. Performance of the proposed channel estimator for a high-rate user.

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