Data Detection Performance of an MTR-UWB Receiver in the Presence of Timing Errors

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***Abstract*— A multiuser transmitted-reference (MTR) ultra wideband (UWB) transceiver is recently proposed to not only increase the data transmission rate, but also enable multiple access effectively. Both features are realized by incorporating frame-rate pseudo-random sequences. This paper further studies detection performance of the MTR receiver when timing is acquired with some random errors from clock synchronization, delay estimation, etc. A timing acquisition scheme is also proposed, by peak-picking the power of the aggregated template estimate.** [1]

I. Introduction

Ultra wideband (UWB) communication technology is promising to establish high rate short-range wireless links. Design of low complexity transceivers in the presence of severe multipath fading has been one of the primary objectives for practical delivery of its various features. Transmitted reference (TR) modulation may be a very effective solution [1], [2], where reference and data pulses are transmitted in pairs. Then, a receiver can utilize a very simple correlation demodulation mechanism. However, the very noisy template has been observed to impose a limitation to the TR receiver performance.

To improve the template estimate, different sample averaging methods have been proposed [2], [3], [4]. In particular, orders of magnitude improvement in the estimated template mean-square-error (MSE) is possible [4], in proportion to the window size, leading to very good bit error (BER) performance. Furthermore, channel utilization is significantly improved. Since those TR schemes are designed for a single user UWB system, a multiuser transmitted reference (MTR) transceiver was recently proposed, introducing frame-rate pseudorandom (PN) spreading codes to individual users [5], [6]. Then, a generalized mean-matching technique was applied to obtain a clean template, assuming coarse time synchronization.

Although TR receivers show a certain degree of immunity to timing errors, detector performance may still be quite sensitive to mistiming [7] or timing jitter [8]. This paper further studies detection performance of the MTR-UWB receiver [5], [6] when random timing jitter from each user's signal is present, and a noisy template is obtained by sample averaging. Timing errors may stem from clock drift, oscillator noise [9], or an imperfect acquisition algorithm. Jitter is modeled as a stochastic stationary process as in [10]. The detector output signal and noise powers are shown to depend on correlations of different waveforms at random time shifts. Following the transformation idea illustrated in standard communication books [11], as practiced in [10], we adopt a frequency domain representation of each received waveform to derive the BER of the MTR receiver. The analytical result is explicitly linked to the characteristic function of each jitter random process, the window size for template estimation, the multiuser interference (MUI), and background noise. Two cases of practical interest are specifically discussed, uniformly and Gaussian distributed jitter. Connections with previous results in the absence of jitter in [5] are illustrated, and a synchronization method for the MTR receiver is suggested along the lines of [12].

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II. An MTR-UWB Transceiver with Timing Jitter

Denote the transmitted pulse of duration T_w by $w(t)$. Each symbol repeats N_f frames of period T_f, leading to symbol period $T_s = N_f T_f$. User k in a K-user system has a binary information sequence $I_{k,n} \in \{\pm 1\}$ to transmit. Then, the transmitted signal with power $\mathcal{P}_k$ from user k has the following form [5]

$$\begin{aligned} s_k(t) &= \sqrt{\frac{\mathcal{P}_k}{2}} \sum_n \big[A_{k,n} w(t - nT_f) \\ &+ I_{k,\lfloor n/N_f \rfloor} B_{k,n} w(t - nT_f - d_k) \big], \end{aligned} \quad (1)$$

where $A_{k,n}$ and $B_{k,n}$ are frame-rate binary PN sequences taking values ± 1, and d_k is the data pulse delay. For notational brevity, we have dropped limits for frame index n. Later, we will follow a similar practice for user index k or l, and new frame indices m' and m. After propagation through multipath channel $g_k(t)$, the received signal is

$$\begin{aligned} r(t) &= \sum_k \sum_n \big[A_{k,n} h_k(t - nT_f - \varepsilon_{k,n}) \\ &+ I_{k,\lfloor n/N_f \rfloor} B_{k,n} h_k(t - nT_f - d_k - \varepsilon_{k,n}) \big] + v(t), \end{aligned} \quad (2)$$

where $h_k(t)$ is the unknown waveform including effects of the front-end filter and transmission power, $\varepsilon_{k,n}$ is the random jitter for the nth frame, and $v(t)$ represents the output of an ideal filter confined in $f \in [-\frac{\mathcal{B}}{2}, \frac{\mathcal{B}}{2}]$ whose input is additive white Gaussian noise (AWGN) with zero-mean and double-sided power spectral density $\frac{N_0}{2}$. The noise power is thus given by $\sigma_v^2 = \frac{N_0}{2}\mathcal{B}$, and noise correlation by [5]

$$E\{v(t)v(\tau)\} = \sigma_v^2 \phi(t-\tau), \quad \phi(t) \triangleq \mathrm{sinc}(\pi \mathcal{B} t). \tag{3}$$

The data model can account for asynchrony of different users and randomness of different propagation delays. The mean of $\varepsilon_{k,n}$ may represent a constant time shift. Let $h_k(t)$ have maximum support in $(0, T_h)$ and assume $T_h < T_f$. Also assume $d_k \ll T_f$, $|\varepsilon_{k,n}| \ll T_f$, although in principle $\varepsilon_{k,n}$ may take an arbitrary value in $(-\infty, \infty)$ in some cases such as a white Gaussian process. All jitters are assumed independent across time and users. Inter-symbol interference (ISI) is neglected in the model. However, the following discussion can be extended to a system with ISI with reasonably increased complexity in the analysis.

At the receiver, a template is first obtained based on the received signal $r(t)$. That template is then time shifted and correlated with the currently received signal in order to estimate $I_{k,n}$. Partition $r(t)$ spanning N_s symbol intervals into $N_p \triangleq N_f N_s$ segments, each of which has frame duration. The m'th ($m' = 1, \cdots, N_p$) segment is defined as $r_{m'}(t) \triangleq r(t + m'T_f)$ for $t \in [0, T_f)$ and $r_{m'}(t) \triangleq 0$ elsewhere. Similarly, define segmented noise $v_{m'}(t)$. Then, in the absence of ISI, $r_{m'}(t)$ has the form

$$\begin{aligned} r_{m'}(t) &= \sum_l \big[A_{l,m'} h_l(t - \varepsilon_{l,m'}) \\ &\quad + I_{l,\lfloor m'/N_f \rfloor} B_{l,m'} h_l(t - d_l - \varepsilon_{l,m'})\big] + v_{m'}(t), \end{aligned} \tag{4}$$

from which $h_k(t)$ is estimated as follows

$$\begin{aligned} &\widehat{h}_k(t) = \frac{1}{N_p}\sum_{m'} A_{k,m'} r_{m'}(t) \\ &= \frac{1}{N_p}\sum_{l,m'} \big[A_{k,m'} A_{l,m'} h_l(t - \varepsilon_{l,m'}) \\ &+ I_{l,\lfloor m'/N_f \rfloor} A_{k,m'} B_{l,m'} h_l(t - d_l - \varepsilon_{l,m'})\big] + A_{k,m'} v_{m'}(t) \end{aligned} \tag{5}$$

This provides an estimate of $h_k(t)$, corrupted by MUI and noise, and includes timing jitter $\varepsilon_{l,m'}$. Once $h_k(t)$ is estimated, we can estimate input $I_{k,n}$ using received signals $r_m(t)$ for $m = nN_f, \cdots, (n+1)N_f - 1$. Next, the reference signal is subtracted to obtain $\widetilde{r}_{k,m}(t) = r_m(t) - A_{k,m}\widehat{h}_k(t)$ as

$$\begin{aligned} \widetilde{r}_{k,m}(t) &= \sum_l \big[A_{l,m} h_l(t - \varepsilon_{l,m}) \\ &\quad + I_{l,n} B_{l,m} h_l(t - d_l - \varepsilon_{l,m})\big] + v_m(t) - A_{k,m}\widehat{h}_k(t). \end{aligned} \tag{6}$$

Then, the data symbol $I_{k,n}$ can be estimated based on N_f correlator outputs in the nth symbol interval by

$$\widehat{I}_{k,n} = \mathrm{sign}(y_k^{(n)}), \;\; y_k^{(n)} = \frac{1}{N_f}\sum_m \int_0^{T_f} \widehat{h}_k(t)\bar{r}_{k,m}(t)dt, \tag{7}$$

where $\bar{r}_{k,m}(t) = B_{k,m}\widetilde{r}_{k,m}(t + d_k)$. Noting (6), the signal and interference plus noise parts of $y_k^{(n)}$ are easily found to be

$$y_s = I_{k,n}\frac{1}{N_f}\sum_m \int_0^{T_f} \widehat{h}_k(t) h_k(t - \varepsilon_{k,m})dt, \tag{8}$$

$$\begin{aligned} y_n &= \frac{1}{N_f}\sum_m \int_0^{T_f} B_{k,m}\widehat{h}_k(t)\Big[A_{k,m} h_k(t + d_k - \varepsilon_{k,m}) \\ &- A_{k,m}\widehat{h}_k(t + d_k) + v_m(t + d_k) \\ &+ \sum_{l \neq k}[A_{l,m} h_l(t + d_k - \varepsilon_{l,m}) \\ &+ I_{l,n} B_{l,m} h_l(t + d_k - d_l - \varepsilon_{l,m})]\Big]dt. \end{aligned} \tag{9}$$

Performance of the receiver depends on jitter statistics, sample size N_p, spreading factor N_f, MUI and AWGN.

III. Performance Study

The receiver output has signal component y_s and noise component y_n. The BER depends on the signal to interference plus noise ratio (SINR). For tractable analysis, we model the PN sequences as independent binary random sequences. Denote the desired signal power by $\mathcal{E}_s = E\{y_s^2\}$, and interference plus noise power by $\mathcal{E}_n = E\{y_n^2\}$. For sufficiently large N_p, y_n can be reasonably approximated as a Gaussian process according to the central limit theorem. Then, the BER is given by $Q(\sqrt{\frac{\mathcal{E}_s}{\mathcal{E}_n}})$, where $Q(x)$ is the Q-function given by $Q(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}} e^{-\frac{u^2}{2}} du$.

From (8), considering $\widehat{h}_k(t)$ and $h_k(t - \varepsilon_{k,m})$ are (or are approximately) independent, we obtain the signal power

$$\mathcal{E}_s = \frac{1}{N_f^2}\sum_{m_1,m_2} \iint E\{\prod_{i=1}^{2} h_k(t_i - \varepsilon_{k,m_i})\} E\{\prod_{i=1}^{2}\widehat{h}_k(t_i)\} dt_1 dt_2,$$

where the double integral is over $(0, T_f)$. Further development requires second order statistics at random delays, and in the following we also require up to fourth-order statistics to evaluate $\mathcal{E}_n$. Motivated by this, we next consider a more general case.

A. Statistics of Waveforms at Random Delays

Consider a frequency domain representation of each waveform at a random time shift, e.g., see [10], as $h_k(t) = \int_{-\infty}^{\infty} H_k(f) e^{j2\pi f t} df$ via the Fourier transform $H_k(f)$. Denote the pth order correlation ($p = 1, \cdots, 4$) of the kth user waveform with the same jitter $\varepsilon_{k,m}$ by

$$\Theta_k^{(p)}(t_1, \cdots, t_p) \triangleq E\{\prod_{i=1}^{p} h_k(t_i - \varepsilon_{k,m})\},$$

and the characteristic function of $\varepsilon_{k,m}$ by $\Phi_k(f) \triangleq E\{e^{j2\pi f \varepsilon_{k,m}}\}$. Then the following can be easily verified

$$\Theta_k^{(p)}(t_1, \cdots, t_p) = \int \!\cdots\! \int \Phi_k(-\sum_{i=1}^{p} f_i) \prod_{i=1}^{p} H_k(f_i) e^{j2\pi f_i t_i} df_i. \tag{10}$$

B. Statistics of Estimated Waveform

With (10) and (5), we can proceed to derive the following correlations of the estimated waveform

$$\Upsilon_k^{(p)}(t_1,\cdots,t_p) \triangleq E\{\prod_{i=1}^{p}\widehat{h}_k(t_i)\},$$

for $p=1,\cdots,4$. Since PN codes, inputs and AWGN all have zero mean, one immediately obtains

$$\Upsilon_k^{(1)}(t_1)=\Theta_k^{(1)}(t_1). \tag{11}$$

To obtain $\Upsilon_k^{(2)}(t_1,t_2)$, we expand $\widehat{h}_k(t_1)\widehat{h}_k(t_2)$ using (5). Due to (3) and the zero mean assumptions previously stated, we obtain

$$\Upsilon_k^{(2)}(t_1,t_2)=(1-\frac{1}{N_p})\prod_{i=1}^{2}\Theta_k^{(1)}(t_i)+\frac{\sigma_v^2}{N_p}\phi(t_1-t_2) + \frac{1}{N_p}\sum_l[\Theta_l^{(2)}(t_1,t_2)+\Theta_l^{(2)}(t_1-d_l,t_2-d_l)]. \tag{12}$$

Similarly, we find that

$$\Upsilon_k^{(3)}(t_1,t_2,t_3)\approx\frac{\boldsymbol{X}_k(t_1,t_2,t_3)}{N_p}+(1-\frac{\sum_{i=1}^{2}i}{N_p})\prod_{i=1}^{3}\Theta_k^{(1)}(t_i) \tag{13}$$

where terms of order higher than $O(\frac{1}{N_p})$ have been ignored,

$$\begin{aligned}\boldsymbol{X}_k(t_1,t_2,t_3) = &\\ &\Theta_k^{(1)}(t_1)\sum_l[\Theta_l^{(2)}(t_2,t_3)+\Theta_l^{(2)}(t_2-d_l,t_3-d_l)]\\ &+\Theta_k^{(1)}(t_2)\sum_l[\Theta_l^{(2)}(t_1,t_3)+\Theta_l^{(2)}(t_1-d_l,t_3-d_l)]\\ &+\Theta_k^{(1)}(t_3)\sum_l[\Theta_l^{(2)}(t_1,t_2)+\Theta_l^{(2)}(t_1-d_l,t_2-d_l)]\\ &+\sigma_v^2[\Theta_k^{(1)}(t_1)\phi(t_2-t_3)+\Theta_k^{(1)}(t_2)\phi(t_1-t_3)]\\ &+\sigma_v^2\Theta_k^{(1)}(t_3)\phi(t_1-t_2).\end{aligned}$$

Similarly, we obtain

$$\Upsilon_k^{(4)}(t_1,\cdots,t_4)\approx\frac{\boldsymbol{Y}_k(t_1,\cdots,t_4)}{N_p}+(1-\frac{\sum_{i=1}^{3}i}{N_p})\prod_{i=1}^{4}\Theta_k^{(1)}(t_i) \tag{14}$$

where

$$\begin{aligned}&\boldsymbol{Y}_k(t_1,\cdots,t_4) =\\ &\Theta_k^{(1)}(t_1)\Theta_k^{(1)}(t_2)\sum_l[\Theta_l^{(2)}(t_3,t_4)+\Theta_l^{(2)}(t_3-d_l,t_4-d_l)]\\ &+\Theta_k^{(1)}(t_1)\Theta_k^{(1)}(t_3)\sum_l[\Theta_l^{(2)}(t_2,t_4)+\Theta_l^{(2)}(t_2-d_l,t_4-d_l)]\\ &+\Theta_k^{(1)}(t_1)\Theta_k^{(1)}(t_4)\sum_l[\Theta_l^{(2)}(t_2,t_3)+\Theta_l^{(2)}(t_2-d_l,t_3-d_l)]\\ &+\Theta_k^{(1)}(t_2)\Theta_k^{(1)}(t_3)\sum_l[\Theta_l^{(2)}(t_1,t_4)+\Theta_l^{(2)}(t_1-d_l,t_4-d_l)]\\ &+\Theta_k^{(1)}(t_2)\Theta_k^{(1)}(t_4)\sum_l[\Theta_l^{(2)}(t_1,t_3)+\Theta_l^{(2)}(t_1-d_l,t_3-d_l)]\\ &+\Theta_k^{(1)}(t_3)\Theta_k^{(1)}(t_4)\sum_l[\Theta_l^{(2)}(t_1,t_2)+\Theta_l^{(2)}(t_1-d_l,t_2-d_l)]\\ &+\sigma_v^2[\Theta_k^{(1)}(t_1)\Theta_k^{(1)}(t_2)\phi(t_3-t_4)+\Theta_k^{(1)}(t_1)\Theta_k^{(1)}(t_3)\phi(t_2-t_4)]\\ &+\sigma_v^2[\Theta_k^{(1)}(t_1)\Theta_k^{(1)}(t_4)\phi(t_2-t_3)+\Theta_k^{(1)}(t_2)\Theta_k^{(1)}(t_3)\phi(t_1-t_4)]\\ &+\sigma_v^2[\Theta_k^{(1)}(t_2)\Theta_k^{(1)}(t_4)\phi(t_1-t_3)+\Theta_k^{(1)}(t_3)\Theta_k^{(1)}(t_4)\phi(t_1-t_2)]\end{aligned}$$

Note that higher order statistics $\Theta_l^{(3)}(t_1,t_2,t_3)$ and $\Theta_l^{(4)}(t_1,\cdots,t_4)$ are not involved. One major cause is that many terms in the order of $O(\frac{1}{N_p^i})$ for $i>1$ have been discarded.

C. Signal Power

Using the above, the signal power is

$$\mathcal{E}_s = \iint\Upsilon_k^{(2)}(t_1,t_2)\Big[\frac{1}{N_f}\Theta_k^{(2)}(t_1,t_2) +(1-\frac{1}{N_f})\prod_{i=1}^{2}\Theta_k^{(1)}(t_i)\Big]dt_1dt_2. \tag{15}$$

Using (12) and (10), and a given distribution of the timing jitter for each user, $\mathcal{E}_s$ can now be evaluated.

D. Interference Plus Noise Power

Starting from (9), a straightforward simplification of $\mathcal{E}_n$ can be carried out. First, expectations on PN codes, inputs and AGWN are taken, yielding

$$\begin{aligned}\mathcal{E}_n = &\frac{1}{N_f}E\Big\{\iint\widehat{h}_k(t_1)\widehat{h}_k(t_2)\\ &\times\Big[\Big(h_k(t_1+d_k-\varepsilon_{k,m})-\widehat{h}_k(t_1+d_k)\Big)\\ &\times\Big(h_k(t_2+d_k-\varepsilon_{k,m})-\widehat{h}_k(t_2+d_k)\Big)\\ &+\sum_{l\neq k}h_l(t_1+d_k-\varepsilon_{l,m})h_l(t_2+d_k-\varepsilon_{l,m})\\ &+\sum_{l\neq k}h_l(t_1+d_k-d_l-\varepsilon_{l,m})h_l(t_2+d_k-d_l-\varepsilon_{l,m})\\ &+\sigma_v^2\phi(t_1-t_2)\Big]dt_1dt_2\Big\}.\end{aligned} \tag{16}$$

Incorporating notation of the waveform statistics, this becomes

$$\begin{aligned}\mathcal{E}_n = &\frac{1}{N_f}\iint\Big[\Upsilon_k^{(4)}(t_1,t_1+d_k,t_2,t_2+d_k)\\ &-\Upsilon_k^{(3)}(t_1,t_1+d_k,t_2)\Theta_k^{(1)}(t_2+d_k)\\ &-\Upsilon_k^{(3)}(t_1,t_2,t_2+d_k)\Theta_k^{(1)}(t_1+d_k)\\ &+\Upsilon_k^{(2)}(t_1,t_2)\sum_l\Theta_l^{(2)}(t_1+d_k,t_2+d_k)\\ &+\Upsilon_k^{(2)}(t_1,t_2)\sum_{l\neq k}\Theta_l^{(2)}(t_1+d_k-d_l,t_2+d_k-d_l)\\ &+\sigma_v^2\Upsilon_k^{(2)}(t_1,t_2)\phi(t_1-t_2)\Big]dt_1dt_2.\end{aligned} \tag{17}$$

Using our results for waveform statistics derived in Section III-A and Section III-B, $\mathcal{E}_n$ can be evaluated.

E. Discussion

Next we examine some special cases for $\mathcal{E}_s$ and $\mathcal{E}_n$.

1) Large sample size N_p: As $N_p \to \infty$, $\mathcal{E}_s$ in (15) becomes

$$\mathcal{E}_s = \iint \prod_{i=1}^{2} \Theta_k^{(1)}(t_i)\Big[\frac{1}{N_f}\Theta_k^{(2)}(t_1,t_2) + (1-\frac{1}{N_f})\prod_{i=1}^{2}\Theta_k^{(1)}(t_i)\Big]dt_1dt_2. \quad (18)$$

Also, we observe from Section III-B in this limiting case that

$$\Upsilon_k^{(p)}(t_1,\cdots,t_p) = \prod_{i=1}^{p}\Theta_k^{(1)}(t_i)$$

for $p = 1,\cdots,4$. Substituting into (17), we obtain

$$\begin{aligned}\mathcal{E}_n = \frac{1}{N_f}\iint \prod_{i=1}^{2}\Theta_k^{(1)}(t_i)\Big[&\Theta_k^{(2)}(t_1+d_k,t_2+d_k) \\ &-\prod_{i=1}^{2}\Theta_k^{(1)}(t_i+d_k) + \sum_{l\neq k}\Theta_l^{(2)}(t_1+d_k,t_2+d_k) \\ &+\sum_{l\neq k}\Theta_l^{(2)}(t_1+d_k-d_l,t_2+d_k-d_l) \\ &+\sigma_v^2\phi(t_1-t_2)\Big]dt_1dt_2. \quad (19)\end{aligned}$$

$\mathcal{E}_s$ depends on $\Theta_k^{(p)}(t_1,\cdots,t_p)$ for $p = 1,2$, that in turn depends on the Fourier transform $H_k(f)$ weighted by the characteristic function of $\varepsilon_{k,m}$ at different frequencies according to (10). $\mathcal{E}_n$ depends on all users channels as well as jitter statistics, as seen from terms $\Theta_l^{(p)}(t_1,\cdots,t_p)$. In addition, AWGN has a contribution scaled by a factor $\prod_{i=1}^{2}\Theta_k^{(1)}(t_i)$ determined by the desired user channel and jitter statistic. Not surprisingly, this factor appears in every term in $\mathcal{E}_s$ and $\mathcal{E}_n$, due to the correlation operation of the receiver, and converging result of the template estimate to its mean $\Theta_k^{(1)}(t)$ as $N_p \to \infty$. The term $\Theta_k^{(2)}(t_1+d_k,t_2+d_k) - \prod_{i=1}^{2}\Theta_k^{(1)}(t_i+d_k)$ also appears in both $\mathcal{E}_s$ and $\mathcal{E}_n$ because of jitter variance.

2) Uniform and Gaussian timing jitters: The general results previously derived require each characteristic function of timing jitter. This takes a simple form for the following two special cases, as also discussed in [10]. If $\varepsilon_{l,m}$ is uniformly distributed, then its probability density function (pdf) is

$$f_l(x) = \{ \begin{array}{ll} \frac{1}{\Delta_l} & \Delta_{l,1} < x < \Delta_{l,2} \\ 0 & \text{otherwise} \end{array}$$

where $\Delta_l = \Delta_{l,2} - \Delta_{l,1}$. Its standard deviation is $\frac{1}{\sqrt{12}}\Delta_l$, and its characteristic function is given by

$$\Phi_l(f) = \mathrm{sinc}(\pi f\Delta_l)e^{j2\pi f\tau_l}$$

where $\tau_l = \frac{1}{2}(\Delta_{l,1} + \Delta_{l,2})$ is its mean. If $\varepsilon_{l,m}$ is Gaussian distributed, then its pdf is

$$f_l(x) = \frac{1}{\sqrt{2\pi}\Delta_l}e^{-\frac{(x-\tau_l)^2}{2\Delta_l^2}}$$

where τ_l is its mean, and Δ_l its standard deviation. Its characteristic function is given by

$$\Phi_l(f) = e^{j2\pi f\tau_l - \frac{1}{2}(2\pi f\Delta_l)^2}.$$

Substituting these characteristic functions into our results, and evaluating multiple integrals, the SINR can be found and BER obtained. For other distributions, the resulting characteristic function may not have a closed form, in which case numerical integration can be employed.

3) Deterministic jitter: Our derived results assuming random jitters can be applied to a system with deterministic jitters as well. Denote the timing error for user l by τ_l. Now, each pdf can be expressed as a delta function $f_l(x) = \delta(x-\tau_l)$. In this case, the characteristic function becomes $\Phi_l(f) = e^{-j2\pi f\tau_l}$. Then, $\Phi_l(-\sum_{i=1}^{p} f_i)$ can be factored as $\prod_{i=1}^{p}\Phi_l(-f_i)$. Consequently, from (10) we obtain

$$\Theta_l^{(p)}(t_1,\cdots,t_p) = \prod_{i=1}^{p} h_l(t_i-\tau_l). \quad (20)$$

Now, $\Theta_l^{(2)}(t_1,t_2) = \prod_{i=1}^{2}\Theta_l^{(1)}(t_i)$, reducing to correlations of users waveforms. Thus $\mathcal{E}_s$ in (15) and $\mathcal{E}_n$ (17) can be found directly from the waveform correlations in this case. No weighted coupling occurs because of the particular form of the characteristic function.

This general observation is consistent with [5], that deals with constant time shifts. To clearly identify the connections, let's consider the special case when $N_p \to \infty$ for which $\mathcal{E}_s$ and $\mathcal{E}_n$ in [5] obey very simple forms. Now, (18) and (19) reduce to

$$\mathcal{E}_s = \iint [h_k(t_1-\tau_k)h_k(t_2-\tau_k)]^2dt_1dt_2, \quad (21)$$

$$\begin{aligned}\mathcal{E}_n = \frac{1}{N_f}\iint &h_k(t_1-\tau_k)h_k(t_2-\tau_k) \\ &\times\Big[\sum_{l\neq k}h_l(t_1+d_k-\tau_l)h_l(t_2+d_k-\tau_l) \\ &+\sum_{l\neq k}h_l(t_1+d_k-d_l-\tau_l)h_l(t_2+d_k-d_l-\tau_l) \\ &+\sigma_v^2\phi(t_1-t_2)\Big]dt_1dt_2. \quad (22)\end{aligned}$$

Performance depends on propagation delays. If we assume all users are synchronized to allow maximal collision of users signals, then $\tau_l = 0$. The results coincide with those in [5], and depend on auto- and cross correlations of different users waveforms.

When the timing error is only due to propagation delay, then detection performance is determined by synchronization errors. In fact, a synchronization algorithm can be developed based on the estimated template at different candidate time shifts in the light of [12]. Re-write the estimate (5) as

$$\begin{aligned}\widehat{h}_k(t) = \frac{1}{N_p}\sum_{l,m'}\big[&A_{k,m'}A_{l,m'}h_l(t-\tau_l) \\ &+I_{l,\lfloor m'/N_f\rfloor}A_{k,m'}B_{l,m'}h_l(t-d_l-\tau_l)\big] + A_{k,m'}v_{m'}(t). \quad (23)\end{aligned}$$

It has a contribution from the desired user in the form of $h_k(t)$. Consider the power of $\widehat{h}_k(t+\tau)$. It achieves its maximum at $\tau = \tau_k$ as $N_p \to \infty$ and interference plus noise is negligible. Therefore, a criterion to estimate τ_k is given by

$$\widehat{\tau}_k = \arg \max_{\tau \in [0, T_h)} \int_0^{T_h} \left[\widehat{h}_k(t+\tau)\right]^2 dt. \quad (24)$$

Peak picking of this function will yield an estimate within a specified resolution, set by a variable grid size.

IV. Simulation

In this section, we present some performance results using our above analysis. Normalized second derivative of Gaussian pulse with duration of $0.7ns$ is used as transmitted monocycles. Frame duration T_f is $30ns$. Each symbol spans two frames. Multipath channels are generated using the IEEE TG802.15.3a CM1 channel model [13] and their tails are truncated to capture 80% of multipath energy. These discrete-time channels have time resolution of $0.03ns$. We use FFT of size 1024 to calculate the channel's frequency response. Four users are active in the system. Each has an integer delay d_k arbitrarily chosen from $1ns$ to $6ns$. Fig. 1 shows the BER with uniformly distributed timing jitter with $\Delta_{l,1} = 0$. Standard deviation of each jitter changes from $0ps$ to $80ps$. All timing jitters for Fig. 2 are Gaussian distributed with zero mean. It can be seen from these two figures that detection performance degrades fast with increasing timing jitter. Compared with perfect timing, performance is about two orders of magnitude worse when jitter deviation is $80ps$ at $E_b/N_0 = 12dB$. Gaussian distributed jitter has less impact on the performance than uniformly distributed jitter with the same jitter deviation. Performance degradation is more sensitive to jitters with deviation in the range from $20ps$ to $60ps$, a similar observation as in [8].

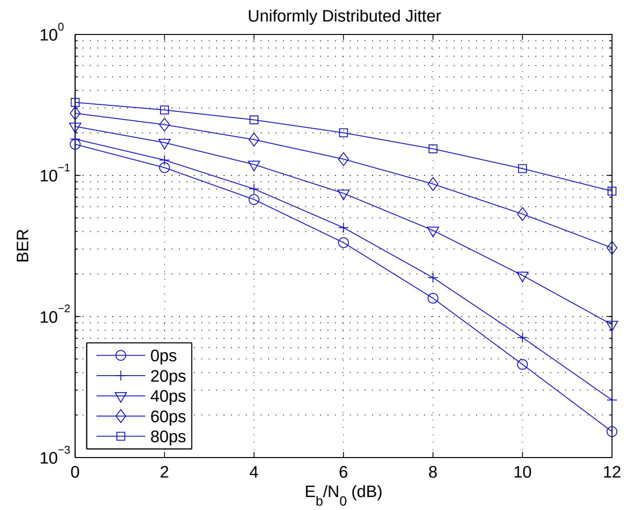

Fig. 1. BER with uniformly distributed timing jitter.

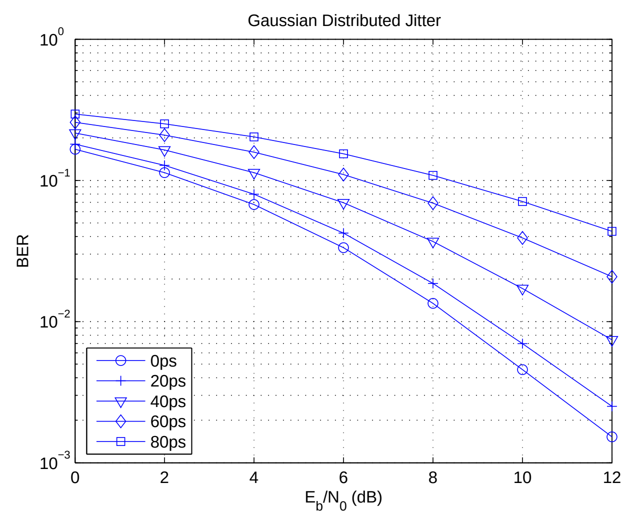

Fig. 2. BER with Gaussian distributed timing jitter.

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