Channel Estimation and Multiuser Detection for Long Code CDMA

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Abstract

In a long code CDMA system, subspace technique is directly applied to downlink channel estimation when spatial/temporal diversity is available. Detection of the desired user is performed by typical linear techniques after channel is estimated. Since the method depends on eigenvalue decomposition on the data covariance matrix which is typically estimated from finite data samples, the optimal solution is then perturbed. Therefore, statistical performance of both the channel estimator and detectors is analyzed based on a perturbation technique. It is further justified by numerical examples.

1 Introduction

Code division multiple access (CDMA) spread spectrum technology has been adopted in the new communication standards [1], [2]. Direct sequence spreading employs aperiodic spreading sequence with period much longer than the symbol duration. Aperiodic (Long) codes bring about various advantages, such as immunity of the system to multiuser interference (MUI) and channel fading on the average, and establishment of a secure communication link in a hostile environment. However, they inevitably destroy cyclostationarity of CDMA signals and cause the system time-varying. This situation disables applicability of many of the existing channel estimation and detection approaches developed for short code CDMA systems.

Study on channel estimation and detection techniques for long code CDMA systems has received considerable attention in recent years. Without channel state information (CSI), least squares (LS) fitting or iterative maximum likelihood (ML) approaches are presented in [3], [4], given transmitted pilot symbols of all users. Semi-blind channel estimation solutions via subspace based data projection for downlink [9] and a deterministic LS fitting channel estimation and sequence detection approach [10] are also derived. Based on a subspace concept, blind downlink channel estimation methods are reported [11], [16]. A blind uplink channel estimation method using correlation matching techniques is proposed in [17]. Low complexity in the channel estimation algorithms can be achieved when statistics of the spreading codes are given or estimated on-line [13]. A computationally efficient minimum mean-square-error (MMSE) detection approach is correspondingly proposed in [12]. Symbol-level and chip-level adaptive MMSE interference suppression and channel equalization schemes have also appeared [5], [6], [8].

In downlink, user specific short Hadamard codes are combined with the base station's long codes to spread the signal spectrum. Their orthogonality is destroyed by multipath propagation. At the receiver, such orthogonality can be restored after channel equalization [7]. Then the user's information sequence can be easily detected by despreading based on orthogonality of different codes. However, most existing channel equalization methods assume perfect CSI, while only focusing on signal detection and performance evaluation. Meanwhile, the channel estimation method [11] requires spreading codes of other users which may not be accessible by a particular mobile user. Our previously proposed downlink channel estimation method [16] is only analyzed by ignoring the particular code structure. Also the performance of joint channel estimation and detection needs to be investigated. Therefore in this paper, we further analyze the channel estimator and linear zero-forcing (ZF) and MMSE detectors when the data covariance is estimated from finite number of received data samples. Based on the results on the second-order statistics of the sample covariance [15], the covariance and mean-square-error (MSE) of the channel estimate are first derived in closed forms. Then performance of those detectors in terms of output signal to interference plus noise ratio (SINR) is analyzed. Simulation study shows high consistency between our analyses and numerical results.

2 CDMA Downlink with Long Codes

Consider a base station communicating with J mobile stations in a CDMA system. The jth user's aperiodic spread-

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ing codes $c_{j,n}(k)$ $(k=0,\ldots,P-1)$, which is the combination of its Hadamard codes and base station's long codes, is used to spread bit $w_j(n)$. Let the chip sequence be transmitted through M subchannels with unknown coefficients $g_m(n)$ for the mth subchannel. Each subchannel is assumed to be FIR and has order q $(q<P)$. Then the chip-rate discrete-time signal due to the mth subchannel is a superposition of signals from J users corrupted by noise [16]

$$y_m(n)=\sum_{j=1}^{J}\sum_{l=0}^{q}g_m(l)s_j(n-l)+v_m(n), \qquad (1)$$

$$s_j(n)=\sum_{k=-\infty}^{\infty}w_j(k)c_{j,k}(n-kP),$$

where $v_m(n)$ is the white Gaussian noise with variance σ_v^2. If we collect P chip rate samples $[y_m(nP),\cdots,y_m(nP+P-1)]$ from each subchannel and collect samples from all M subchannels in a big vector, then the received data becomes

$$\boldsymbol{y}(n)=\mathcal{G}\boldsymbol{b}(n)+\boldsymbol{v}(n),\quad \boldsymbol{b}(n)=\sum_{j=1}^{J}\boldsymbol{b}_j(n) \qquad (2)$$

where $\mathcal{G}$ is a $MP\times(P+q)$ block Toeplitz matrix whose first block row is $[\boldsymbol{G},\boldsymbol{0}]$ with $\boldsymbol{G}$ given by

$$\boldsymbol{G}=\begin{bmatrix} g_1(q) & \cdots & g_1(0)\\ \vdots & & \vdots\\ g_M(q) & \cdots & g_M(0)\end{bmatrix},$$

$$\boldsymbol{b}_j(n)=[w_j(n-1)\boldsymbol{c}_{j,n-1}^T(P-q:P-1),w_j(n)\boldsymbol{c}_{j,n}^T]^T, \qquad (3)$$

and $\boldsymbol{c}_{j,n}=[c_{j,n}(0),\cdots,c_{j,n}(P-1)]^T$. $\boldsymbol{b}(n)$ is a sum signal from all users and has $P+q$ elements.

3 Multiuser Detection in CDMA Downlink

According to [16], $\mathcal{G}$ is a tall matrix when $M\geq 2$ and $q<P$. If all subchannels have no common zeros, then subspace technique is directly applicable to estimate all subchannels up to a scalar ambiguity. Denote the transmission power of each user's symbol as σ_w^2, and the covariance of the base station's random codes as σ_c^2. According to (2), the data covariance matrix is derived as

$$\boldsymbol{R}=E\{\boldsymbol{y}(n)\boldsymbol{y}(n)^H\}=\rho\mathcal{G}\mathcal{G}^H+\sigma_v^2\boldsymbol{I}, \qquad (4)$$

where $\rho=J\sigma_w^2\sigma_c^2$. The columns of $\mathcal{G}$ span the signal subspace of $\boldsymbol{R}$, and are orthogonal to the noise subspace: $\boldsymbol{U}_n^H\mathcal{G}=\boldsymbol{0}$. Let h_i $(i=-q,\ldots P-1)$ denote columns of $\mathcal{G}$, and $\boldsymbol{h}$ as

$$\begin{aligned}\boldsymbol{h} &= [g_1(0),\cdots,g_M(0),g_1(1),\cdots,g_M(1),\cdots,\\ &\quad g_1(q),\cdots,g_M(q)]^T. \end{aligned} \qquad (5)$$

Then $h_i=\boldsymbol{J}^{iM}\boldsymbol{A}\boldsymbol{h}$, where $\boldsymbol{A}=[\boldsymbol{I}_{M(q+1)},\boldsymbol{0}]^T$ and $\boldsymbol{J}$ is a Jordan matrix with all $1's$ in the first sub-diagonal. Hence, $\boldsymbol{h}$ can be estimated by [16]

$$\boldsymbol{h}=\arg\min_{||\boldsymbol{\alpha}||=1}\boldsymbol{\alpha}^H\boldsymbol{X}\boldsymbol{\alpha} \qquad (6)$$

where $\boldsymbol{X}=\sum_{i=-q}^{P-1}\boldsymbol{A}_i^H\boldsymbol{U}_n\boldsymbol{U}_n^H\boldsymbol{A}_i$, $\boldsymbol{A}_i\triangleq\boldsymbol{J}^{iM}\boldsymbol{A}$.

Once channel vector $\boldsymbol{h}$ is estimated, an MMSE equalizer or a ZF equalizer can be applied to the received data vectors, yielding an approximated sum signal. The desired symbol $w_1(n)$ is then detected by despreading sum signal using the desired user's codes. These equalizers obey the following forms

$$\boldsymbol{f}_{mmse}=\boldsymbol{R}^{-1}\boldsymbol{A}\boldsymbol{h}, \qquad (7)$$

$$\boldsymbol{f}_{zf}=\mathcal{G}(\mathcal{G}^H\mathcal{G})^{-1}\boldsymbol{e}, \qquad (8)$$

where $\boldsymbol{e}$ is a unitary vector with its $(q+1)$th element as 1. Applying each equalizer to P consecutive data vectors $\boldsymbol{Y}(n)=[\boldsymbol{y}(n),\ldots,\boldsymbol{y}(n+P-1)]$ and despreading the equalized data by the desired user's code vector $\boldsymbol{c}_{1,n}$, the symbol $w_1(n)$ is estimated respectively as

$$\widehat{w}_{1,mmse}(n)=\boldsymbol{f}_{mmse}^H\boldsymbol{Y}(n)\boldsymbol{c}_{1,n}, \qquad (9)$$

$$\widehat{w}_{1,zf}(n)=\boldsymbol{f}_{zf}^H\boldsymbol{Y}(n)\boldsymbol{c}_{1,n}. \qquad (10)$$

Their performance will be evaluated next.

4 Performance Analysis

As observed from (6), (7) and (8), both channel estimator and equalizers depend on $\boldsymbol{R}$. They will be perturbed if $\boldsymbol{R}$ is estimated from N vectors as $\widetilde{\boldsymbol{R}}=\frac{1}{N}\sum_{n=1}^{N}\boldsymbol{y}(n)\boldsymbol{y}^H(n)$. In this section, we will study the statistical performance of the channel estimator and equalizers in terms of channel MSE and output SINRs.

4.1 Perturbation in channel estimate

This perturbation has been investigated in [16] in terms of the perturbation of $\boldsymbol{y}(n)$ by noise. Here we will derive a more accurate form in terms of the perturbation of $\boldsymbol{R}$. Denote the perturbation by preceding the corresponding quantity by δ, and the perturbed quantity with $\tilde{}$, i.e., $\delta\boldsymbol{h}=\widetilde{\boldsymbol{h}}-\boldsymbol{h}$. Since $\boldsymbol{R}$ is perturbed by $\delta\boldsymbol{R}$, its null space $\boldsymbol{U}_n$ is perturbed by [14]

$$\delta\boldsymbol{U}_n\approx-\frac{1}{\rho}(\mathcal{G}\mathcal{G}^H)^{\dagger}\delta\boldsymbol{R}\boldsymbol{U}_n,$$

which results in a perturbation to $\boldsymbol{X}$

$$\delta\boldsymbol{X}\approx\sum_{i=-q}^{P}\boldsymbol{A}_i^H(\boldsymbol{U}_n\delta\boldsymbol{U}_n^H+\delta\boldsymbol{U}_n\boldsymbol{U}_n^H)\boldsymbol{A}_i. \qquad (11)$$

Due to $\delta\boldsymbol{X}$, $\boldsymbol{h}$ obtained from (6) is perturbed with perturbation $\delta\boldsymbol{h}$ [14]

$$\delta\boldsymbol{h} \approx -\boldsymbol{X}^{\dagger}\delta\boldsymbol{X}\boldsymbol{h}. \tag{12}$$

After substituting (11) in (12), applying $\delta\boldsymbol{U}_n$ and noticing that $\boldsymbol{U}_n^H h_i = \boldsymbol{0}$, we obtain the perturbation of channel estimate

$$\delta\boldsymbol{h} \approx \frac{1}{\rho}\sum_{i=-q}^{P-1}\boldsymbol{T}_i\boldsymbol{U}_n^H\delta\boldsymbol{R}\boldsymbol{t}_i \tag{13}$$

where $\boldsymbol{T}_i$ and $\boldsymbol{t}_i$ are deterministic quantities

$$\boldsymbol{T}_i = \boldsymbol{X}^{\dagger}\boldsymbol{A}_i^H\boldsymbol{U}_n, \quad \boldsymbol{t}_i = (\mathcal{G}\mathcal{G}^H)^{\dagger}\boldsymbol{A}_i\boldsymbol{h}.$$

Therefore, the covariance of $\delta\boldsymbol{h}$ becomes

$$\text{Cov}_h \approx \frac{1}{\rho^2}\sum_{i,j}\boldsymbol{T}_i\boldsymbol{U}_n^H E\{\delta\boldsymbol{R}\boldsymbol{t}_i\boldsymbol{t}_j^H\delta\boldsymbol{R}\}\boldsymbol{U}_n\boldsymbol{T}_j^H, \tag{14}$$

and the mean-square-error is equal to the trace of Cov_h. Both quantities depend on the term $E\{\delta\boldsymbol{R}\boldsymbol{t}_i\boldsymbol{t}_j^H\delta\boldsymbol{R}\}$. Hence, it suffices to determine a matrix in a form

$$\boldsymbol{\Psi} = E\{\delta\boldsymbol{R}\boldsymbol{D}\delta\boldsymbol{R}\}, \tag{15}$$

where $\boldsymbol{D}$ can be replaced by corresponding deterministic quantities. It is shown in [15] that if all quantities are real, then

$$\boldsymbol{\Psi} = \frac{\kappa_{4b}}{N}\mathcal{G}[\boldsymbol{I}\odot(\mathcal{G}^T\boldsymbol{D}\mathcal{G})]\mathcal{G}^T + \frac{1}{N}tr(\boldsymbol{R}\boldsymbol{D})\boldsymbol{R} + \frac{1}{N}\boldsymbol{R}\boldsymbol{D}^T\boldsymbol{R}, \tag{16}$$

where $\odot$ represents element-wise multiplication, κ_{4b} is the fourth-order cumulant of the sum signal $\boldsymbol{b}(n)$. For a complex system,

$$\boldsymbol{\Psi} = \frac{\kappa_{4b}}{N}\mathcal{G}[\boldsymbol{I}\odot(\mathcal{G}^H\boldsymbol{D}\mathcal{G})]\mathcal{G}^H + \frac{1}{N}tr(\boldsymbol{R}\boldsymbol{D})\boldsymbol{R}. \tag{17}$$

Therefore, for a given data model, statistical properties of the inputs and additive noise, $\boldsymbol{\Psi}$ can always be evaluated. Applying (16) or (17), one can verify that in both cases (14) further reduces to

$$\text{Cov}_h \approx \frac{\sigma_v^2}{N\rho^2}\sum_{i,j}(\boldsymbol{t}_j^H\boldsymbol{R}\boldsymbol{t}_i)\boldsymbol{T}_i\boldsymbol{T}_j^H. \tag{18}$$

It is interesting to note from (18) that better performance of channel estimation can be achieved when there are more active users in the system.

4.2 Perturbation in SINR

Since both equalizers filter the data vector $\boldsymbol{Y}(n)\boldsymbol{c}_{1,n}$ by (9) and (10), it is essential to express the data vector explicitly in terms of desired symbol, interfering symbols and noise in order to evaluate SINR. If we define $\boldsymbol{B}_n = [\boldsymbol{b}(n),\ldots,\boldsymbol{b}(n+P-1)]$ and $\boldsymbol{V}_n = [\boldsymbol{v}(n),\ldots,\boldsymbol{v}(n+P-1)]$, then

$$\boldsymbol{Y}_n\boldsymbol{c}_{1,n} = \mathcal{G}\boldsymbol{B}_n\boldsymbol{c}_{1,n} + \boldsymbol{V}_n\boldsymbol{c}_{1,n}.$$

After substituting $\boldsymbol{b}(n),\ldots,\boldsymbol{b}(n+p-1)$ with (2) and (3), $\boldsymbol{B}_n\boldsymbol{c}_{1,n}$ can be split as $\boldsymbol{B}_n\boldsymbol{c}_{1,n} = \boldsymbol{s}_1 w_1(n) + \boldsymbol{H}_{int}\boldsymbol{w}_{int}(n)$, where $\boldsymbol{s}_1$ and $\boldsymbol{H}_{int}$ have particular structures of users' codes. Applying the random property of both the base station's codes and noise, and invoking the orthogonality of Hadamard codes, one can verify that

$$E\{\boldsymbol{V}_n\boldsymbol{c}_{1,n}\boldsymbol{c}_{1,n}^H\boldsymbol{V}_n^H\} = \gamma\boldsymbol{I},$$

$$E\{\boldsymbol{s}_1\boldsymbol{s}_1^H\} = \sigma_c^4 diag\{(P-q):(P-1), P^2, (P-1):1\}, \tag{19}$$

$$\begin{aligned} E\{\boldsymbol{H}_{int}\boldsymbol{H}_{int}^H)\} &= (J-1)diag\{P,\ldots,P,0,P,\ldots,P\}) \\ &+ \sigma_c^4(diag\{q:1,0:P-1\}, \end{aligned} \tag{20}$$

where $\gamma = P\sigma_c^2\sigma_v^2$, ":" represents succession of integers, 0 is in the $(q+1)$th position of each of the last two diagonal matrices.

Based on the above results, SINR is then computed as

$$\text{SINR} = \frac{\boldsymbol{f}^H\boldsymbol{R}_1\boldsymbol{f}}{\boldsymbol{f}^H\boldsymbol{R}_{int}\boldsymbol{f}} \tag{21}$$

where

$$\boldsymbol{R}_1 = \sigma_w^2\mathcal{G}E\{\boldsymbol{s}_1\boldsymbol{s}_1^H\}\mathcal{G}^H, \tag{22}$$

$$\boldsymbol{R}_{int} = \sigma_w^2\mathcal{G}E\{\boldsymbol{H}_{int}\boldsymbol{H}_{int}^H\}\mathcal{G}^H + \gamma\boldsymbol{I}. \tag{23}$$

Obviously, the perturbation of $\boldsymbol{R}$ will cause perturbations of equalizers and finally the SINR perturbed to be

$$\widetilde{\text{SINR}} \approx \frac{\boldsymbol{f}^H\boldsymbol{R}_1\boldsymbol{f} + E\{\delta\boldsymbol{f}^H\boldsymbol{R}_1\delta\boldsymbol{f}\}}{\boldsymbol{f}^H\boldsymbol{R}_{int}\boldsymbol{f} + E\{\delta\boldsymbol{f}^H\boldsymbol{R}_{int}\delta\boldsymbol{f}\}}. \tag{24}$$

Since perturbations of MMSE and ZF receivers are rather distinct, we will study the corresponding SINRs separately in the following. For shorter notations, we will drop subscripts for both receivers later.

4.2.1 MMSE receiver

Using (7) and noticing that $\widetilde{\boldsymbol{R}} = \boldsymbol{R} + \delta\boldsymbol{R}$, the perturbation in the receiver can be found to be $\delta\boldsymbol{f} \approx \boldsymbol{R}^{-1}\boldsymbol{A}\delta\boldsymbol{h} - \boldsymbol{R}^{-1}\delta\boldsymbol{R}\boldsymbol{R}^{-1}\boldsymbol{A}\boldsymbol{h}$. After substituting (13), it is related to $\delta\boldsymbol{R}$ by

$$\delta\boldsymbol{f} \approx \sum_{i=-q}^{P-1}\boldsymbol{Q}_i\delta\boldsymbol{R}\boldsymbol{t}_i - \boldsymbol{R}^{-1}\delta\boldsymbol{R}\boldsymbol{f}, \tag{25}$$

where $\boldsymbol{Q}_i = \boldsymbol{R}^{-1}\boldsymbol{A}\boldsymbol{T}_i\boldsymbol{U}_n^H$. Two quantities in (24) follow the same form $E\{\delta\boldsymbol{f}^H\boldsymbol{\Phi}\delta\boldsymbol{f}\}$ with corresponding substitution of a deterministic matrix $\boldsymbol{\Phi}$. Using (25), we have

$$E\{\delta\boldsymbol{f}^H\boldsymbol{\Phi}\delta\boldsymbol{f}\} \approx \sum_{i,j}\boldsymbol{t}_i^H E\{\delta\boldsymbol{R}\boldsymbol{Q}_i^H\boldsymbol{\Phi}\boldsymbol{Q}_j\delta\boldsymbol{R}\}\boldsymbol{t}_j$$

$$
\begin{aligned}
&+ \quad \boldsymbol{f}^H E\{\delta \boldsymbol{R}\boldsymbol{R}^{-1}\boldsymbol{\Phi}\boldsymbol{R}^{-1}\delta\boldsymbol{R}\}\boldsymbol{f} \\
&- \quad \sum_i \boldsymbol{t}_i^H E\{\delta \boldsymbol{R}\boldsymbol{Q}_i^H\boldsymbol{\Phi}\boldsymbol{R}^{-1}\delta\boldsymbol{R}\}\boldsymbol{f} \\
&- \quad \boldsymbol{f}^H \sum_i E\{\delta \boldsymbol{R}\boldsymbol{R}^{-1}\boldsymbol{\Phi}\boldsymbol{Q}_i\delta\boldsymbol{R}\}\boldsymbol{t}_i
\end{aligned}
$$

where each term in the form of $E\{\delta \boldsymbol{R}\boldsymbol{D}\delta\boldsymbol{R}\}$ can be directly obtained from (15) or (16).

4.2.2 ZF receiver

Using (8) and noticing that the perturbed term $(\widetilde{\mathcal{G}}^H\widetilde{\mathcal{G}})^{-1}$ can be obtained by applying Taylor expansion, the perturbation of the ZF equalizer is derived as

$$
\delta \boldsymbol{f} \approx \boldsymbol{\Pi}^{\perp}\delta\mathcal{G}(\mathcal{G}^H\mathcal{G})^{-1}\boldsymbol{e} - (\mathcal{G}^{\dagger})^H\delta\mathcal{G}^H(\mathcal{G}^{\dagger})^H\boldsymbol{e} \qquad (26)
$$

where $\boldsymbol{\Pi}^{\perp} \triangleq \boldsymbol{I} - \mathcal{G}\mathcal{G}^{\dagger}$, $\mathcal{G}^{\dagger} \triangleq (\mathcal{G}^H\mathcal{G})^{-1}\mathcal{G}^H$. Similarly, we need to evaluate $E\{\delta\boldsymbol{f}^H\boldsymbol{R}_1\delta\boldsymbol{f}\}$ and $E\{\delta\boldsymbol{f}^H\boldsymbol{R}_{int}\delta\boldsymbol{f}\}$ to obtain $\widetilde{SINR}$.

Because $\boldsymbol{\Pi}^{\perp}\mathcal{G} = \boldsymbol{0}$ and $\boldsymbol{R}_1$ in (22) has a particular structure, it can be found that

$$
E\{\delta\boldsymbol{f}^H\boldsymbol{R}_1\delta\boldsymbol{f}\} \approx \boldsymbol{e}^H\mathcal{G}^{\dagger}E\{\delta\mathcal{G}\mathcal{G}^{\dagger}\boldsymbol{R}_1(\mathcal{G}^{\dagger})^H\delta\mathcal{G}^H\}(\mathcal{G}^{\dagger})^H\boldsymbol{e}.
$$

Thus, to obtain this quantity, it suffices to evaluate a typical form $E\{\delta\mathcal{G}\boldsymbol{\Phi}_1\delta\mathcal{G}^H\}$ and then replace $\boldsymbol{\Phi}_1$ by $\mathcal{G}^{\dagger}\boldsymbol{R}_1(\mathcal{G}^{\dagger})^H$. Noticing the definition of $\mathcal{G}$, $\delta\mathcal{G}$ is related to $\delta\boldsymbol{h}$ as the following

$$
\delta\mathcal{G} = [\boldsymbol{A}_{-q}\delta\boldsymbol{h}, \ldots, \boldsymbol{A}_{P-1}\delta\boldsymbol{h}].
$$

Then

$$
E\{\delta\mathcal{G}\boldsymbol{\Phi}_1\delta\mathcal{G}^H\} = \sum_{i,j=1}^{P+q} \boldsymbol{\Phi}_1^{(i,j)}\boldsymbol{A}_{i-1-q}\mathrm{Cov_h}\boldsymbol{A}_{j-1-q}^H, \quad (27)
$$

where $\boldsymbol{\Phi}_1^{(i,j)}$ is the (i,j)th element of $\boldsymbol{\Phi}_1$, and $\mathrm{Cov_h}$ has been derived in (18).

To simplify $E\{\delta\boldsymbol{f}^H\boldsymbol{R}_{int}\delta\boldsymbol{f}\}$, let us define the first term of $\boldsymbol{R}_{int}$ in (23) as $\boldsymbol{\Sigma}$

$$
\boldsymbol{\Sigma} = \sigma_w^2\mathcal{G}E\{\boldsymbol{H}_{int}\boldsymbol{H}_{int}^H\}\mathcal{G}^H.
$$

It can be verified that

$$
\begin{aligned}
E\{\delta\boldsymbol{f}^H\boldsymbol{R}_{int}\delta\boldsymbol{f}\} \quad &\approx \quad \boldsymbol{e}^H\mathcal{G}^{\dagger}E\{\delta\mathcal{G}\mathcal{G}^{\dagger}\boldsymbol{\Sigma}(\mathcal{G}^{\dagger})^H\delta\mathcal{G}^H\}(\mathcal{G}^{\dagger})^H\boldsymbol{e} \\
&+ \ \gamma\boldsymbol{e}^H\mathcal{G}^{\dagger}E\{\delta\mathcal{G}(\mathcal{G}^H\mathcal{G})^{-1}\delta\mathcal{G}^H\}(\mathcal{G}^{\dagger})^H\boldsymbol{e} \\
&+ \ \gamma\boldsymbol{e}^H(\mathcal{G}^H\mathcal{G})^{-1}E\{\delta\mathcal{G}^H\boldsymbol{\Pi}^{\perp}\delta\mathcal{G}\}(\mathcal{G}^H\mathcal{G})^{-1}\boldsymbol{e}.
\end{aligned}
$$

The first and second terms can be derived by following (27). To obtain the third term, we need $E\{\delta\mathcal{G}^H\boldsymbol{\Pi}^{\perp}\delta\mathcal{G}\}$. Considering the ith column of $\delta\mathcal{G}$ is $\boldsymbol{A}_{i-1-q}\delta\boldsymbol{h}$, the (i,j)th element of this matrix is $E\{\delta\boldsymbol{h}^H\boldsymbol{\Theta}_{i,j}\delta\boldsymbol{h}\}$ where $\boldsymbol{\Theta}_{i,j} = \boldsymbol{A}_{i-1-q}^H\boldsymbol{\Pi}^{\perp}\boldsymbol{A}_{j-1-q}$. Substituting (13) for $\delta\boldsymbol{h}$, we have

$$
E\{\delta\boldsymbol{h}^H\boldsymbol{\Theta}_{i,j}\delta\boldsymbol{h}\} \approx \frac{1}{\rho^2}\boldsymbol{t}_i^H E\{\delta\boldsymbol{R}\boldsymbol{U}_n\boldsymbol{T}_i^H\boldsymbol{\Theta}_{i,j}\boldsymbol{T}_j\boldsymbol{U}_n^H\delta\boldsymbol{R}\}\boldsymbol{t}_j. \qquad (28)
$$

The expectation on the right hand side is readily obtained according to (15) and (16). Therefore, $\widetilde{SINR}$ follows.

5 Simulation Examples

In this section, we show the performance of the proposed channel estimator, receivers, and also verify our analysis presented in the previous section. The transmitted sequences are drawn from a binary constellation $[-1, 1]$. Each user's short codes are Hadamard codes, which are then multiplied by base station's binary random codes. Spreading factor is set to be 16, the number of active users is assumed to be 10 and $M = 2$ is set for the simulated system. Totally 100 Monte Carlo simulations are performed to obtain the average results. Fig. 1 shows the MSEs of the proposed channel estimator over different N. Fig. 2 and Fig. 3 compare experimental SINRs of the MMSE and ZF receivers with their corresponding analytical ones respectively. Satisfactory performance can be observed for both channel estimation, and SINRs of the two different receivers according to these figures. Meanwhile, the experimental results of both the MSE and SINRs are highly consistent with their analytical values for large N, which verifies our analysis.

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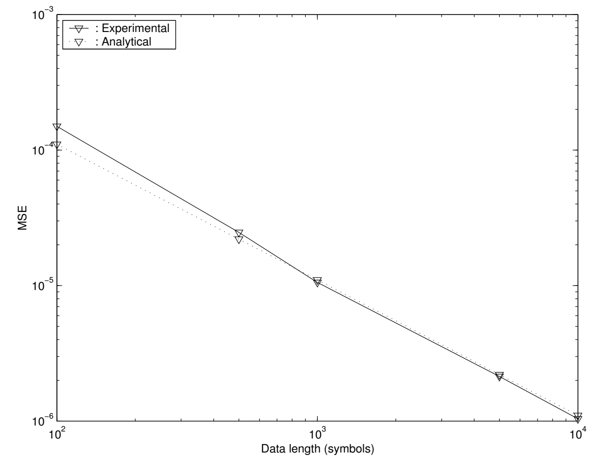

Fig. 1. MSE of channel estimation.

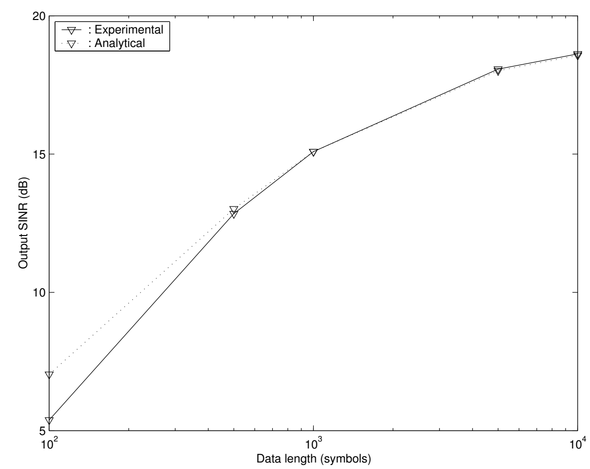

Fig. 2. Output SINR for the MMSE receiver.

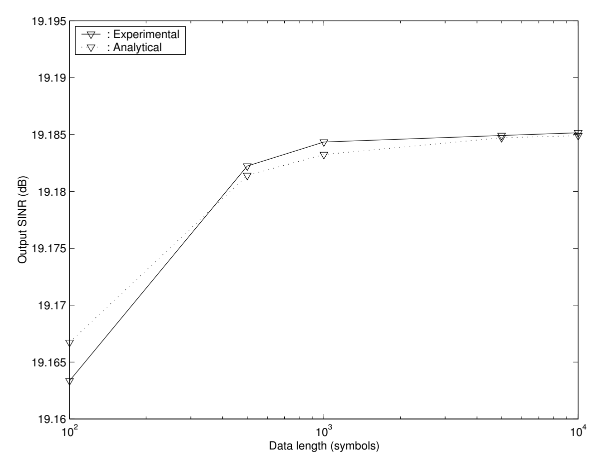

Fig. 3. Output SINR for the ZF receiver.