Subspace-Based Channel Estimation for CDMA Downlink with Aperiodic Spreading Codes and Multiple Subchannels

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Abstract

In this paper we consider a downlink CDMA system with aperiodic spreading codes and multipath propagation. Assisted by either multiple antennas or oversampling, multiple subchannels are created. Then subspace technique can be directly applied to estimate all channel parameters, based only on the channel output. The proposed method does not require either the spreading codes of the users or pre-processing of the received data. It is also applicable to an overloaded system. These appealing features make our method significantly different from existing subspace approaches.

1 Introduction

Code division multiple access (CDMA) systems with aperiodic spreading codes (long codes) have recently received considerable attention. Aperiodic codes are employed to more evenly spread transmitted signals over the available spectrum and maximize system capacity. They help ensure a secure communication link between communication parties in a hostile environment, protecting users' information against intentional interception. Together with other merits, long codes have been adopted in the third generation CDMA systems and proposed for future networks as well. However, the difficulty in multiuser detection arises from the user's time-varying signature. In a multipath communication environment, this signature is not known *a priori*, making direct design of the detector intractable.

Because of this important challenge, a number of studies on such systems have appeared. Based on the finite alphabet property of the input, an iterative method to estimate the FIR channels and the transmitted symbols is presented in [8]. A Toeplitz displacement method for multipath channel estimation is proposed in [4]. Given the pilot symbols of all users, the least squares (LS) fitting or iterative maximum likelihood (ML) approaches are presented in [2], [3]. A blind uplink channel estimation method using correlation matching techniques is proposed in [10], based only on the correlation of the channel's output conditioned on the long spreading codes. A computationally efficient algorithm has been reported for downlink channel estimation in [12] and uplink channel estimation in [11]. Low complexity in the channel estimation and multiuser detection algorithms can be achieved when the statistics of the spreading codes are given [13], [14].

Besides the correlation matching approaches, subspace concepts [1], [6] have been adopted to estimate the downlink multipath channel with a precoding structure [7]. In a long code CDMA system, the noise subspace usually does not exist in the received data vector space because the aperiodic spreading codes virtually increase the number of sources and make the system appear overloaded. Therefore, group-based decorrelation is performed in [9] by first obtaining a new data vector which generates the noise subspace. The spreading codes of all users in the desired group are needed to perform a computationally expensive matrix inverse operation at each evaluation step. However, in downlink communications, the desired user may have no knowledge of the spreading codes of other users, rendering the direct application of that method questionable.

In this paper, we seek a channel estimation solution based on the channel output only. It will be revealed that to directly apply the subspace technique, spatial/temporal diversity should be employed to generate multiple subchannels. This diversity extends the dimension of the operational space and creates more degrees of freedom for the noise subspace. If we treat each transmitted chip sample as an input, then the signature vector of each chip input is characterized only by the unknown channel parameters. Thus by minimizing its projection onto the estimated noise subspace, the channel vector can be estimated within a scalar ambiguity. Based on perturbation analysis [5], the covariance and mean square error of the channel estimator are derived. Simulation examples are provided for verification.

2 CDMA Downlink with Aperiodic Spreading Codes

Consider a base station communicating with J mobile stations in a CDMA system [12]. The jth user is assigned aperiodic spreading codes $c_{j,n}(k)$ $(k = 0, \ldots, P-1)$ of length P to spread bit $w_j(n)$ at time n. After multiplexing, the base station transmits the combined signals for all users to each mobile station through a common multipath channel. Assume the channel is FIR and has order q $(q < P)$ with $q+1$ chip rate coefficients. Also assume that at the mobile station we either have M antenna elements for signal reception or we oversample the channel output to generate M subchannels. If we use $g_m(n)$ to represent the mth composite subchannel impulse response, including the transmitter, the physical channel and the receiver, then the output of the mth subchannel due to user j is

$$y_{j,m}(n) = \sum_{l=0}^{q} g_m(l) s_j(n-l) \tag{1}$$

where

$$s_j(n) = \sum_{k=-\infty}^{\infty} w_j(k) c_{j,k}(n-kP).$$

Signals for different users arrive at the receiver simultaneously. The received signal is a superposition of signals from J users corrupted by noise

$$y_m(n) = \sum_{j=1}^{J} y_{j,m}(n) + v_m(n) \tag{2}$$

where $v_m(n)$ is the white Gaussian noise with variance $\sigma_v^2 = E\{\|v_m(n)\|^2\}$.

If we collect P chip rate samples $y_m(nP), \cdots, y_m(nP+P-1)$ from each subchannel during the nth bit interval and put samples from all M subchannels in a big vector

$$\begin{aligned}\boldsymbol{y}(n) &= [y_1(nP), \cdots, y_M(nP), \cdots, y_1(nP+P-1), \\ &\quad \cdots, y_M(nP+P-1)]^T,\end{aligned}$$

then from (1) to (2), $\boldsymbol{y}(n)$ can be found to be

$$\boldsymbol{y}(n) = \mathcal{G}\boldsymbol{b}(n) + \boldsymbol{v}(n), \quad \boldsymbol{b}(n) = \sum_{j=1}^{J} \boldsymbol{b}_j(n). \tag{3}$$

In (3), $\mathcal{G}$ is a $MP \times (P+q)$ block Toeplitz matrix. It is partitioned into P subblocks row by row with each block row containing M rows. Its first block row is $[\boldsymbol{G}\ \boldsymbol{0}]$

$$\boldsymbol{G} = \begin{bmatrix} g_1(q) & \cdots & g_1(0) \\ \vdots & & \vdots \\ g_M(q) & \cdots & g_M(0) \end{bmatrix}.$$

Other successive block rows are obtained by shifting the previous row to the right by one column. All entries in $\mathcal{G}$ are zero except entries of the form $\boldsymbol{G}$. $\boldsymbol{b}_j(n)$ includes the spreading codes and bits of user j at time n and $n-1$ as follows

$$\begin{aligned}\boldsymbol{b}_j(n) &= [w_j(n-1)[c_{j,n-1}(P-q), \cdots, c_{j,n-1}(P-1)], \\ &\quad w_j(n)[c_{j,n}(0), \cdots, c_{j,n}(P-1)]]^T.\end{aligned}$$

$\boldsymbol{b}(n)$ can be treated as a combined input from all users with $P+q$ elements. The signature of each element is the corresponding column vector in $\mathcal{G}$. If $M > 1$, the model virtually represents a single input multiple output (SIMO) system [6]. In this case, $\mathcal{G}$ becomes a tall matrix. Then the signal subspace is invariant and well defined. Thus the subspace method is readily applicable to estimate all subchannels. However, it can be easily observed that if we focus on the signature waveform of the transmitted bit which is the channel convolved with spreading codes, then the signal subspace varies from bit to bit, rendering the subspace method not directly applicable.

3 Subspace-Based Downlink Channel Estimation

From our previous analysis, all columns of $\mathcal{G}$ span the signal subspace which is time invariant. We define its columns as h_i for $i = -q, \cdots, 0, \cdots, P-1$. And collect the unknown channel parameters in a vector

$$\begin{aligned}\boldsymbol{h} &= [g_1(0), \cdots, g_M(0), g_1(1), \cdots, g_M(1), \cdots, \\ &\quad g_1(q), \cdots, g_M(q)]^T.\end{aligned} \tag{4}$$

Then $h_0 = [\boldsymbol{h}^T, 0, \cdots, 0]^T$. If we define

$$\boldsymbol{A} = \begin{bmatrix} \boldsymbol{I}_{M(q+1)} \\ \boldsymbol{0} \end{bmatrix},$$

then $h_0 = \boldsymbol{A}\boldsymbol{h}$. Other columns h_i also contain partial/full copies of vector $\boldsymbol{h}$ together with some zeros. To explore this relationship, we introduce a Jordan matrix $\boldsymbol{J}$ with all $1's$ in the first sub-diagonal. For convenience, we also use the symbol $\boldsymbol{J}^{-1}$ to denote $\boldsymbol{J}^T$ although $\boldsymbol{J}$ is singular: $\boldsymbol{J}^{-1} \triangleq \boldsymbol{J}^T$ and define $\boldsymbol{J}^0$ as an identity matrix. Then h_i can be obtained by shifting all elements of h_0 up or down by iM positions

$$h_i = \boldsymbol{J}^{iM} h_0 = \boldsymbol{J}^{iM} \boldsymbol{A}\boldsymbol{h}. \tag{5}$$

To use the subspace technique, we form a data matrix $\boldsymbol{Y}$ by concatenating N observation vectors

$$\boldsymbol{Y} = [\boldsymbol{y}(n)\ \boldsymbol{y}(n+1) \cdots \boldsymbol{y}(n+N-1)] = \mathcal{G}\boldsymbol{B} + \boldsymbol{\Psi}$$

where $\boldsymbol{B}$ similarly contains N vectors of $\boldsymbol{b}(n), \cdots, \boldsymbol{b}(n+N-1)$, $\boldsymbol{\Psi}$ is a noise matrix. Applying singular value decomposition (SVD) to a noise free data matrix $\widetilde{\boldsymbol{Y}}$ yields

$$\widetilde{\boldsymbol{Y}} = \mathcal{G}\boldsymbol{B} = \left[\begin{array}{cc} \boldsymbol{U}_s & \boldsymbol{U}_n \end{array}\right] \left[\begin{array}{cc} \boldsymbol{\Sigma}_s & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} \end{array}\right] \left[\begin{array}{c} \boldsymbol{V}_s^H \\ \boldsymbol{V}_n^H \end{array}\right].$$

It is well known that $\boldsymbol{U}_s$ spans the signal subspace which is the column span of $\mathcal{G}$. $\boldsymbol{U}_n$ spans the noise subspace with dimension $MP - P - q$. Therefore $\boldsymbol{U}_n^H \mathcal{G} = \boldsymbol{0}$. Hence we obtain our channel estimation method by minimizing $||\boldsymbol{U}_n^H \mathcal{G}||^2$ where $||\cdot||$ refers to the Frobenius norm of a matrix. After considering (5), the channel $\boldsymbol{h}$ is estimated from noisy data by

$$\widehat{\boldsymbol{h}} = \arg \min_{||\boldsymbol{h}||=1} \boldsymbol{h}^H \left[\sum_{i=-q}^{P-1} \boldsymbol{A}^T \boldsymbol{J}^{-iM} \widehat{\boldsymbol{U}}_n \widehat{\boldsymbol{U}}_n^H \boldsymbol{J}^{iM} \boldsymbol{A} \right] \boldsymbol{h} \tag{6}$$

where $\widehat{\boldsymbol{U}}_n$ is an estimate of $\boldsymbol{U}_n$. Therefore $\widehat{\boldsymbol{h}}$ is the eigenvector of the inner matrix corresponding to its minimum eigenvalue.

It can be observed that it is required that $MP > P + q$ to guarantee the existence of $\boldsymbol{U}_n$. If $q < P$, then we need to choose $M \geq 2$ for multiple subchannels. As q increases, more subchannels are needed for the channel to be identifiable. The key contribution of this paper lies in the formulation of the downlink channel estimation problem for aperiodically spreading CDMA systems into one which is solvable by the subspace method directly from the channel output.

4 Performance Analysis

We begin by developing the perturbation to the noise subspace due to an imperfect estimate of the received data vector. Let the noisy data matrix be decomposed

$$\boldsymbol{Y} = \widetilde{\boldsymbol{Y}} + \boldsymbol{\Psi} = (\widehat{\boldsymbol{U}}_s \, \widehat{\boldsymbol{U}}_n) \left(\begin{array}{cc} \widehat{\boldsymbol{\Sigma}}_s & \boldsymbol{0} \\ \boldsymbol{0} & \widehat{\boldsymbol{\Sigma}}_n \end{array} \right) \left(\begin{array}{c} \widehat{\boldsymbol{V}}_s^H \\ \widehat{\boldsymbol{V}}_n^H \end{array} \right) \tag{7}$$

where $\widehat{\boldsymbol{U}}_s = \boldsymbol{U}_s + \Delta \boldsymbol{U}_s$ and $\widehat{\boldsymbol{U}}_n = \boldsymbol{U}_n + \Delta \boldsymbol{U}_n$ are the perturbed signal and noise subspaces. Similarly $\widehat{\boldsymbol{\Sigma}}_s$ and $\widehat{\boldsymbol{\Sigma}}_n$ are the perturbed versions of $\boldsymbol{\Sigma}_s$ and $\boldsymbol{\Sigma}_n = \boldsymbol{0}$ respectively. It can be shown that the first order perturbation of the noise subspace due to noise has a form [5]

$$\Delta \boldsymbol{U}_n \approx \boldsymbol{U}_s \boldsymbol{\Sigma}_s^{-1} \boldsymbol{V}_s^H \boldsymbol{\Psi}^H \boldsymbol{U}_n = -(\widetilde{\boldsymbol{Y}}^\dagger)^H \boldsymbol{\Psi}^H \boldsymbol{U}_n \tag{8}$$

where $\dagger$ denotes the left pseudo-inverse. In the absence of $\Delta \boldsymbol{U}_n$, (6) will provide an exact channel vector which is the unique null vector of the following objective matrix

$$\boldsymbol{X} \equiv \sum_{i=-q}^{P-1} \boldsymbol{A}^T \boldsymbol{J}^{-iM} \boldsymbol{U}_n \boldsymbol{U}_n^H \boldsymbol{J}^{iM} \boldsymbol{A}. \tag{9}$$

However, the appearance of noise introduces $\Delta \boldsymbol{U}_n$ and $\boldsymbol{X}$ is perturbed. According to (9), the first order perturbation to $\boldsymbol{X}$ can be easily found to be

$$\begin{aligned} \Delta \boldsymbol{X} \quad &\approx \quad \sum_{i=-q}^{P-1} (\boldsymbol{A}^T \boldsymbol{J}^{-iM} \Delta \boldsymbol{U}_n \boldsymbol{U}_n^H \boldsymbol{J}^{iM} \boldsymbol{A} \\ &+ \quad \boldsymbol{A}^T \boldsymbol{J}^{-iM} \boldsymbol{U}_n \Delta \boldsymbol{U}_n^H \boldsymbol{J}^{iM} \boldsymbol{A}). \end{aligned} \tag{10}$$

When the perturbed cost function is minimized, the perturbation in the channel estimate $\Delta \boldsymbol{h}$ can be computed as

$$\Delta \boldsymbol{h} \approx -\boldsymbol{X}^\dagger \Delta \boldsymbol{X} \boldsymbol{h}. \tag{11}$$

After substituting (8) in (10) and then (10) in (11), we obtain

$$\Delta \boldsymbol{h} \approx \sum_{i=-q}^{P-1} \boldsymbol{X}^\dagger \boldsymbol{A}^T \boldsymbol{J}^{-iM} \boldsymbol{U}_n \boldsymbol{U}_n^H \boldsymbol{\Psi} \widetilde{\boldsymbol{Y}}^\dagger \boldsymbol{J}^{iM} \boldsymbol{A} \boldsymbol{h}. \tag{12}$$

where the orthogonality between $\boldsymbol{U}_n^H$ and $\boldsymbol{h}$ has been applied. For convenience, we define

$$\boldsymbol{B}_i = \boldsymbol{X}^\dagger \boldsymbol{A}^T \boldsymbol{J}^{-iM} \boldsymbol{U}_n \boldsymbol{U}_n^H, \quad \boldsymbol{a}_i = \widetilde{\boldsymbol{Y}}^\dagger h_i.$$

(12) becomes

$$\Delta \boldsymbol{h} \approx \sum_{i=-q}^{P-1} \boldsymbol{B}_i \boldsymbol{\Psi} \boldsymbol{a}_i. \tag{13}$$

Now the covariance can be computed in a compact form

$$E\{\Delta \boldsymbol{h} \Delta \boldsymbol{h}^H\} \approx E \left\{ \sum_{i,j=-q}^{P-1} \boldsymbol{B}_i \boldsymbol{\Psi} \boldsymbol{a}_i \boldsymbol{a}_j^H \boldsymbol{\Psi}^H \boldsymbol{B}_j^H \right\}. \tag{14}$$

To gain some insight into the covariance, we will simplify it by looking closely at the statistics of the noise matrix $\boldsymbol{\Psi}$. If we partition $\boldsymbol{\Psi}$ row by row, then under the assumption that the noise is additive white with variance σ_v^2, it can be shown that $E\{\boldsymbol{\Psi} \boldsymbol{a}_i \boldsymbol{a}_j^H \boldsymbol{\Psi}^H\} = \sigma_v^2 \boldsymbol{a}_j^H \boldsymbol{a}_i \boldsymbol{I}_{MP}$. Therefore (14) becomes

$$E\{\Delta \boldsymbol{h} \Delta \boldsymbol{h}^H\} \approx \sigma_v^2 E \left\{ \sum_{i,j=-q}^{P-1} \boldsymbol{a}_j^H \boldsymbol{a}_i \boldsymbol{B}_i \boldsymbol{B}_j^H \right\}. \tag{15}$$

Further simplification can be achieved by making an imprecise assumption. Observe that only $\boldsymbol{a}_i$ and $\boldsymbol{a}_j$ are explicitly input-dependent. If we assume $\boldsymbol{a}_j^H \boldsymbol{a}_i$ and $\boldsymbol{B}_i \boldsymbol{B}_j^H$ are approximately independent, then

$$E\{\Delta \boldsymbol{h} \Delta \boldsymbol{h}^H\} \approx \sigma_v^2 \sum_{i,j=-q}^{P-1} E\left\{\boldsymbol{a}_j^H \boldsymbol{a}_i\right\} E\{\boldsymbol{B}_i \boldsymbol{B}_j^H\}. \tag{16}$$

Though this equation is not so accurate, computer simulation can be used to quantify how good this assumption is. The remaining two expectations are dependent on the statistics of the received data vector and can be derived as follows. Recalling the definition of $\boldsymbol{a}_i$, we find

$$E\left\{\boldsymbol{a}_j^H \boldsymbol{a}_i\right\} = h_j^H E\left\{\left(\widetilde{\boldsymbol{Y}}\widetilde{\boldsymbol{Y}}^H\right)^\dagger\right\} h_i \quad (17)$$

$$= h_j^H E\left\{\left(\mathcal{G}\boldsymbol{B}\boldsymbol{B}^H\mathcal{G}^H\right)^\dagger\right\} h_i \quad (18)$$

where the final term comes from the definition of $\widetilde{\boldsymbol{Y}}$. Note that this expression has explicitly separated the signature elements in $\mathcal{G}$ from the coded source data in the $\boldsymbol{B}$ matrices. Define $\widehat{\boldsymbol{R}}_B$ as the sample correlation matrix of the spread input sequence. Then we have

$$E\left\{\boldsymbol{a}_j^H \boldsymbol{a}_i\right\} = h_j^H (\mathcal{G}^\dagger)^H E\{\widehat{\boldsymbol{R}}_B^{-1}\}\mathcal{G}^\dagger h_i. \quad (19)$$

From the statistics of input sequences, the central term reduces to [5]

$$E\{\widehat{\boldsymbol{R}}_B^{-1}\} = \frac{1}{JN\sigma_w^2\sigma_c^2}\boldsymbol{I}_{P+q} \quad (20)$$

where factor J results from the variance of a typical element in $\boldsymbol{b}(n)$ which is superimposed by J independent variables (see eq. (3)). Substituting (20) in (19) and then (19) in (16), a final form for the covariance is approximated by

$$E\{\Delta\boldsymbol{h}\Delta\boldsymbol{h}^H\} \approx \frac{\sigma_v^2}{JN\sigma_w^2\sigma_c^2}\sum_{i,j=-q}^{P-1} h_j^H(\mathcal{G}^\dagger)^H\mathcal{G}^\dagger h_i E\{\boldsymbol{B}_i\boldsymbol{B}_j^H\}. \quad (21)$$

This form has desirable characteristics as N becomes large. It also shows how the system load, the power of the noise, code, and input sequences effect the channel coefficient estimate. With more users in the system, lower noise power or higher signal power, the performance of our estimator improves.

From $E\{\Delta\boldsymbol{h}\Delta\boldsymbol{h}^H\}$ either given by (15) or approximated by (21), the total mean square error of all estimated elements in $\boldsymbol{h}$ becomes the trace of this variance

$$MSE = trace\left(E\{\Delta\boldsymbol{h}\Delta\boldsymbol{h}^H\}\right).$$

This quantity can be used to justify the performance of our channel estimator.

5 Simulations

We simulate a CDMA system and test several channel estimators based on our proposed method (solid lines), the method of Weiss and Friedlander (WF) [9] modified for multiple channels (dashed lines), and the high and low fidelity analytical approximations presented in (15) ($\circ$'s) and (21) ($\times$'s). The transmitted sequences and spreading codes are drawn from a binary constellation $[-1, 1]$ and two real subchannels ($M = 2$) of order $q = 4$ are created randomly for each simulation. Figures 1-3 are obtained based on the average of 100 Monte Carlo simulations. The mean square error is adopted as a performance metric

$$MSE_{dB} = 10\, log(\|\frac{\boldsymbol{h}}{\|\boldsymbol{h}\|} - \widehat{\boldsymbol{h}}\|^2).$$

We compare estimator convergence as a function of transmitted bit period in Fig. 1. The WF method converges quickly to a low error level, but this is achieved at the high cost of having to know all user spreading codes. It requires moderate system load as in the current situation with only 5 users. It is also observed that both of our analytical results match well with each other. The proposed method tends to approach the analytical performance as N becomes large. Fig. 2 shows the performance of the estimators across a large range of signal to noise ratios (SNRs). The proposed method performs more closely to the analytical approximation with increasing SNR. Fig. 3 shows the estimator performance under a variety of user loading conditions. The WF method suffers severely with increased number of users. However, increasing the system load can significantly improve the proposed channel estimator.

6 Conclusions

We have presented a method for blind estimation of a downlink CDMA communication channel with aperiodic spreading codes. The orthogonality of the noise subspace and virtual channel signatures proves to be a reliable basis for estimating the coefficients of the dispersive channel. The proposed method demonstrated good performance even for a heavily loaded system, agreeing with our first order perturbation analysis for large N and/or high SNR levels. It requires no knowledge of the user spreading codes and no pre-processing of the received data. Adaptive methods for tracking the noise subspace and more efficiently obtaining the channel estimate are under investigation to reduce computations.

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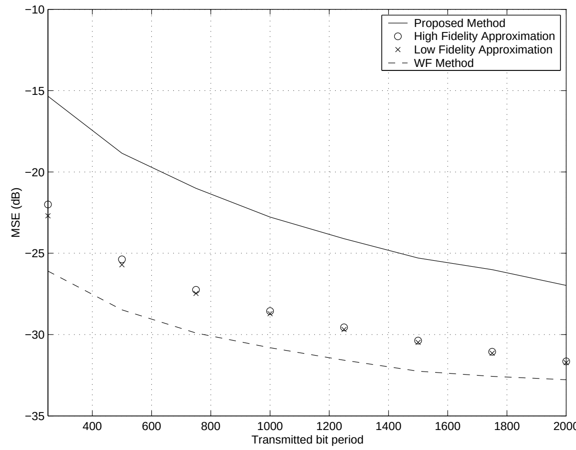

Fig. 1. Estimator convergence over time with SNR=15 dB and five users.

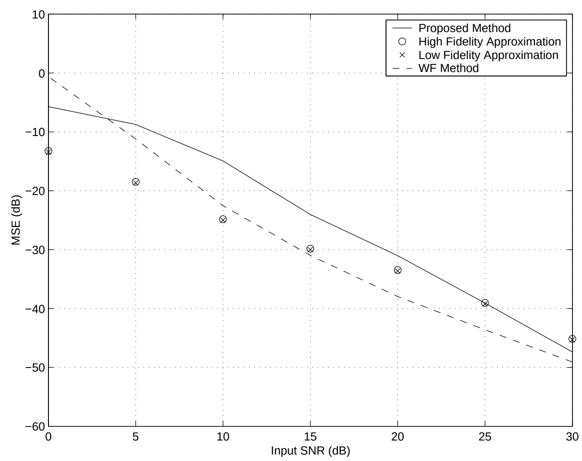

Fig. 2. Impact of SNR on estimator performance with five users after 1000 bit periods.

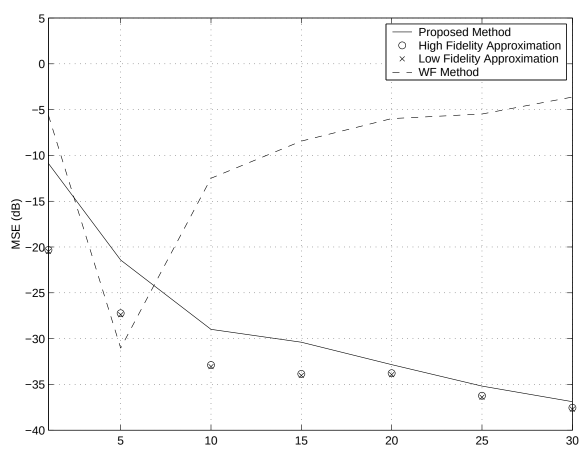

Fig. 3. Impact of system load on estimator performance with SNR=15dB after 1000 bit periods.