Towards Closing the Gap Between MOE and Subspace Methods

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Abstract

In this paper, we propose a Power of R (POR) technique to significantly improve the performance of the MOE receiver. The new receiver is shown to asymptotically converge to the MMSE receiver. The convergence is established either under high SNR, or with large exponent raised in the power of the covariance matrix. Connection between our POR method and a subspace method is investigated, and their asymptotic equivalence is also established. However, the POR receiver does not require either a computationally expensive subspace decomposition of a data covariance matrix, or estimation of dimension of either the signal subspace or the noise subspace.

1 Introduction

The blind minimum output energy (MOE) method has been applied to multiuser detection in direct-sequence (DS) code-division multiple access (CDMA) systems. Without multipath fading, by using the code waveform, the MOE receiver provides a direct MMSE detection solution [2]. If multipath is present and unknown, a bank of MOE receivers with different pre-selected constraints are used to obtain the outputs, which are then combined according to some criteria such as maximum ratio combining [5], [7]. To achieve optimality for the receiver, [8] parameterizes the constraints and further optimizes them. It is shown that the method not only provides a multiuser receiver, but also a blind channel estimator. Subsequently developed robust method [10] provides direct adaptive receiver design techniques. The MOE idea has also been successfully applied to a space-time coded CDMA system [3].

However, because of AWGN, it is shown in [8] that the optimal constraint vector is a biased estimate of the channel vector. The channel estimation error induces some loss in the performance of the MOE receiver when compared with the ideal MMSE receiver. To alleviate the noise effect, some modifications are suggested in [11] and [12]. Though performance improvement is guaranteed, residual error still exists in [11]. If noise power is not perfectly known, [12] can only perform subtraction in an ad-hoc manner by properly choosing a close candidate from its estimate.

In this paper, we propose a power of R (POR) technique to raise the power of the data covariance matrix $\boldsymbol{R}$ to a positive integer m in the MOE cost function to alleviate the noise effect. According to our analysis based on a perturbation technique, the channel estimation error decreases as m increases. When $m \to \infty$, the proposed method is equivalent to the subspace method. It is also shown that the proposed MMSE receiver built on the estimated channel parameters when $m \geq 2$ asymptotically converges to the optimal MMSE receiver in terms of the output signal to interference plus noise ratio (SINR), eliminating a penalty in the MOE method [8] where $m = 1$.

2 System Model and MOE Receiver

Consider a CDMA system with J users. User j ($j = 1, \cdots, J$) is assigned spreading codes $c_j(k)$ ($k = 0, \ldots, P-1$) of length P to spread its information symbol $w_j(n)$. Let the chip sequence be transmitted through a linear channel with a baseband chip rate discrete-time impulse response $g_j(n)$. Then the received discrete-time signal at the chip-rate receiver is

$$y(n) = \sum_{j=1}^{J} \sum_{l=-\infty}^{\infty} w_j(l) h_j(n - d_j - lP) + v(n). \qquad (1)$$

where $w_j(n)$ is assumed to have zero-mean and variance $\sigma_{w_j}^2$, $h_j(n) = \sum_{m=-\infty}^{\infty} g_j(m) c_j(n-m)$ is a waveform sequence, d_j is the delay of user j in chip periods.

The discrete-time model can be easily formulated into a matrix/vector representation. Assume the receiver is synchronized to our desired user - user 1 ($d_1 = 0$). All channels $g_j(n)$ have maximum order $q \ll P$. After collecting measurements corresponding to ν symbol periods in a vector $\boldsymbol{y}(n)$, then

$$\boldsymbol{y}(n) = \boldsymbol{h}_1 w_1(n + \nu_0) + \boldsymbol{H}_{int} \boldsymbol{w}_{int}(n) + \boldsymbol{v}(n) \qquad (2)$$

where $\boldsymbol{h}_1 \triangleq \mathcal{C}_1 \boldsymbol{g}_1$ is the signature vector of symbol $w_1(n + \nu_0)$ ($0 \leq \nu_0 \leq \nu$), $\mathcal{C}_1$ is the code filtering matrix whose first $\nu_0 P$ rows are all zeros, $\boldsymbol{g}_1$ is the channel vector, $\boldsymbol{w}_{int}(n)$ is the interference vector including ISI and MUI, $\boldsymbol{H}_{int}$ is the corresponding signature matrix and $\boldsymbol{v}(n)$ is the noise component. This structure of the desired user's signature waveform has been exploited in [8] to design an MOE receiver when $\boldsymbol{g}_1$ is unknown.

The MOE receiver $\boldsymbol{f}$ and the constraint vector $\boldsymbol{g}$ are obtained by the following max/min criterion [8]

$$\max_{\boldsymbol{g}} \min_{\boldsymbol{f}} \mathcal{J} = \boldsymbol{f}^H \boldsymbol{R} \boldsymbol{f}, \text{ subject to } \mathcal{C}_1^H \boldsymbol{f} = \boldsymbol{g} \quad (3)$$

where $\boldsymbol{R} = \mathrm{E}\{\boldsymbol{y}(n)\boldsymbol{y}^H(n)\}$ is the data covariance matrix. The above criterion is equivalent to optimizing $\boldsymbol{g}$ as [8],

$$\boldsymbol{g}_{moe} = \arg \min_{\|\boldsymbol{g}\|=1} \boldsymbol{g}^H \boldsymbol{B} \boldsymbol{g}, \;\; \boldsymbol{B} = \mathcal{C}_1^H \boldsymbol{R}^{-1} \mathcal{C}_1. \quad (4)$$

and then constructing the receiver as $\boldsymbol{f}_{moe} = \boldsymbol{R}^{-1}\mathcal{C}_1 \boldsymbol{g}_{moe}$ after ignoring a scalar. By (4), the optimal constraint vector $\boldsymbol{g}_{moe}$ is the eigenvector of $\boldsymbol{B}$ associated with its minimum eigenvalue.

This max/min blind approach exhibits near-optimal performance [8] compared with the MMSE receiver because $\boldsymbol{g}_{moe}$ is a biased channel estimator in the presence of noise. If the constraint vector approaches $\boldsymbol{g}_1$, then the MOE receiver conforms to the MMSE receiver. Since the subspace method can provide an accurate estimate of the channel, we will propose a novel POR technique to bridge the gap between the MOE and subspace methods.

3 Proposed POR Approach

In order to develop an improved receiver, we start from a key result on the channel estimation error from [8]. If $\boldsymbol{R}$ is decomposed by EVD as

$$\boldsymbol{R} = \begin{bmatrix} \boldsymbol{U}_s & \boldsymbol{U}_n \end{bmatrix} \begin{bmatrix} \boldsymbol{\Lambda}_s + \sigma_v^2 \boldsymbol{I} & \boldsymbol{0} \\ \boldsymbol{0} & \sigma_v^2 \boldsymbol{I} \end{bmatrix} \begin{bmatrix} \boldsymbol{U}_s^H \\ \boldsymbol{U}_n^H \end{bmatrix} \quad (5)$$

where $\boldsymbol{\Lambda}_s = diag\{\lambda_1^2, \ldots, \lambda_\xi^2\}$, $\boldsymbol{U}_s$ and $\boldsymbol{U}_n$ represent the signal and noise subspaces respectively, and if $\boldsymbol{B}$ is expanded as a power series of small noise power σ_v^2 (see eqs. (35) and (48) in [8]), then the MOE channel estimation error becomes [8]

$$\boldsymbol{g}_{moe} - \frac{\boldsymbol{g}_1}{\|\boldsymbol{g}_1\|} \approx -\sigma_v^2 \boldsymbol{A}_0^\dagger \boldsymbol{A}_1 \frac{\boldsymbol{g}_1}{\|\boldsymbol{g}_1\|} \quad (6)$$

where † represents pseudo-inverse,

$$\boldsymbol{A}_0 = \mathcal{C}_1^H \boldsymbol{U}_n \boldsymbol{U}_n^H \mathcal{C}_1, \quad \boldsymbol{A}_1 = \mathcal{C}_1^H \boldsymbol{U}_s \boldsymbol{\Lambda}_s^{-1} \boldsymbol{U}_s^H \mathcal{C}_1.$$

It is observed that this error is proportional to σ_v^2 which is the minimum eigenvalue of $\boldsymbol{R}$. Since $\boldsymbol{R}$ is a key element in $\boldsymbol{B}$ which affects $\boldsymbol{g}_{moe}$, if we raise the power of $\boldsymbol{R}$ in the MOE cost function (4), then the minimum eigenvalue of the resulting matrix will decrease exponentially. Therefore, the channel estimation error is expected to decrease as well. Following this direction, we thus modify the MOE criterion (4) and consider the following POR cost function to find a better constraint vector

$$\boldsymbol{g}_{por} = \arg \min_{\|\boldsymbol{g}\|=1} \boldsymbol{g}^H \boldsymbol{A} \boldsymbol{g}, \;\; \boldsymbol{A} = \mathcal{C}_1^H \boldsymbol{R}^{-m} \mathcal{C}_1 \quad (7)$$

where m is a positive integer. Matrix $\boldsymbol{R}$ in $\boldsymbol{A}$ has been raised to a power m if compared with $\boldsymbol{R}$ in $\boldsymbol{B}$. Once $\boldsymbol{g}_{por}$ is obtained, the proposed receiver takes an MMSE-like form

$$\boldsymbol{f}_{por} = \boldsymbol{R}^{-1} \mathcal{C}_1 \boldsymbol{g}_{por}. \quad (8)$$

Although performance improvement in channel estimation of this POR method has been intuitively illustrated, it still needs theoretical justification. Meanwhile, the performance of the corresponding receiver built upon this constraint vector should be investigated as well.

4 Performance Analysis

Since performance of the POR method is affected by the additive noise (σ_v^2), the power parameter m, and finite data length, the quantitative effects of these elements on the performance will be evaluated in this section. Due to lack of space, their proofs are omitted.

4.1 Effect of noise

We begin with our channel estimator obtained from (7). $\boldsymbol{g}_{por}$ is the eigenvector of $\boldsymbol{A}$ corresponding to its minimum eigenvalue γ_{por}. A perturbation by noise in $\boldsymbol{R}$ causes a perturbation in $\boldsymbol{A}$. Thus a perturbation in $\boldsymbol{g}_{por}$ occurs. For convenience, assume $\|\boldsymbol{g}_1\| = 1$. To facilitate analysis, we first establish the following lemma.

Lemma 1: Under a small noise assumption, $\boldsymbol{A}$ can be expressed as a power series of σ_v^2 as follows

$$\sigma_v^{2m} \boldsymbol{A} = \boldsymbol{A}_0 + \sigma_v^{2m} \boldsymbol{A}_m - \sigma_v^{2m+2} m \boldsymbol{A}_{m+1} + \boldsymbol{O}(\sigma_v^{2m+4}) \quad (9)$$

where

$$\boldsymbol{A}_i = \mathcal{C}_1^H \boldsymbol{U}_s \boldsymbol{\Lambda}_s^{-i} \boldsymbol{U}_s \mathcal{C}_1, \;\; i = 1, \cdots, m, m+1, \cdots.$$

Based on *Lemma 1*, channel estimation error perturbed by a small noise σ_v^2 can be evaluated.

Lemma 2: Under assumption that $[\mathcal{C}_1, \boldsymbol{H}_{int}]$ is of full column rank and for small σ_v^2, the POR method with $m > 1$ has channel estimation error given by

$$\Delta \boldsymbol{g} = \boldsymbol{g}_{por} - \boldsymbol{g}_1 = -\sigma_v^{2m} \boldsymbol{A}_0^\dagger \boldsymbol{A}_m \boldsymbol{g}_1 + \boldsymbol{o}(\sigma_v^{2m+2}). \quad (10)$$

To quantify the performance of a receiver $\boldsymbol{f}$, we adopt its output signal to interference plus noise ratio (SINR) which is defined as $\mathrm{SINR} = \frac{\sigma_{w_1}^2 |\boldsymbol{f}^H \boldsymbol{h}_1|^2}{\boldsymbol{f}^H \boldsymbol{R} \boldsymbol{f} - \sigma_{w_1}^2 |\boldsymbol{f}^H \boldsymbol{h}_1|^2}$. The perturbation of SINR of the POR receiver is then compared with that of the ideal MMSE receiver in the following lemma.

Lemma 3: For small σ_v^2, the SINRs of the MMSE receiver and POR receiver are both given by

$$\frac{1 + O(\sigma_v^2)}{\sigma_v^2 \sigma_{w_1}^2 \boldsymbol{g}_1^H \boldsymbol{A}_2 \boldsymbol{g}_1 + O(\sigma_v^4)}.$$

According to *Lemma 2* and *Lemma 3*, the following theorem can be established after taking the limit $\sigma_v^2 \to 0$.

Theorem 1: As $\sigma_v^2 \to 0$, the following holds for any fixed m and $m \geq 2$,

$$\boldsymbol{g}_{por} \to \boldsymbol{g}_1, \quad \frac{\mathrm{SINR}_{mmse}}{\mathrm{SINR}_{por}} \to 1. \tag{11}$$

□

It is clear that the channel estimate asymptotically converges to the true channel vector. The output SINR of the POR receiver asymptotically converges to that of the ideal MMSE receiver, making the proposed receiver significantly different from the MOE receiver where a penalty exists.

4.2 Effect of parameter m

The POR method depends on the parameter m. A theorem similar to *Theorem 1* can be proved.

Theorem 2: As $m \to \infty$, (11) also holds for any fixed small σ_v^2 (high SNR).

4.3 Effect of finite number of data samples

As observed from (7) and (8), both the estimated channel vector $\boldsymbol{g}_{por}$ and the POR receiver $\boldsymbol{f}_{por}$ depend on $\boldsymbol{R}$. They will be perturbed if $\boldsymbol{R}$ is perturbed. We will denote a perturbation by preceding the corresponding quantity by δ, and perturbed quantity with $\tilde{}$. For example, $\delta\boldsymbol{R} = \widetilde{\boldsymbol{R}} - \boldsymbol{R}$, where $\widetilde{\boldsymbol{R}} = \frac{1}{N}\sum_{n=1}^{N} \boldsymbol{y}(n)\boldsymbol{y}^H(n)$. Although $\widetilde{\boldsymbol{R}}$ converges to $\boldsymbol{R}$ as $N \to \infty$, a perturbation $\delta\boldsymbol{R}$ due to finite N will cause $\boldsymbol{A}$ perturbed and finally POR channel estimator and receiver perturbed. Here, we are interested in a perturbed eigenvector of $\boldsymbol{A}$ after $\boldsymbol{R}$ is perturbed, and investigate how N affects the performance of the channel estimator and POR receiver.

Lemma 4: For large N, the covariance of the perturbation of channel estimation becomes

$$\mathrm{Cov}(\delta\boldsymbol{g}) \approx \sum_{k_1,k_2=1}^{m} \boldsymbol{T}_{k_1} E\{\delta\boldsymbol{R}\boldsymbol{t}_{k_1}\boldsymbol{t}_{k_2}^H \delta\boldsymbol{R}\}\boldsymbol{T}_{k_2}^H \tag{12}$$

where $\boldsymbol{T}_k$ and $\boldsymbol{t}_k$ are deterministic quantities

$$\boldsymbol{T}_k = (\boldsymbol{A} - \gamma_{por}\boldsymbol{I})^{\dagger}\mathcal{C}_1^H \boldsymbol{R}^{-k}, \quad \boldsymbol{t}_k = \boldsymbol{R}^{-(m-k)}\boldsymbol{f}_{por}.$$

The mean-square-error is then equal to the trace of $\mathrm{Cov}(\delta\boldsymbol{g})$. For a real system, the general term $\boldsymbol{\Psi} = E\{\delta\boldsymbol{R}\boldsymbol{D}\delta\boldsymbol{R}\}$ for any deterministic $\boldsymbol{D}$ is given by [13]

$$\boldsymbol{\Psi} = \frac{\kappa_{4w}}{N}\boldsymbol{H}[\boldsymbol{I}\odot(\boldsymbol{H}^T\boldsymbol{D}\boldsymbol{H})]\boldsymbol{H}^T + \frac{tr(\boldsymbol{R}\boldsymbol{D})}{N}\boldsymbol{R} + \frac{1}{N}\boldsymbol{R}\boldsymbol{D}^T\boldsymbol{R} \tag{13}$$

where κ_{4w} denotes the fourth order cumulant, $\odot$ represents element-wise multiplication and $\boldsymbol{H} \triangleq [\boldsymbol{h}_1, \boldsymbol{H}_{int}]$. For a complex system, it becomes

$$\boldsymbol{\Psi} = \frac{\kappa_{4w}}{N}\boldsymbol{H}[\boldsymbol{I}\odot(\boldsymbol{H}^H\boldsymbol{D}\boldsymbol{H})]\boldsymbol{H}^H + \frac{1}{N}tr(\boldsymbol{R}\boldsymbol{D})\boldsymbol{R}. \tag{14}$$

Combining *Lemma 2* and *Lemma 4*, we further obtain the channel estimation error perturbed by both noise and finite data symbols.

Lemma 5: For small σ_v^2 and large N, channel estimation error $E\{||\widetilde{\boldsymbol{g}} - \boldsymbol{g}_1||^2\}$ is given by $||\boldsymbol{g}_{por} - \boldsymbol{g}_1||^2 + E\{||\widetilde{\boldsymbol{g}} - \boldsymbol{g}_{por}||^2\}$ where the first term is obtained from (10) and the second term by the trace of (12).

The perturbation of $\boldsymbol{R}$ causes $\delta\boldsymbol{f}$ to $\boldsymbol{f}_{por}$ and then finally the perturbation of $SINR$, which is obtained in the following lemma.

Lemma 6: For large N, the perturbation of SINR is approximated by

$$\widetilde{\mathrm{SINR}} \approx \sigma_{w_1}^2 \frac{||\boldsymbol{f}_{por}^H\boldsymbol{h}_1||^2 + E\{\delta\boldsymbol{f}^H\boldsymbol{h}_1\boldsymbol{h}_1^H\delta\boldsymbol{f}\}}{\boldsymbol{f}_{por}^H\boldsymbol{R}_i\boldsymbol{f}_{por} + E\{\delta\boldsymbol{f}^H\boldsymbol{R}_i\delta\boldsymbol{f}\}} \tag{15}$$

where $\boldsymbol{R}_i = \boldsymbol{R} - \sigma_{w_1}^2\boldsymbol{h}_1\boldsymbol{h}_1^H$, expected values can be evaluated from a general form $E\{\delta\boldsymbol{f}^H\boldsymbol{X}\delta\boldsymbol{f}\}$ with $\boldsymbol{X}$ replaced by $\boldsymbol{h}_1\boldsymbol{h}_1^H$ or $\boldsymbol{R}_i$

$$\begin{aligned} E\{\delta\boldsymbol{f}^H\boldsymbol{X}\delta\boldsymbol{f}\} \approx & \sum_{k_1,k_2} \boldsymbol{t}_{k_1}^H E\{\delta\boldsymbol{R}\boldsymbol{Q}_{k_1}^H\boldsymbol{X}\boldsymbol{Q}_{k_2}\delta\boldsymbol{R}\}\boldsymbol{t}_{k_2} \\ & + \boldsymbol{f}_{por}^H E\{\delta\boldsymbol{R}\boldsymbol{R}^{-1}\boldsymbol{X}\boldsymbol{R}^{-1}\delta\boldsymbol{R}\}\boldsymbol{f}_{por} \\ & - \sum_{k} \boldsymbol{t}_k^H E\{\delta\boldsymbol{R}\boldsymbol{Q}_k^H\boldsymbol{X}\boldsymbol{R}^{-1}\delta\boldsymbol{R}\}\boldsymbol{f}_{por} \\ & - \boldsymbol{f}_{por}^H \sum_{k} E\{\delta\boldsymbol{R}\boldsymbol{R}^{-1}\boldsymbol{X}\boldsymbol{Q}_k\delta\boldsymbol{R}\}\boldsymbol{t}_k, \end{aligned}$$

and $\boldsymbol{Q}_k = \boldsymbol{R}^{-1}\mathcal{C}_1\boldsymbol{T}_k$.

5 Connections with the Subspace Method

The subspace channel estimation method is based on minimizing $\boldsymbol{g}^H\boldsymbol{A}_0\boldsymbol{g}$ with a norm constraint [9]. Under some identifiability conditions, unique solution up to a scalar is ensureed. According to *Lemma 1*, the difference between the proposed and subspace criteria lies in the fact that the proposed cost function includes several terms affected by the noise power. However, as $m \to \infty$, the proposed cost function converges to the subspace one, closing the gap between these two methods.

Since $\boldsymbol{A}_0$ involves the noise subspace $\boldsymbol{U}_n$, it is usually obtained in the subspace method from EVD of the sample covariance matrix $\widetilde{\boldsymbol{R}}$. Due to effects of noise and finite samples, the dimension of the noise subspace has to be first determined by some detection techniques such as Akaike's information theoretic criterion (AIC) [1], minimum description length (MDL) [6] or fixed ratio detection. Those techniques highly depend on the SNR and number of available samples [4]. Their performance relies heavily on the distribution of the small eigenvalues of $\widetilde{\boldsymbol{R}}$ (not $\boldsymbol{R}$) and is not always satisfactory. If the dimension is correctly estimated, then the subspace method is one of the best second-order based approaches. However, the proposed approach avoids estimating that dimension directly. Instead, it approximates the noise subspace of $\boldsymbol{R}$ based on $\boldsymbol{R}^{-m}$. Accordingly,

$$\sigma_v^{2m}\boldsymbol{R}^{-m} = \boldsymbol{U}_n\boldsymbol{U}_n^H + \boldsymbol{U}_s diag\{(\frac{\sigma_v^2}{\lambda_i^2+\sigma_v^2})^m\}\boldsymbol{U}_s^H.$$

Clearly, each null basis has a unity weight, while the i-th signal basis has a weight of $(\frac{\sigma_v^2}{\lambda_i^2+\sigma_v^2})^m$, which is less than 1 irrespective of the noise power. $(\frac{\sigma_v^2}{\lambda_i^2+\sigma_v^2})^m$ can converge to zero for sufficiently large m. Theoretically, $\lim_{m\to\infty} \sigma_v^{2m}$ $\boldsymbol{R}^{-m}$=$\boldsymbol{U}_n\boldsymbol{U}_n^H$, irrespective of the noise power. Therefore, estimating the noise subspace from $\boldsymbol{R}^{-m}$ has an obvious advantage. It eliminates a need to obtain the corresponding dimension and thus mitigates the effects of noise and finite samples by certain degree.

6 Simulation Results

We consider a CDMA system, where each user in the system transmits BPSK signals through an individual channel of four equally powered Gaussian paths. Each user's spreading sequence, channel and delay are randomly generated in each of 100 Monte Carlo realizations except that the desired user is synchronized. $\nu = 5$ and $\nu_0 = 3$ are set for all simulations. We first demonstrate effects of both m and σ_v^2. Fig. 1 and Fig. 2 plot normalized mean squared error (NMSE) and SINR in the case of $N = \infty$ respectively. As expected, at each SNR, NMSE monotonically decreases as m increases. On the other hand, NMSEs of the proposed method converge very well to their analytical values at high SNRs. In Fig. 2, the SINR ratios for the receivers with $m > 1$ all converge to 1 as $SNR \to \infty$. For a fixed SNR, convergence is also observed as m increases. These observations justify our *Theorem 1* and *Theorem 2*. By contrast, the SINR of the receiver with $m = 1$ (the MOE receiver) converges to a constant smaller than 1 [8]. We then study the effect of finite data length with NMSE presented in Fig. 3 and SINR in Fig. 4. Clearly, the experimental results all converge to their corresponding analytical values when N is large. Moreover, the SINRs of the POR receiver with $m > 1$ also converges to that of the ideal MMSE receiver. We also compare the POR method with the MOE method [8], and the subspace method [9] in different loading situations. The subspace method using AIC and MDL criteria for rank estimation are termed correspondingly as subspace (AIC) and subspace (MDL). 500 symbols are used to obtain the receivers for all methods and independent 6000 symbols are used to calculate BERs, which are plotted in Fig. 5. The superiority of the POR receivers to the MOE receiver can be observed for all loading cases. When there are fewer than 8 users in the system, only slight differences are found among the proposed receivers and the subspace based receivers. As the number of active users increases, the identifiability is hard to satisfy for all receivers. Although performance of all approaches degrades apparently, the proposed POR receivers with both $m = 2$ and $m = 4$ exhibit much slower degradation rates than all other receivers.

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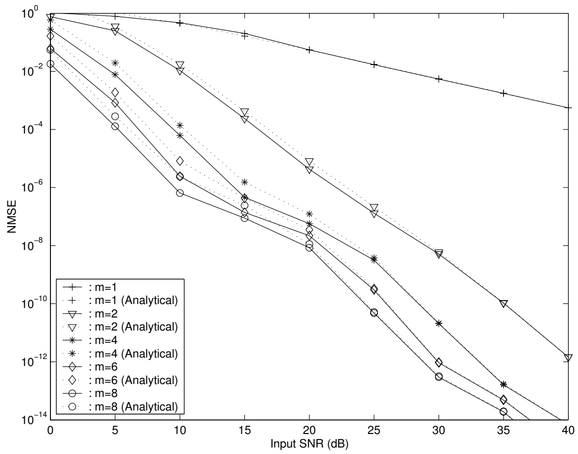

Fig. 1. NMSE v.s. SNR ($P = 8$, $J = 3$).

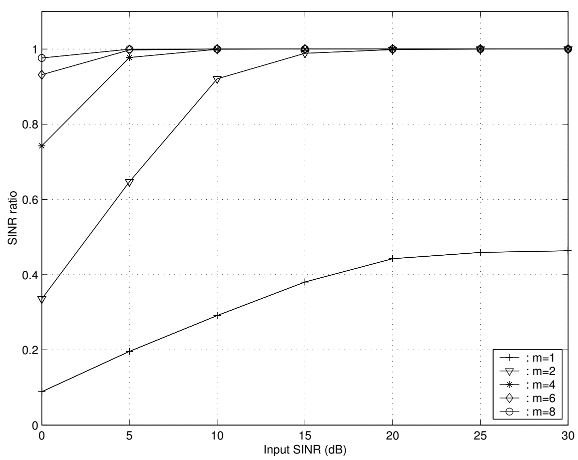

Fig. 2. SINR ratios v.s. SNR ($P = 8$, $J = 3$).

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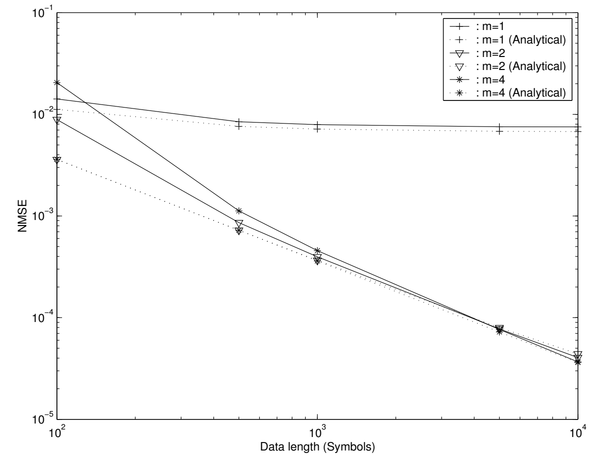

Fig. 3. Effect of data record length N on NMSE ($P = 8$, $J = 3$).

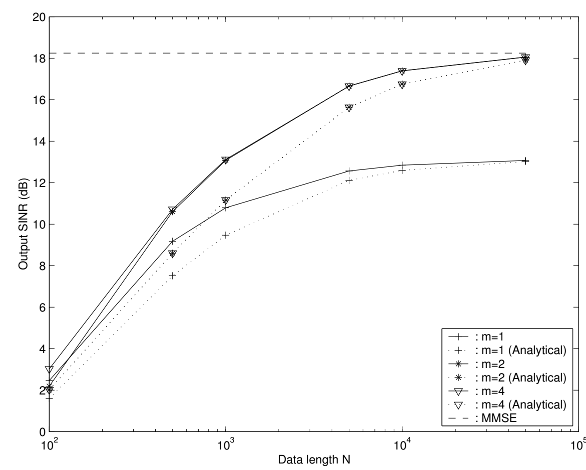

Fig. 4. Effect of data length N on output SINR ($P = 8$, $J = 3$).

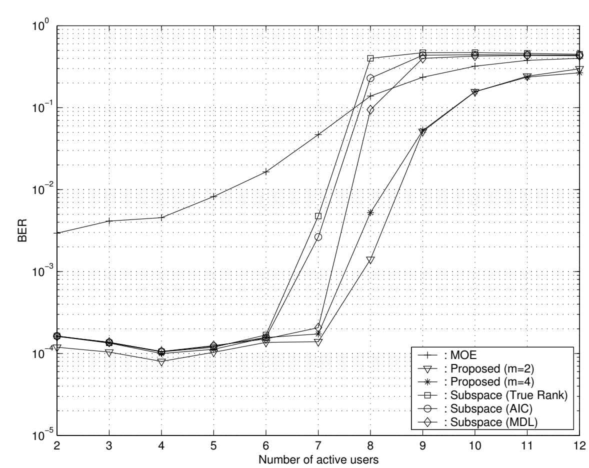

Fig. 5. BER v.s. users ($P = 12$, $N = 500$ and $SNR = 25dB$).