Blind Detection of CDMA Signals Based on Kalman Filter

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Abstract

This paper studies detection of CDMA signals based on Kalman filter. In the absence of both training symbols and the knowledge about multipath channel parameters, the system can be exactly fit into the Kalman model. By defining combination of channel and transmitted signals as state variables, the state transition matrix and measurement matrix are derived without approximations. The covariance of process noise in the proposed algorithm is recursively estimated from the estimated states, different from all existing approaches where channel parameters are assumed known or estimated by other methods. Therefore the proposed structure is much simpler and easy to implement.

1 Introduction

Kalman filter has been studied for several decades and applied to solve various problems [2]. Recently, there is increasing interest in applying it to multiuser detection. Applications are based on the fact that a CDMA system can fit the Kalman model exactly/approximately in terms of measurement equation and state transition equation. Various approaches have been proposed according to different assumptions and different assignment of state variables. When training symbols are available, the error covariance matrix required in the Kalman filter can be estimated before symbol detection [11]. If the Rayleigh fading channel is generated from a lowpass Butterworth filter with known statistics, then adaptive Bayesian detection can be performed [1]. When the fading channel follows auto-regressive (AR) model with given or estimated model parameters, the Kalman filter can also be run for user detection [3], [4]. For slowly fading channels and given channel coefficients, the Kalman filter can compete finite length minimum mean-square-error (MMSE) receiver [6], [7]. In the absence of knowledge about channel, channel parameters have to be estimated by other methods such as decision feedback estimation [5] or least mean square estimation [8].

It can be observed that most of those existing approaches require the knowledge of channel parameters. They are either assumed given or estimated from other methods in order to apply the Kalman filtering idea. In other methods where they are not required, new state vector is defined as a combination of channel and the input symbols [5], [10]. However, certain assumptions at the symbol boundaries are made to obtain the covariance of the process noise. Thus the model developed does not precisely describe the system.

In this paper, we develop a unified Kalman filter model which can accurately capture the property of the CDMA system both at the symbol boundaries and within the symbol intervals. We similarly define the state vector as in [5]. With the knowledge of timing and spreading codes for each symbol, both the measurement matrix and state transition matrix can be easily derived. Hence, application of Kalman filter requires only the covariances of process noise and measurement noise. It is found that the second order statistics of the process noise are related to those of the state vector. Therefore, instead of pursuing other methods to estimate the channel, we propose to recursively estimate the required covariance of the process noise directly from the state vector. As a common practice, the measurement noise is assumed to be additive white Gaussian (AWGN) with unknown power. However, the mismatch of noise power will not incur significant performance degradation [5] within certain range of the noise level. We thus pre-select it as a constant. Therefore, an integrated implementation of Kalman filter is achievable.

2 CDMA System Model

For simplicity of presentation, we assume a synchronous CDMA system with J users. Our discussion can be easily extended to an asynchronous system as long as arrival time instants for all users are known or have been estimated. User j ($j = 1, \cdots, J$) is assigned periodic spreading codes $c_j(k)$ ($k = 0, \ldots, P-1$) of length P to transmit P chips per information symbol. Let the chip sequence be transmitted through a linear channel with a chip rate impulse response $g_j(n)$. Then the received discrete-time signal $y(n)$ at the chip rate receiver is a superposition of the signals from all users plus noise $v(n)$ [9]

$$y(n) = \sum_{j=1}^{J} y_j(n) + v(n) \tag{1}$$

where

$$y_j(n) = \sum_{l=-\infty}^{\infty} w_j(l) h_j(n - lP),$$

$$h_j(n) = \sum_{m=-\infty}^{\infty} g_j(m) c_j(n - m)$$

$w_j(n)$ is a zero-mean, i.i.d. information bearing sequence of user j with unit power, $h_j(n)$ is its signature (the convolution of the code with the channel), and $v(n)$ is assumed to be AWGN with zero-mean and variance $\sigma_v^2 = \mathrm{E}\{\|v(n)\|^2\}$. Assume the channel $g_j(n)$ has maximum order $q \ll P$ [9]. After collecting P measurements in a vector $\boldsymbol{y}(n) = [y(nP), \cdots, y(nP + P - 1)]^T$ and defining channel vector $\boldsymbol{g}_j = [g_j(0), \cdots, g_j(q)]^T$, then

$$\boldsymbol{y}(n) = \sum_{j=1}^{J} \sum_{k=-1}^{0} \boldsymbol{C}_{j,k} \boldsymbol{g}_{j,k} w_j(n + k) + \boldsymbol{v}(n) \tag{2}$$

where

$$\boldsymbol{C}_{j,0} = \begin{bmatrix} c_j(0) & & \mathbf{0} \\ \vdots & \ddots & \\ \vdots & & c_j(0) \\ \vdots & & \vdots \\ c_j(P-1) & \cdots & c_j(P-q-1) \end{bmatrix}, \tag{3}$$

$$\boldsymbol{C}_{j,-1} = \begin{bmatrix} c_j(P-1) & \cdots & c_j(P-q) \\ & \ddots & \vdots \\ & & c_j(P-1) \\ & \mathbf{0} & \end{bmatrix} \tag{4}$$

$$\boldsymbol{g}_{j,0} = \boldsymbol{g}_j, \quad \boldsymbol{g}_{j,-1} = [g_j(1), \cdots, g_j(q)]^T. \tag{5}$$

In (2), there are contributions from not only current symbols $w_j(n)$, but also interfering symbols $w_j(n-1)$. Since we collect P samples in the data vector $\boldsymbol{y}(n)$, we have $E\{\boldsymbol{v}(n_1) \boldsymbol{v}^H(n_2)\} = \sigma_v^2 \delta(n_1 - n_2) \boldsymbol{I}_P$ which means different noise vectors are uncorrelated. All transmitted symbols together with the channel coefficients are unknown and need to be estimated. We will seek our solutions based on the Kalman filter.

3 Kalman Filter Formulation

In this section, we will formulate the detection problem into Kalman filtering.

3.1 State space representation

Since both inputs and channel coefficients are unknown, we define combined quantities as our states $\boldsymbol{x}(n) = [\boldsymbol{x}_0^T(n), \boldsymbol{x}_{-1}^T(n)]^T$ where $\boldsymbol{x}_k(n)$ $(k = 0, -1)$ is a subvector depending on the channel and the inputs at $n + k$

$$\boldsymbol{x}_k(n) = [\boldsymbol{g}_{1,k}^T w_1(n+k), \cdots, \boldsymbol{g}_{J,k}^T w_J(n+k)]^T \tag{6}$$

Then the measurement equation follows from (2)

$$\boldsymbol{y}(n) = \boldsymbol{H}\boldsymbol{x}(n) + \boldsymbol{v}(n) \tag{7}$$

where

$$\boldsymbol{H} = [\boldsymbol{C}_{1,0}, \cdots, \boldsymbol{C}_{J,0}, \boldsymbol{C}_{1,-1}, \cdots, \boldsymbol{C}_{J,-1}]$$

Under our definition for the state vector, the state transition equation can be written as

$$\boldsymbol{x}(n) = \boldsymbol{F}\boldsymbol{x}(n-1) + \boldsymbol{u}(n) \tag{8}$$

where the $J(2q+1) \times J(2q+1)$ transition matrix has the form

$$\boldsymbol{F} = \begin{bmatrix} & \mathbf{0}_{J(q+1) \times J(2q+1)} & \\ \mathbf{0}_{q \times 1} \; \boldsymbol{I}_q & & \\ & \ddots & \mathbf{0}_{Jq \times Jq} \\ & \mathbf{0}_{q \times 1} \; \boldsymbol{I}_q & \end{bmatrix}$$

and the process noise is

$$\boldsymbol{u}(n) = [\boldsymbol{x}_0^T(n), \mathbf{0}^T]^T$$

Here we take into account the symbol boundaries and the symbol intervals. This transition equation applies to all symbol intervals. No approximations have been made. This point makes our method more attractive than other approaches [5], [10]. The process noise $\boldsymbol{u}(n)$ has covariance

$$\boldsymbol{Q} = E\{\boldsymbol{u}(n)\boldsymbol{u}^H(n)\} = \begin{bmatrix} \boldsymbol{Q}_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}$$

where non-zero block $\boldsymbol{Q}_1$ is unknown instead of being assumed unitary in some work. However, $\boldsymbol{Q}_1$ can be easily observed to be related to the partial state vector by

$$\boldsymbol{Q}_1 = E\{\boldsymbol{x}_0(n)\boldsymbol{x}_0^H(n)\}$$

Therefore, we propose an iterative method to estimate $\boldsymbol{Q}_1$ at time n from $\boldsymbol{x}(n)$ and its previous value

$$\boldsymbol{Q}_1(n) = (1-\gamma)\boldsymbol{Q}_1(n-1) + \gamma \boldsymbol{x}_0(n)\boldsymbol{x}_0^H(n)$$

with forgetting factor γ, where $\boldsymbol{x}_0(n)$ is taken from the state vector $\boldsymbol{x}(n)$ based on our definition. With initialization for $\boldsymbol{x}(n)$ and pre-selected power $\widehat{\sigma}_v^2$ for the measurement noise, $\boldsymbol{x}(n)$ can be estimated from Kalman filtering. The algorithm for mean square estimation of $\boldsymbol{x}(n)$ is described as follows [2]:

Input vector process:
Observations $= \{\boldsymbol{y}(1), \boldsymbol{y}(2), \cdots, \boldsymbol{y}(N)\}$

Known parameters
State transition matrix $= \boldsymbol{F}$
Measurement matrix $= \boldsymbol{H}$
Covariance matrix of measurement noise
$= \sigma_v^2 \boldsymbol{I}$ (or priorly chosen)

Computation: $n = 1, 2, 3, \cdots$

$$\boldsymbol{K}_{n,n-1} = \boldsymbol{F}\boldsymbol{K}(n-1)\boldsymbol{F}^H + \boldsymbol{Q}(n-1)$$

$$\boldsymbol{G}(n) = \boldsymbol{K}_{n,n-1}\boldsymbol{H}^H[\boldsymbol{H}\boldsymbol{K}_{n,n-1}\boldsymbol{H}^H + \sigma_v^2\boldsymbol{I}]^{-1}$$

$$\boldsymbol{K}(n) = \Big[\boldsymbol{I} - \boldsymbol{G}(n)\boldsymbol{H}\Big]\boldsymbol{K}_{n,n-1}$$

$$\boldsymbol{\alpha}(n) = \boldsymbol{y}(n) - \boldsymbol{H}\boldsymbol{F}\widehat{\boldsymbol{x}}(n-1)$$

$$\widehat{\boldsymbol{x}}(n) = \boldsymbol{F}\widehat{\boldsymbol{x}}(n-1) + \boldsymbol{G}(n)\boldsymbol{\alpha}(n)$$

$$\widehat{\boldsymbol{x}}_0(n) = \Big[\boldsymbol{I} \;\; \boldsymbol{0}\Big]\widehat{\boldsymbol{x}}(n)$$

$$\boldsymbol{Q}_1(n) = (1-\gamma)\boldsymbol{Q}_1(n-1) + \gamma\widehat{\boldsymbol{x}}_0(n)\widehat{\boldsymbol{x}}_0^H(n)$$

$$\boldsymbol{Q}(n) = \begin{bmatrix} \boldsymbol{Q}_1(n) & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} \end{bmatrix}$$

Initial conditions:

$$\widehat{\boldsymbol{x}}(0) = \boldsymbol{0}$$

$$\boldsymbol{Q}_1(0) = \boldsymbol{I}$$

$$\boldsymbol{K}(0) = \boldsymbol{I}$$

$$0 < \gamma < 1$$

In the previous steps, $\boldsymbol{G}(n)$ is the Kalman gain, $\boldsymbol{K}(n-1)$, $\boldsymbol{K}_{n,n-1}$ and $\boldsymbol{K}(n)$ are the previous *a posteriori* covariance, the *a priori* covariance, and the *a posteriori* covariance matrices of the estimated state vector respectively. $\widehat{\boldsymbol{x}}_0(n)$ is a subvector of $\widehat{\boldsymbol{x}}(n)$ obtained by a selection operation. It is used to update the covariance of the process noise.

In order to overcome a possible problem that the matrix $\boldsymbol{K}(n)$ loses symmetry or positive definiteness, another form to update $\boldsymbol{K}(n)$ can be adopted as a substitute [2]

$$\boldsymbol{K}(n) = \Big[\boldsymbol{I} - \boldsymbol{G}(n)\boldsymbol{H}\Big]\boldsymbol{K}_{n,n-1}\Big[\boldsymbol{I} - \boldsymbol{G}(n)\boldsymbol{H}\Big]^H + \sigma_v^2\boldsymbol{G}(n)\boldsymbol{G}^H(n)$$

The forgetting factor γ in estimating the covariance of the process noise will affect the number of state vectors effective in the estimation. It will also affect the convergence rate and error level. Its choice will be studied by simulation examples. Once the state vector is estimated at each time interval, $\widehat{\boldsymbol{x}}_0(n)$ which includes channel vectors and inputs can be used for either channel or symbol estimation (see (6)). Its j-th entry serves as an estimate for $\boldsymbol{g}_j w_j(n)$. If the input power is unknown, each channel vector can be estimated up to a scalar ambiguity (including amplitude and phase). On the other hand, if the phase of any element in the channel vector is known, then the input can be estimated. In the case of differential encoding, the information symbol can be decoded from two successive state vectors.

3.2 Observability

The observability of the state defined as combination of inputs and channel parameters depends on the rank of the observability matrix for our given linear system model

$$\boldsymbol{O} = \begin{bmatrix} \boldsymbol{H} \\ \boldsymbol{H}\boldsymbol{F} \\ \vdots \\ \boldsymbol{H}\boldsymbol{F}^{\nu-1} \end{bmatrix} \tag{9}$$

where ν is the dimension of $\boldsymbol{x}(n)$ and equals $J(2q+1)$. The measurement matrix $\boldsymbol{H}$ has dimension $P \times \nu$. If different users are assigned different spreading codes, it is reasonable to argue that any of its sub-matrix of $P \times P$ has full rank since we exclude $g_j(0)w_j(n-1)$ from our state vector to avoid a zero column in $\boldsymbol{C}_{j,-1}$. Meanwhile, due to the particular structure of the transition matrix with Jq non-zero columns, post-multiplying $\boldsymbol{H}$ by $\boldsymbol{F}$ will shift some columns in $\boldsymbol{H}$ to the left. The resulting matrix will have Jq non-zero sub-vectors while all others are zero sub-vectors. These Jq non-zero sub-vectors are appended beneath $\boldsymbol{H}$. If we define $\boldsymbol{O}_s$ as a sub-matrix of $\boldsymbol{O}$ with only first two block entries $\boldsymbol{H}$ and $\boldsymbol{H}\boldsymbol{F}$, then $\boldsymbol{O}_s$ is $2P \times \nu$. Under our assumption that $P \geq J(q+1)$, it has full column rank. The rank of $\boldsymbol{O}$ does not alter with other entries $\boldsymbol{H}\boldsymbol{F}^i$ (for $i > 1$) appended. Therefore, $\boldsymbol{O}$ is of full column rank.

3.3 Steady state solution

Kalman filter provides recursive estimate of the state vector. It is interesting to investigate its stability and convergence. In the current system, the transition matrix $\boldsymbol{F}$ is asymptotically stable since all its eigenvalues lie within the unit circle. Therefore, given any positive semidefinite symmetric initial condition $\boldsymbol{K}(0)$, $\lim_{n\to\infty} \boldsymbol{K}_{n,n-1} = \boldsymbol{S}$,

$\lim_{n\to\infty} \boldsymbol{K}(n-1) = \lim_{n\to\infty} \boldsymbol{K}(n) = \boldsymbol{K}, \lim_{n\to\infty} \boldsymbol{G}(n) = \boldsymbol{G}$, where $\boldsymbol{S}$, $\boldsymbol{K}$ and $\boldsymbol{G}$ satisfy the following equations [6]

$$\boldsymbol{S} = \boldsymbol{F}\boldsymbol{K}\boldsymbol{F}^H + \boldsymbol{Q} \tag{10}$$

$$\boldsymbol{G} = \boldsymbol{S}\boldsymbol{H}^H[\boldsymbol{H}\boldsymbol{S}\boldsymbol{H}^H + \sigma_v^2\boldsymbol{I}]^{-1} \tag{11}$$

$$\boldsymbol{K} = \Big[\boldsymbol{I} - \boldsymbol{G}\boldsymbol{H}\Big]\boldsymbol{S} \tag{12}$$

However, it is not trival to solve either $\boldsymbol{G}$ or $\boldsymbol{K}$ from (10)-(12) due to their high non-linarity. Based on our simulation results, it is observed that $\boldsymbol{G}$ converges very fast. In such a case, one may fix it after some iterations without further update, in order to reduce computations.

4 Simulations

In this section, some simulation results are presented to demonstrate the applicability of the proposed method. A DS/CDMA system is simulated with Gold sequence of length 31 as spreading codes. The average mean-square-error (MSE) of estimate for the state vector from 500 realizations is adopted for performance evaluation. Without further definitions, 4 users, channel length 4 and input SNR $20dB$ are used as default conditions. Each channel coefficient is randomly selected from the interval $[-1, 1]$ in each realization. The channel vector is then normalized. Fig. 1 investigates the impact of the forgetting factor γ. As analyzed in previous part, it is shown in our simulation that the estimated state covariances will reach a certain steady level after some iterations. The thick solid line represents the ideal case when channels are assumed known. The other lines indicate cases when channel vectors are to be estimated. It can be observed that the estimation error decreases with the increasing of γ, γ = 0.2, 0.4, 0.6 and 0.8, respectively. This comes with the expense of an accordingly slower converging speed. To achieve an appropriate balance between a fast converging speed and a relatively low estimation error, we choose $\gamma = 0.5$ in the following simulations.

Fig. 2 shows the performance with different number of users, varying from 2 to 8. Fig. 3 displays the performance under different channel orders from 2 to 7. It is shown that when other statistics remain the same, the smaller the user number or channel order is, the better the performance. This is consistent with our prediction, since both the increase of user number and channel order result in an expansion of the state vector size, and consequently a decline on the estimation accuracy.

Fig. 4 is the bit error rate (BER) curve based on 10000 bits under varying input SNR. BER starts falling rapidly after SNR goes above $5dB$ and reaches a satisfactorily low level for 10dB and higher SNR. Since in practice, the power of measurement noise is often not available, we investigate the sensitivity of performance to such mismatch in Fig. 5 for a large range of SNR from $0dB$ to $40dB$. The thick line represents the result when the noise power is assumed known. Other lines indicate the performance when the input SNRs are assumed to be $10dB$, $20dB$, $30dB$ and $40dB$ respectively. We can see that the performance deviation from the ideal result is rather small when the estimated input SNR is between its actual value and $10dB$ above. It is thus advisable to select SNR in the proposed method to be slightly higher than its actual value, preferably less than $10dB$ difference. The performance is relatively insensitive to SNR mismatch within certain range.

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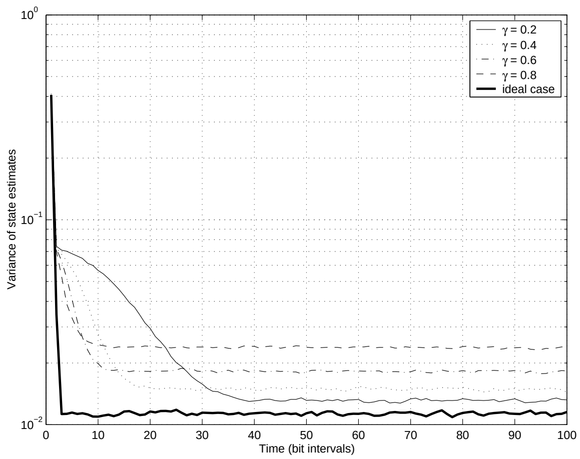

Fig. 1. Performance comparision with estimated and true covariance of the process noise.

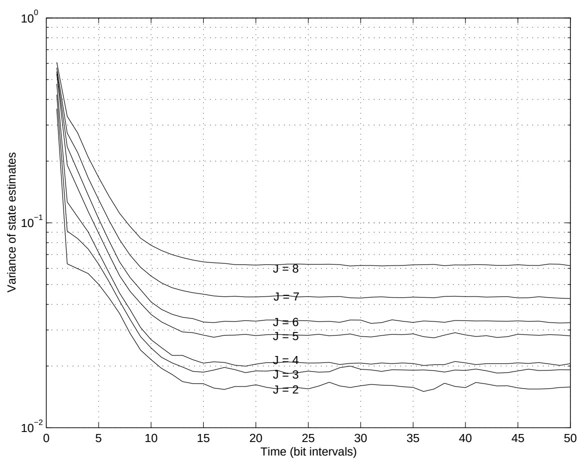

Fig. 2. Performance with different user numbers.

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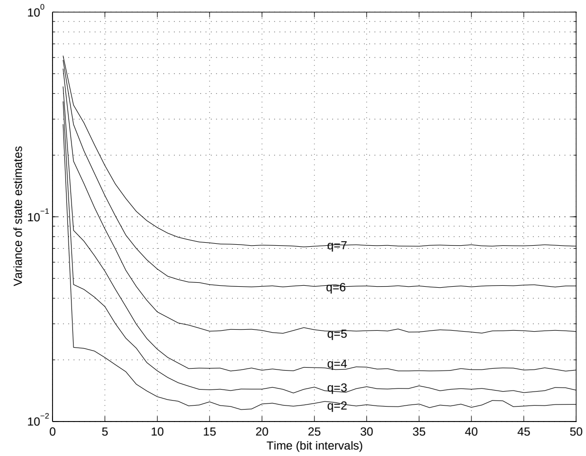

Fig. 3. Performance with different channel orders.

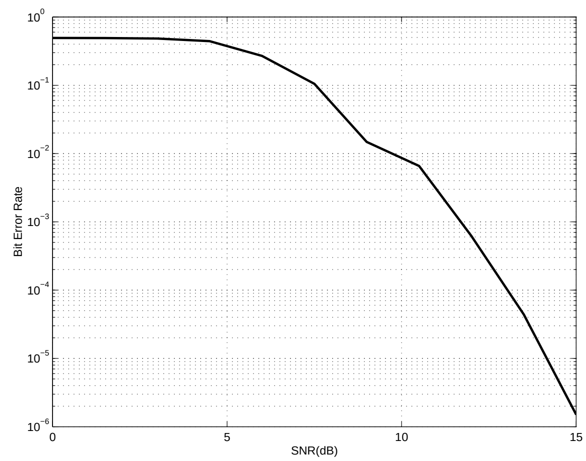

Fig. 4. Bit Error Rate (BER) with varying input SNR.

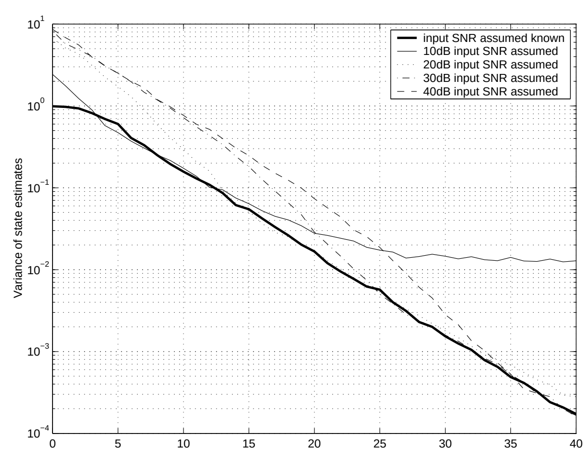

Fig. 5. Effect of SNR mismatch on state estimation.