A Globally Convergent CMA-Based Approach to Blind Multiuser Detection

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Abstract

In this paper, we study blind multiuser detection in a CDMA system in the presence of multipath propagation. We force the receiver to follow a minimum mean-square-error (MMSE) type parameterized by a channel-like constraint vector. Then we minimize the Godard's CMA cost function with respect to such a constraint vector in a much smaller dimensional space. Global convergence of the proposed approach is established. Performance of the channel estimator and receiver is also analyzed, together with comparisons with existing methods. Simulation results show that the proposed method performs well by both batch and adaptive implementations.

1 Introduction

Multiuser interference (MUI) is a typical obstacle to be obviated in detection of input signals in a DS/CDMA system. Substantial efforts have focused on design of blind linear multiuser detectors due to their low complexity, ease of implementation and acceptable performance.

Recently, there emerges significant interest in studying high order statistics (HOS) based multiuser detection techniques which may yield better detection performance. In a single-user system, the received signal is corrupted by intersymbol interference (ISI) due to multipath propagation. The constant modulus algorithm (CMA) proposed by Godard [2] can be implemented at the receiver to effectively suppress ISI. The CMA algorithm has also been successfully applied to a multiple-input multiple-output (MIMO) system with its convergence analyzed as well [8]. A CDMA system is a special class of MIMO systems. When the HOS based techniques are applied to multiuser detection, different methods can be divided into two categories. One is to design a bank of detectors with each one detecting one user [1], [5]. Thus all users can be detected at the same time. This multiuser detection scheme can be implemented in the base station which is capable of processing large amount of data in parallel. The other one is to consider only the desired user in a flat fading channel [11] or multipath environment [4], [9], [12], such as in the mobile station. With given spreading codes of the desired user, the detector is forced to satisfy a linear constraint such that signals from the user of interest are detected.

However, those algorithms have shown certain disadvantages. It is clear that approaches suitable for flat channels suffer from signature mismatch. Among those capable of multipath mitigation, [4] exhibits local minima and inability to optimally combine signal components from different paths. The approach [9] is a batch iterative algorithm based on a batch processing of a block of data. Their global convergence has not been established analytically. Our previously developed CMA-based method [12] requires proper initialization for the constraint vector.

In this paper, we propose a novel CMA-based criterion to detect a user of interest in a CDMA system in the presence of multipath distortion. The convergence problem in [12] motivates us to modify the constraints for the receiver. We still adopt the Godard's CMA cost function, but constrain the receiver to take a MMSE form with an unknown channel-like vector. Therefore our cost function is indeed parameterized by the constraint vector. Through minimization either with or without norm constraint, the constraint vector is shown to globally converge to the desired channel vector within a phase ambiguity in a small noise system.

2 CDMA System Model

Consider a direct sequence (DS) CDMA system with J users. User j is assigned a periodic spreading sequence $c_j(k)$ $(k = 0, \dots, P-1)$. Let the chip sequence be transmitted through a multipath channel with unknown coefficients $g_j(n)$. Then the received chip-rate discrete-time signal $y(n)$ has the form [13]

$$y(n) = \sum_{j=1}^{J} \sum_{l=-\infty}^{\infty} w_j(l) h_j(n - d_j - lP) + v(n),$$

$$h_j(n) = \sum_{m=-\infty}^{\infty} g_j(m) c_j(n-m) \qquad (1)$$

where $w_j(n)$ is the n-th information symbol from user j and assumed to take either $+1$ or -1 with equal probability, $h_j(n)$ is a signature sequence, and $v(n)$ is zero-mean additive white Gaussian noise (AWGN). All quantities in (1) are assumed to be real in this paper, and the maximum channel order for all users is denoted as q. Without loss of generality, user 1 is treated as the user of interest. The receiver is assumed to be synchronized to this user. After collecting $L = \nu P$ measurements in a vector $\boldsymbol{y}_n = [y(nP), \ldots, y(nP + L - 1)]^T$, the received data vector becomes [9]

$$\boldsymbol{y}_n = \boldsymbol{H}\boldsymbol{w}(n) + \boldsymbol{v}_n = \boldsymbol{h}_1 w_1(n+\nu_0) + \boldsymbol{H}_{int}\boldsymbol{w}_{int}(n) + \boldsymbol{v}_n, \tag{2}$$

where $\boldsymbol{H} = [\boldsymbol{h}_1, \boldsymbol{H}_{int}]$, $\boldsymbol{h}_1 \triangleq \mathcal{C}_1 \boldsymbol{g}_1$ is the signature vector of the desired symbol $w_1(n + \nu_0)$ with a time offset $0 \leq \nu_0 \leq \nu$, $\mathcal{C}_1$ is the code filtering matrix whose first $\nu_0 P$ rows are all zeros, $\boldsymbol{g}_1$ is the channel vector, $\boldsymbol{w}(n) = [w_1(n + \nu_0), \boldsymbol{w}_{int}^T(n)]^T$, $\boldsymbol{w}_{int}(n)$ is an interference vector including ISI and multiuser interference (MUI), $\boldsymbol{H}_{int}$ is the corresponding signature matrix, $\boldsymbol{v}_n$ is the noise vector. The particular structure of $\boldsymbol{h}_1$ will be exploited to derive a blind detector which is capable of combating multipath distortion and suppressing both ISI and MUI. Throughout this paper, we assume: (**AS1**) $[\mathcal{C}_1,\ \boldsymbol{H}_{int}]$ has full column rank, which is also required by the subspace method [10] and the MOE method [7].

3 Constrained CMA-Based Receivers

The CMA cost function [2] is adopted to obtain a receiver subject to an MMSE-form constraint

$$\min_{\boldsymbol{f}} \mathcal{J}_{CMA} = E\{(|z_n|^2 - 1)^2\}, \ \text{ s.t. } \boldsymbol{f} = \boldsymbol{R}^{-1}\mathcal{C}_1\boldsymbol{g} \tag{3}$$

where $z_n = \boldsymbol{f}^H \boldsymbol{y}_n$, $\boldsymbol{R}$ is the autocorrelation of $\boldsymbol{y}_n$. The constraint on $\boldsymbol{f}$ aims at removing user and delay ambiguity. Constraint vector $\boldsymbol{g}$ is a parameterized channel-like vector. Applying the constraint of $\boldsymbol{f}$ directly to the output yields $z_n = \boldsymbol{g}^H \mathcal{C}_1^H \boldsymbol{R}^{-1} \boldsymbol{y}_n$. In this way, (3) is transformed to the following unconstrained one

$$\min_{\boldsymbol{g}} \mathcal{J}(\boldsymbol{g}) = E\{(\boldsymbol{g}^H \mathcal{C}_1^H \boldsymbol{R}^{-1} \boldsymbol{y}_n \boldsymbol{y}_n^H \boldsymbol{R}^{-1} \mathcal{C}_1 \boldsymbol{g} - 1)^2\}. \tag{4}$$

Once the optimal $\boldsymbol{g}$ is obtained from (4), the receiver $\boldsymbol{f}$ can be constructed as (3). In practice, unconstrained optimization w.r.t. $\boldsymbol{g}$ may suffer from slow convergence or divergence in the presence of model imperfections. To enhance the robustness of the algorithm, we introduce a norm constraint on $\boldsymbol{g}$. The use of norm constraint for solving some practical implementation problems has been discussed in [6]. Since $\boldsymbol{g}$ is expected to converge to $\boldsymbol{g}_1$, the norm constraint is naturally chosen as $||\boldsymbol{g}|| = ||\boldsymbol{g}_1||$. The new norm-constrained optimization problem is formulated as

$$\min_{\boldsymbol{g}} \mathcal{J}(\boldsymbol{g}) = E\{(\boldsymbol{g}^H \mathcal{C}_1^H \boldsymbol{R}^{-1} \boldsymbol{y}_n \boldsymbol{y}_n^H \boldsymbol{R}^{-1} \mathcal{C}_1 \boldsymbol{g} - 1)^2\} \quad \text{s.t. } ||\boldsymbol{g}|| = ||\boldsymbol{g}_1||. \tag{5}$$

If $||\boldsymbol{g}_1||$ is unknown, we relax the norm constraint to a norm bound, which results in the norm-bounded optimization

$$\min_{\boldsymbol{g}} \mathcal{J}(\boldsymbol{g}) = E\{(\boldsymbol{g}^H \mathcal{C}_1^H \boldsymbol{R}^{-1} \boldsymbol{y}_n \boldsymbol{y}_n^H \boldsymbol{R}^{-1} \mathcal{C}_1 \boldsymbol{g} - 1)^2\} \quad \text{s.t. } \gamma_{min} < ||\boldsymbol{g}|| < \gamma_{max}. \tag{6}$$

The bounds γ_{min} and γ_{max} should be chosen properly such that $||\boldsymbol{g}_1||$ lies in the interval $(\gamma_{min}, \gamma_{max})$. One choice is given by

$$\gamma_{min} = \sqrt{\frac{||\boldsymbol{h}_1||^2}{\max(eig(\mathcal{C}_1^H \mathcal{C}_1))}},\ \gamma_{max} = \sqrt{\frac{||\boldsymbol{h}_1||^2}{\min(eig(\mathcal{C}_1^H \mathcal{C}_1))}}$$

when the power of the desired user is known by using power control. Otherwise, they can be pre-selected for the interval to span a certain range. Due to its loose constraint, the norm-bounded optimization is expected to achieve a better performance than the norm-constrained one.

Since the cost functions are all nonlinear, closed-form solutions are impossible. Gradient descent method is thus applied to obtain optimal $\boldsymbol{g}$ and then $\boldsymbol{f}$ for each approach. Both batch iterative and adaptive implementations can be adopted. In the batch iterative implementation, all expectations are estimated by the average over a block of data, while in the adaptive one, they are approximated by instantaneous values except that $\boldsymbol{R}^{-1}$ is estimated using a smoothing window.

4 Performance Analysis

We will establish the global convergence of the proposed method for both unconstrained and norm-constrained cases. We will also present analytical results for the channel estimation error and output signal to interference plus ratio (SINR) of the receivers. Their proofs are omitted due to lack of space. Without loss of generality, we assume $||\boldsymbol{g}_1|| = 1$ and consider a real system throughout our analysis.

4.1 Convergence Analysis

To proceed, we first perform singular value decomposition (SVD) on $\boldsymbol{H}$

$$\boldsymbol{H} = [\boldsymbol{U}_s\ \ \boldsymbol{U}_n] \begin{bmatrix} \boldsymbol{\Lambda}_H \\ \boldsymbol{0} \end{bmatrix} \boldsymbol{V}^T \tag{7}$$

where $\boldsymbol{\Lambda}_H = diag\{\lambda_1,\ \ldots,\ \lambda_\zeta\}$, $\boldsymbol{U}_s$ and $\boldsymbol{U}_n$ span the signal and noise subspaces respectively, $\boldsymbol{V}$ is a unitary matrix. Then $\boldsymbol{R}$ has the following decomposition

$$\boldsymbol{R} = \boldsymbol{U}_s(\boldsymbol{\Lambda}_s + \sigma_v^2 \boldsymbol{I})\boldsymbol{U}_s^T + \sigma_v^2 \boldsymbol{U}_n \boldsymbol{U}_n^T \tag{8}$$

where $\boldsymbol{\Lambda}_s = \boldsymbol{\Lambda}_H^2 = diag\{\lambda_1^2, \ldots, \lambda_\zeta^2\}$.

The global convergence for both unconstrained and norm-constrained approaches can be established based on (4) and

(5) respectively. Direct analysis of the stationary points of (4) and (5) in a very noisy environment is very complicated, since both equations contain higher order cross terms. Therefore, we only study the convergence when $\sigma_v^2 \to 0$.

Proposition 1 (Global convergence): Under (**AS1**) and when $\sigma_v^2 \to 0$, the optimal $\boldsymbol{g}$ obtained by the unconstrained optimization (4) or the norm-constrained optimization (5) will converge to the multipath channel $\boldsymbol{g}_1$ up to ± 1. The receiver $\boldsymbol{f} = \boldsymbol{R}^{-1}\mathcal{C}_1\boldsymbol{g}$ will converge to the MMSE receiver. □

4.2 Channel Estimation Error

According to our previous proposition, the constraint vector converges to the channel vector up to ± 1 at high SNR. In this section, we analyze the noise effect on the channel estimation if $\boldsymbol{g}$ is treated as a channel estimate. To facilitate the analysis, we restrict our attention to the neighborhood of the optimal point of $\boldsymbol{g} = \boldsymbol{g}_1$ when the noise power is small ($\sigma_v^2 << 1$). For another optimal solution $-\boldsymbol{g}_1$, perturbation due to noise can be similarly analyzed. The estimated channel, denoted as $\boldsymbol{g} = \boldsymbol{g}_1 + \Delta\boldsymbol{g}_1 + \Delta\boldsymbol{g}_1^{\perp}$, has a small deviation from $\boldsymbol{g}_1$ where $\Delta\boldsymbol{g}_1$ represents the in-space estimation error and $\Delta\boldsymbol{g}_1^{\perp}$ the error in the orthogonal space. It can be easily verified that $\Delta\boldsymbol{g}_1$ is much smaller than $\Delta\boldsymbol{g}_1^{\perp}$ when perturbation is small. Therefore, only $\Delta\boldsymbol{g}_1^{\perp}$ will be considered in the analysis. The corresponding channel estimation results are presented in the following proposition.

Proposition 2 (Channel estimation error): For small σ_v^2, channel estimation error of the unconstrained approach is approximated by

$$\Delta\boldsymbol{g}_1^{\perp} \approx \sigma_v^8 \boldsymbol{A}_0^{\dagger}\mathcal{C}_1^T\boldsymbol{U}_s\boldsymbol{\Lambda}_s^{-1}\boldsymbol{U}_s^T\sum_{i=2}^{M}(\boldsymbol{h}_1^T\boldsymbol{U}_s\boldsymbol{\Lambda}_s^{-2}\boldsymbol{U}_s^T\boldsymbol{h}_i)^3\boldsymbol{h}_i. \tag{9}$$

For the norm-constrained approach, channel estimation error is approximated by

$$\Delta\boldsymbol{g}_1^{\perp} \approx -1.5\sigma_v^4\boldsymbol{g}_1^T\boldsymbol{A}_2\boldsymbol{g}_1\boldsymbol{A}_0^{\dagger}\boldsymbol{A}_1\boldsymbol{g}_1 \tag{10}$$

where $\boldsymbol{h}_i$ is the ith column of $\boldsymbol{H}$,

$$\boldsymbol{A}_0 = \mathcal{C}_1^T\boldsymbol{U}_n\boldsymbol{U}_n^T\mathcal{C}_1, \quad \boldsymbol{A}_1 = \mathcal{C}_1^T\boldsymbol{U}_s\boldsymbol{\Lambda}_s^{-1}\boldsymbol{U}_s^T\mathcal{C}_1,$$
$$\boldsymbol{A}_2 = \mathcal{C}_1^T\boldsymbol{U}_s\boldsymbol{\Lambda}_s^{-2}\boldsymbol{U}_s^T\mathcal{C}_1.$$

□

4.3 Output SINR

For a linear receiver, the output SINR can be defined as

$$\text{SINR} = \frac{(\boldsymbol{f}^T\boldsymbol{h}_1)^2}{\boldsymbol{f}^T\boldsymbol{R}\boldsymbol{f} - (\boldsymbol{f}^T\boldsymbol{h}_1)^2}. \tag{11}$$

Then, the asymptotic SINR ratio of the unconstrained or norm-constrained approach compared with the ideal MMSE receiver is presented in the following proposition.

Proposition 3 (Asymptotic SINR ratio): If the receiver is $\boldsymbol{f}_{new} = \boldsymbol{R}^{-1}\mathcal{C}_1\boldsymbol{g}$, where $\boldsymbol{g}$ is obtained from (5) or (4), then

$$\frac{\text{SINR}_{\text{new}}}{\text{SINR}_{\text{mmse}}} \to 1 \text{ as } \sigma_v^2 \to 0. \tag{12}$$

□

Compared with the MOE method, the proposed methods give much smaller channel estimation errors (at least in the level σ_v^4 vs. σ_v^2). Also the receivers' performance exhibits asymptotic superiority according to the previous proposition.

5 Comparison with TL's Approach

TL's approach [9] constructs the receiver by the following optimization

$$\min_{\boldsymbol{f}} \mathcal{J}_{CMA}(\boldsymbol{f}), \text{ s.t. } (\mathcal{C}_1^{\perp})^T\boldsymbol{R}\boldsymbol{f} = 0 \tag{13}$$

where $\mathcal{C}_1^{\perp} = null(\mathcal{C}_1)$. The receiver $\boldsymbol{f}$ is iteratively updated by

$$\boldsymbol{f}_{TL}(n+1) = \boldsymbol{f}_{TL}(n) - \mu_f\boldsymbol{V}_n\boldsymbol{V}_n^T\nabla_{\boldsymbol{f}}\mathcal{J}_{CMA} \tag{14}$$

where $\nabla_{\boldsymbol{f}}\mathcal{J}_{CMA}$ is the gradient of Godard's cost function and $\boldsymbol{V}_n$ contains all basis of the null space of $\boldsymbol{R}\mathcal{C}_1^{\perp}$. To compare with TL's approach, we first obtain the update of $\boldsymbol{g}$ for the proposed unconstrained algorithm

$$\boldsymbol{g}(n+1) = \boldsymbol{g}(n) - \mu_g\mathcal{C}_1^T\boldsymbol{R}^{-1}\nabla_{\boldsymbol{f}}\mathcal{J}_{CMA}. \tag{15}$$

Since $\boldsymbol{f}(n) = \boldsymbol{R}^{-1}\mathcal{C}_1\boldsymbol{g}(n)$, left multiplying $\boldsymbol{R}^{-1}\mathcal{C}_1$ yields the following equivalent update w.r.t. $\boldsymbol{f}$

$$\boldsymbol{f}(n+1) = \boldsymbol{f}(n) - \mu_g\boldsymbol{R}^{-1}\mathcal{C}_1\mathcal{C}_1^T\boldsymbol{R}^{-1}\nabla_{\boldsymbol{f}}\mathcal{J}_{CMA}. \tag{16}$$

From (14) and (16), it is observed that the difference between the proposed unconstrained method and TL's method lies in the transformation matrix $\boldsymbol{R}^{-1}\mathcal{C}_1\mathcal{C}_1^T\boldsymbol{R}^{-1}$ and $\boldsymbol{V}_n\boldsymbol{V}_n^T$. From the subsequent analysis, we shall see that the two matrices are related to each other in the sense that they share the same basis and differ in singular values. Since $(\mathcal{C}_1^{\perp})^T\boldsymbol{R}\boldsymbol{R}^{-1}\mathcal{C}_1 = \boldsymbol{0}$ and $\mathcal{C}_1^T\boldsymbol{R}^{-1}\mathcal{C}_1$ has rank $q+1$, $\boldsymbol{R}^{-1}\mathcal{C}_1$ contains $q+1$ independent null vectors of $\boldsymbol{R}\mathcal{C}_1^{\perp}$. On the other hand, the null space of $\boldsymbol{R}\mathcal{C}_1^{\perp}$ has rank $q+1$, which implies that $\boldsymbol{R}^{-1}\mathcal{C}_1$ constitutes a null span of $\boldsymbol{R}\mathcal{C}_1^{\perp}$, that is $\boldsymbol{V}_n = span\{\boldsymbol{R}^{-1}\mathcal{C}_1\}$. Therefore

$$\boldsymbol{R}^{-1}\mathcal{C}_1\mathcal{C}_1^T\boldsymbol{R}^{-1} = \boldsymbol{V}_n\boldsymbol{\Sigma}_{CR}\boldsymbol{V}_n^T \tag{17}$$

where $\boldsymbol{\Sigma}_{CR}$ is a diagonal matrix with the singular values of $\boldsymbol{R}^{-1}\mathcal{C}_1\mathcal{C}_1^T\boldsymbol{R}^{-1}$ on the diagonal. From (14), (16) and

(17), it is clear that, during the update of the receiver, TL's method tries to project CMA's gradient onto the null space of $\boldsymbol{R}\mathcal{C}_1^{\perp}$, while the proposed one projects CMA's gradient to the weighted null space. Although simulation results in the following section show very close performance between these two approaches under batch implementation, the proposed approach has been shown to have global convergence in the case of $\sigma_v^2 \to 0$. The analytical channel estimation error and asymptotic output SINR ratio are also derived for the proposed approach. In addition, the proposed method can be adaptively implemented to track system variations due to channel fading and dynamic loading.

6 Simulations

Several simulation examples are presented to demonstrate the applicability and performance of the proposed approaches. Each user's spreading codes, delay and multipath channel parameters are randomly generated in each of 100 realizations except that the desired user is synchronized. $\nu = 5$ and $\nu_0 = 3$ in all simulations. $\boldsymbol{g}$ is initialized to be $[1,\ \mathbf{0}]^T$ for both the proposed and the MOE methods.

First, we adopt an ideal cost function which is equivalent to $N = \infty$ data vectors and test our analysis. Consider $P = 8$ and 3 equal power users. Each channel has 4 paths. The first path has zero delay and unit power. The remaining paths are Gaussian distributed of variance 0.3 with delays uniformly distributed from 1 to $P - 1$. Fig. 1 shows the experimental normalized channel estimation mean square errors (NMSEs) of the proposed and the MOE methods in solid lines. The corresponding analytical NMSEs are also plotted in dashed lines for comparison. It can be observed that the experimental NMSE of the proposed norm-constrained method converges to its analytical result as SNR increases. Due to the accuracy limitation of Matlab, the results of the unconstrained and norm-bounded approaches are only tested up to $20dB$ SNR. For the unconstrained approach, similar convergence is observed. As expected, the experimental NMSE of the norm-bounded approach, better than that of the norm-constrained one, converges to the NMSE of the unconstrained one at high SNR. In addition, all proposed approaches have much lower NMSE levels than the MOE method. Fig. 2 illustrates the SINR ratio. The ratio of the MOE method converges to a constant smaller than 1, while the counterparts of all the three proposed approaches converge to 1 as $SNR \to \infty$.

We also compare our methods with TL's method [9] and the MOE method [7] in a batch iterative way. 400 data symbols are used for constructing receivers for all methods, then 6000 independent symbols are used to calculate BERs. $P = 12$. Fig. 3 shows the BERs w.r.t. variable active user numbers from 2 to 12. The proposed and TL's methods have very similar BERs when there are fewer than 8 users. As the number of active users increases to 8 and up, (**AS1**) is hard to satisfy. As a result, the BERs of the proposed and the MOE algorithms increase apparently. TL's approach, though having less restrictive identifiability conditions, shows larger performance degradation than the proposed ones for the current simulation setup. The last experiment shows the performance of the proposed method implemented adaptively in a channel fading environment, and compares it with the ideal MMSE receiver, the adaptive MOE method [13], the CLS receiver [3] which assumes perfect channel knowledge. Each user's channel is assumed to be two-ray Raleigh fading [13] with the same power for different paths. The delay of the second path is set to 3 chip periods. A square pulse shaping function is employed. $f_d T = 2 \times 10^{-3}$, where f_d represents the maximum Doppler shift and T denotes the symbol period. Fig. 4 demonstrates the steady state BERs over various SNRs from $5dB$ to $15dB$. It is seen that the BER level of the proposed approach converge to that of the CLS method as SNR increases, and the proposed method exhibits better tracking ability than the MOE method.

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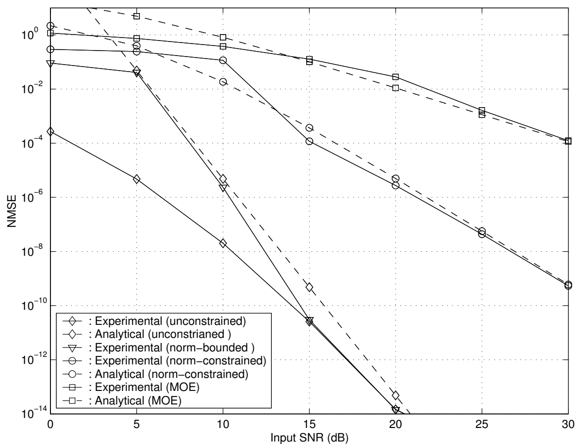

Fig. 1. Channel estimation MSE vs. input SNR for $N = \infty$.

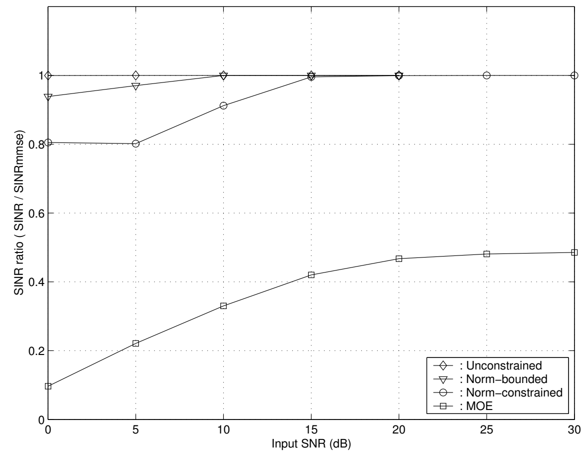

Fig. 2. SINR ratio vs. input SNR for $N = \infty$.

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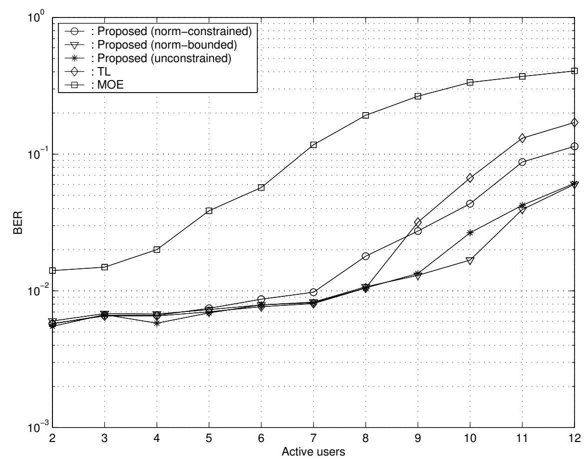

Fig. 3. BER vs. number of active users by batch iterative implementation with $P = 12$, $q = 11$, $SNR = 20dB$.

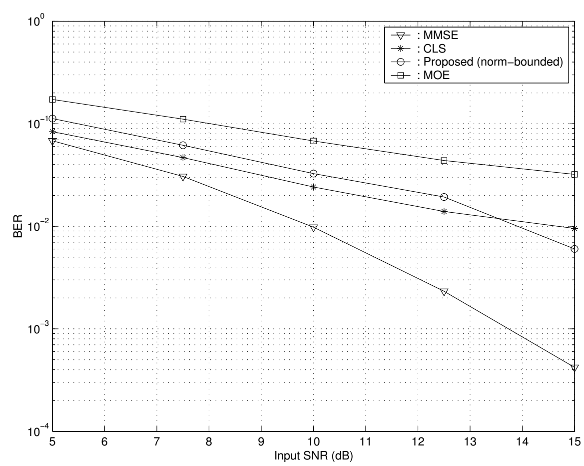

Fig. 4. BER vs. input SNR by adaptive implementation with $P = 8$, $q = 3$, $J = 3$, $f_d T = 2 \times 10^{-3}$.