Blind Channel Estimation for Precoded Variable Bit-Rate Multiuser Systems

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Abstract

Precoder has been shown to be able to provide source diversity and more freedom in design. In this paper we employ precoding techniques for block transmission based on a multirate filterbank structure. To meet multiuser communication requirement for various high data-rates, different filterbanks are used as precoders with corresponding coefficients and up/down sampling rates. However, due to high speed communication in the presence of unknown multipath, different interferences exist in the received data, such as multiuser interference, intersymbol interference and interblock interference. To estimate the multipath channel for a desired user, we generalize the subspace method by employing all signatures associated with this channel in the current system. In this way a better performance for channel estimate can be achieved. The delay for that user can also be jointly estimated together with channel estimation.

1. Introduction

There is increasingly significant interest in designing new wireless communication networks which can provide multimedia services. The new network will support not only traditional voice communication, but also image, video and data transmission at a much higher rate. To share the channel resource among different users, multiuser communication schemes have to be deployed such as TDMA, FDMA, CDMA or hybrid schemes. Under bandwidth constraint, CDMA provides larger capacity. It may be well noticed that CDMA builds more redundancy into the channel input. Each bit information is repeated many times (equal to the processing gain) before transmission which equivalently creates source diversity. This highly redundant scheme offers the low-rate system a unique capability to combat multipath effects and makes ISI negligible. However, new wireless networks require accommodation of multirate sources from different users. Most current proposals suggest either multicode (MC) or multiple processing gain (MPG) mechanism [4], while restricting themselves to data rates of integral multiples of some basic low-rate. In order to support variable bit rate transmission however, a comprehensive scheme needs to be investigated.

From signal processing perspective, multirate requirement can be easily satisfied by designing an appropriate multirate filterbank at the transmitter with adjustable upsampling rate P and downsampling rate M (assume $M \leq P$). These precoders usually deal with block transmissions, where data stream is divided into consecutive equal size blocks. Such transmitters with precoding have been derived jointly with the receiver by minimizing the MSE subject to a constraint on the transmit power [7]. With precoders, various modulation schemes can be unified [7], [8]. Precoders themselves can be orthogonal to each other, or in a Vandermonde structure to guarantee successful equalization of the channel irrespective of channel zero locations [8].

In this work, we propose a more general scheme to gain more flexibility in system design and unify various multiple access methods. Different from [8], we explicitly consider in the received data the interblock interference (IBI) in addition to multiuser interference (MUI) and intersymbol interference (ISI), which is a typical scenario for high speed communications in the presence of unknown multipath. For such a system, the multipath channel can be estimated by conventional subspace method. To obtain a better estimate, we generalize the subspace method by employing all signatures of the desired user's bits and build a new cost function based on the noise subspace. Similar idea has been used in [10] in a different scenario based on subspace intersection technique for a long observation window equal to several symbol periods. Its extension to multirate CDMA systems is briefly discussed therein but with some restrictions on the length of the channel. Our approach is more general and applicable to variable bit rate communication system. The proposed cost function is parameterized by both unknown channel and time delay for the desired user. By minimizing this function, these unknowns can be obtained. The identifiability conditions are also investigated. Due to finite data samples in estimating the noise subspace, the minimum mean square error of the channel estimate is provided as a function of the number of data samples based on perturbation analysis. Simulation results show that significant improvement over the conventional subspace method is observed.

2. Multirate precoding framework in multiuser communications

Let us assume K users are simultaneously provided services in a wireless communication system. User k has i.i.d. bit stream $b_k(n)$ to transmit (see Figure 1). This stream is partitioned into consecutive blocks with each block M_k bits and converted into M_k parallel sub-streams. In the n-th block, let $b_{k,m_k}(n)$ denote the m_k-th bit $b_k(nM_k + m_k)$. The m_k-th branch is followed by a downsampler by factor M_k and an upsampler by factor P with filter coefficients $s_{k,m_k}(n)$. By design, the precoding filters are assumed to be FIR with length P for all users and linear time-

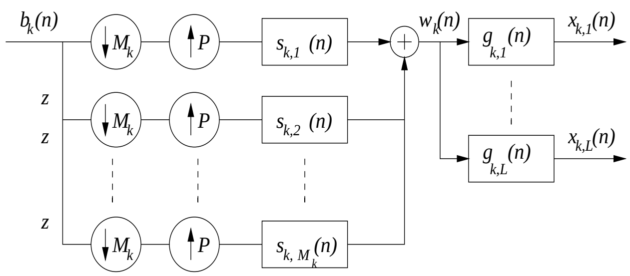

Fig. 1. Multirate precoder followed by antenna diversity

invariant (LTI). We adopt common upsampling rate P for all users but different downsampling rates ($M_k < P$). In such a way variable bit rates of users' information are converted into a fixed transmission rate to reach channel's capacity under practical bandwidth requirement. For simplicity of description, we still term signals after precoders by "chips" as in CDMA communications. If we denote the bit period for user k by T_{b_k} and common chip period by T_c, then within M_k bit periods, there are P chips to be transmitted. By simple calculation, we can find that $M_k = \frac{T_c}{T_{b_k}} P$, or equivalently $M_k = \frac{R_{b_k}}{R_c} P$ in terms of the bit rate and chip rate. By adjusting M_k for each user, its information rate can be easily satisfied.

The structure in Figure 1 is a typical multirate filterbank. Its output is a superposition of signals from M_k branches (e.g., [8])

$$w_k(n) = \sum_{m=1}^{M_k} w_k^{(m)}(n),\ \ w_k^{(m)}(n) = \sum_{i=-\infty}^{\infty} b_{k,m}(i) s_{k,m}(n-iP) \tag{1}$$

These coded chips are then sent to the destination through a wireless environment. In the current work, the physical channel is assumed to be FIR and LTI. If L antennas are employed at the receiver, the composite channel impulse response for the l-th subchannel including physical channel and the antenna response can be combined together and denoted by $g_{k,l}(n)$. Also we assume $g_{k,l}(n)$ has order q_k and factor it by P as $q_k = Q_k P + \eta_k$ where Q_k and η_k are both integers with $0 \leq \eta_k < P$. Hence the noise-free data from user k of l-th sub-channel and m-th branch can be expressed by

$$x_{k,l}^{(m)}(n) = \sum_{q=0}^{Q_k P+\eta_k} g_{k,l}(q) \sum_{i=-\infty}^{\infty} b_{k,m}(i) s_{k,m}(n-q-iP) \tag{2}$$

To explicitly reveal the channel input/output (I/O) relationship, we will employ matrix representation. Let us collect ν chip samples of $x_{k,l}^{(m)}(n)$ in a vector $\mathrm{x}_{k,l}^{(m)}(n)$ from $x_{k,l}^{(m)}(nP)$ to $x_{k,l}^{(m)}(nP+\nu-1)$. Since the channel spans at most $Q_k P + P - 1$ chip periods, ν is chosen as $\nu = QP$ with its minimum to be $\max(P + Q_k P + P) = \max(Q_k + 2)P$. We also arrange channel coefficients and precoding filtering coefficients in different vectors $\mathrm{g}_{k,l} = [g_{k,l}(0), \cdots, g_{k,l}(q_k)]^T$, $\mathrm{s}_{k,m} = [s_{k,m}(0), \cdots, s_{k,m}(P-1)]^T$. After careful calculation, the convolution in (2) can be nicely written in a matrix form as $\mathrm{x}_{k,l}^{(m)}(n) = \mathrm{U}_{k,m}(n)\mathrm{g}_{k,l}$ where $\mathrm{U}_{k,m}(n)$ has the structure in Fig. 2 with subblocks (in dashed dotted box) associated with different bits. Each subblock is the product of the corresponding bit and a shift version of $\nu \times (q_k+1)$ filtering matrix

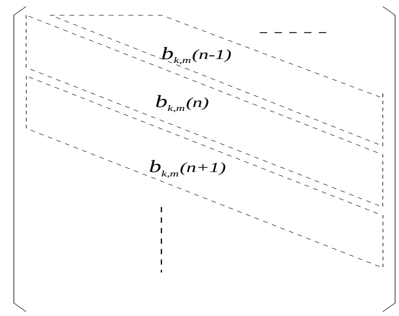

Fig. 2. structure of matrix $\mathrm{U}_{k,m}(n)$

$\mathrm{S}_{k,m}$ with shift (up or down) to be a multiple of P rows

$$\mathrm{S}_{k,m} = \begin{bmatrix} s_{k,m}(0) & & 0 \\ \vdots & \ddots & s_{k,m}(0) \\ s_{k,m}(P-1) & & \vdots \\ 0 & \ddots & s_{k,m}(P-1) \\ \vdots & 0 & 0 \end{bmatrix} \tag{3}$$

The shift operation can be made by a $\nu \times \nu$ Jordan matrix J with all $1's$ in the first diagonal below the main diagonal. For convenience, we will use the symbol J^{-1} to denote J^T although J is singular ([7]): $\mathrm{J}^{-1} \triangleq \mathrm{J}^T$ and define J^0 as an identity matrix. Assume the k-th user arrives at the receiver in a quasi-synchronous way with delay δ_k in chip periods. Considering M_k branches of $\mathrm{x}_{k,l}^{(m)}(n)$ for user k and putting L data vectors of $\mathrm{x}_{k,l}^{(m)}(n)$ successively in a big vector and similarly for the channel sub-vectors $\mathrm{g}_{k,l}$ respectively, it can be shown that the received data vector has the following form

$$\mathrm{y}(n) = \sum_{k=1}^{K} \sum_{m=1}^{M_k} \sum_{j=-(Q_k+1)}^{Q-1} \mathrm{A}_{k,j,m} \mathrm{g}_k b_{k,m}(n+j) + \mathrm{v}(n) \tag{4}$$

where $\mathrm{A}_{k,j,m}$ is a code filtering matrix

$$\mathrm{A}_{k,j,m} = \mathrm{I}_L \otimes (\mathrm{J}^{Pj+\delta_k} \mathrm{S}_{k,m}) \tag{5}$$

"$\otimes$" represents the Kronecker product, and $\mathrm{v}(n)$ is AWGN. The I/O model (4) shows that $\mathrm{y}(n)$ includes not only ISI within a block (indexed by m), but also IBI (indexed by j) from the same user. The signatures $\mathrm{A}_{k,j,m}\mathrm{g}_k$ of user k for different j and m all contain the channel information g_k. Based on this framework and subspace method, we will derive a cost function to blindly estimate channel vector g_k and delay δ_k next. Without loss of generality, user 1 is assumed to be the desired user.

3. Blind channel estimation

It is well known that subspace method is a powerful tool for blind channel estimation. It was originally proposed by [6], and has been successfully applied to single rate CDMA systems [2]. The method has been improved based on subspace intersection idea [10] for a long observation window equal to several symbol periods. Its extension to multirate CDMA systems is briefly discussed therein but with some restrictions on the length of the channel. It has been shown that it is also applicable to block equalization [7] in a precoded system with limited IBI and fixed data rate. We will extend this idea to channel and delay estimation in variable high rate multiuser communication scenarios.

To guarantee identifiability of channel vector g_1, some constraints on the size of related vectors and matrices have to be imposed. It is observed that the data vector $\mathrm{y}(n)$ in (4) is $LQP \times 1$ to which user k contributes $M_k(Q+Q_k+1)$ signatures corresponding to bits from both inter- and intra-blocks (summation in terms of j and m). The signature corresponding to bit $b_{k,m}(n+j)$ is $\mathrm{A}_{k,m,j}\mathrm{g}_k$ and denoted by $\mathrm{c}_{k,m,j}$. It is the product of the code filtering matrix and channel vector. If we collect these vectors in a matrix C_k column by column

$$\mathrm{C}_k = [\mathrm{c}_{k,m,j}], \;\; j = -(Q_k+1), \cdots, Q-1; \;\; m = 1, \cdots, M_k \tag{6}$$

and also put $b_{k,m}(n+j)$ in a vector $\mathrm{b}_k(n)$ at time n, then the output (4) is simplified as

$$\mathrm{y}(n) = \sum_{k=1}^{K} \mathrm{C}_k \mathrm{b}_k(n) + \mathrm{v}(n) \tag{7}$$

Without loss of generality, we assume signature waveforms $\{\mathrm{C}_k\}_{k=1}^{K}$ are linearly independent. The data correlation matrix R is computed from (7)

$$\mathrm{R} = E\{\mathrm{y}(n)\mathrm{y}^H(n)\} = \sum_{k=1}^{K} \zeta_k \mathrm{C}_k \mathrm{C}_k^H + \sigma_v^2 \mathrm{I} \tag{8}$$

where ζ_k is the signal power of user k, σ_v^2 is the noise power. To obtain the noise subspace, eigen decomposition of R yields

$$\mathrm{R} = \mathrm{U}_s \mathbf{\Lambda}_s \mathrm{U}_s^H + \mathrm{U}_n \mathbf{\Lambda}_n \mathrm{U}_n^H \tag{9}$$

$$\mathbf{\Lambda}_s = diag(\lambda_i^2 + \sigma_v^2), \;\; \mathbf{\Lambda}_n = \sigma_v^2 \mathrm{I}$$

It is well known that U_s spans the signal subspace which is also a range space of $[\mathrm{C}_1, \cdots, \mathrm{C}_K]$ and U_n spans the noise subspace. The dimension of the noise subspace is $\mu \triangleq LQP - \sum_{k=1}^{K} M_k(Q+Q_k+1)$. To guarantee the existence of noise subspace, we should have

$$LQP > \sum_{k=1}^{K} M_k(Q+Q_k+1) \tag{10}$$

The separation of noise subspace from signal subspace facilitates the estimation of channel vector g_1. For user 1, all signatures $\mathrm{c}_{1,m,j}$ lie in the signal subspace, thus they are orthogonal to U_n which leads to

$$\mathrm{U}_n^H \mathrm{c}_{1,m,j} = \mathrm{U}_n^H \mathrm{A}_{1,m,j} \mathrm{g}_1 = \mathbf{0} \tag{11}$$

for all possible j and m. It is a conventional subspace approach by picking a particular j and m. To guarantee a unique solution, it is required that $\mathrm{U}_n^H \mathrm{A}_{1,m,j}$ have full column rank. This matrix has dimension $\mu \times L(q_1+1)$. Thus $\mu \geq L(q_1+1)$. Together with (10), we require

$$L(QP - q_1 - 1) > \sum_{k=1}^{K} M_k(Q+Q_k+1) \tag{12}$$

(12) provides us a flexible choice of parameters based on the trade-off between design complexity and system capacity. If some users have low rate information sources, then the system can accommodate more users. If we have more antennas (large L), then we need short data records (small Q). Since conventional subspace method ignores rest of the signatures from the desired user, we propose to employ all possible information related to the unknown channel. Based on (11), we can similarly build our cost function with unknown channel vector [6], but consider all signatures of the desired user

$$\xi_1(\mathrm{g}) = \mathrm{g}^H [\sum_{m=1}^{M_1} \sum_{j=-(Q_1+1)}^{Q-1} \mathrm{A}_{1,m,j}^H \mathrm{U}_n \mathrm{U}_n^H \mathrm{A}_{1,m,j}] \mathrm{g} \tag{13}$$

Under some conditions (similar to those in the subspace method) and a unit norm constraint, for any given delay, minimization of the cost function (13) will guarantee a unique solution g_1 within a scalar ambiguity. This is established in the following theorem.

Theorem 1: Let S be a code matrix constructed by stacking $\mathrm{A}_{1,m,j}$ for all j, m in the row-wise and similarly H be an interference matrix whose columns consist of signatures from all interferers

$$\mathrm{S} = [\mathrm{A}_{1,m,j}], \;\; \mathrm{H} = [\mathrm{C}_2, \cdots, \mathrm{C}_K]$$

Under (10) and if $[\mathrm{S} \; \mathrm{H}]$ has full column rank, then g_1 is a unique minimum point of (13) within a scalar ambiguity.

Proof: For simplicity of presentation and without loss of generality, we may assume $L = 1$, $M_1 = 1$. The proof can be easily extended to general scenarios. Then we can denote $\mathrm{A}_{1,j,m}$ by A_j and $\mathrm{S}_{1,m}$ by S_1 for notational convenience.

The proof can proceed by contradiction [7],[9]. Suppose there is another vector $\widetilde{\mathrm{g}}_1$ which makes $\mathrm{A}_j \widetilde{\mathrm{g}}_1 \in \mathrm{U}_s$ for $\forall j$. Since span$\{\mathrm{U}_s\}$ = span$\{[\mathrm{C}_1 \; \mathrm{H}]\}$, $\mathrm{A}_j \widetilde{\mathrm{g}}_1$ can be expressed by different linear combinations of column vectors in $\{[\mathrm{C}_1 \; \mathrm{H}]\}$ for different j. By stacking vectors $\mathrm{A}_j \widetilde{\mathrm{g}}_1$ for all j in a matrix, then we can conclude that there exists matrices D_1 and D such that

$$\mathrm{S} \; diag(\widetilde{\mathrm{g}}_1) = \mathrm{S} \; diag(\mathrm{g}_1) \, \mathrm{D}_1 + \mathrm{H} \, \mathrm{D} \tag{14}$$

where $\mathrm{C}_1 = \mathrm{S} \; diag(\mathrm{g}_1)$ has been used and $diag(\cdot)$ represents a block diagonal matrix with diagonal entry to be the argument inside. Since $[\mathrm{S} \; \mathrm{H}]$ has full column rank, from (14) we have

$$diag(\widetilde{\mathrm{g}}_1) = diag(\mathrm{g}_1) \, \mathrm{D}_1 \tag{15}$$

By equating each sub-block of both sides of (15) we obtain $\mathrm{D}_1 = \alpha \mathrm{I}$ and $\widetilde{\mathrm{g}}_1 = \alpha \mathrm{g}_1$. □

After considering all j and m, eq. (11) becomes a generalized subspace method applicable to the current precoded multirate multiuser communication systems. Different from previous approaches, it employs all available information associated with the unknown channel. However, the delay τ_1 for user 1 is not a priori known and thus needs to be estimated. Taking into account (13), the joint estimation of the delay and channel vector for the desired user can be formulated as follows

$$(\widehat{\tau}_1, \widehat{\mathrm{g}}_1) = \arg\min_{\tau, \mathrm{g}} \mathrm{g}^H [\sum_{m=1}^{M_1} \sum_{j=-(Q_1+1)}^{Q-1} \mathrm{A}_{1,m,j}^H \mathrm{U}_n \mathrm{U}_n^H \mathrm{A}_{1,m,j}] \mathrm{g} \tag{16}$$

where $\mathbf{A}_{1,m,j}$ is a function of the delay and given by (5). The norm constraint on $\mathbf{g}$ should be imposed to avoid trivial solution. Since chip synchronization is assumed, it only requires checking all possible integers in $[0, P-1]$ for the delay. For each possible value, the normalized channel vector is the eigenvector of the matrix inside the brackets corresponding to its minimum eigenvalue within a phase ambiguity. Due to the unknown multipath distortion, the estimation of an arbitrary delay is more involved and needs further investigation.

4. Asymptotic Performance Analysis

It can be observed that to perform channel estimation, the noise subspace is required. It is obtained from eigen decomposition of $\mathbf{R}$ which is estimated from N data sample vectors $\widehat{\mathbf{R}} = \frac{1}{N}\sum_{n=1}^{N}\mathbf{y}(n)\mathbf{y}^H(n)$. The finite data samples will determine the accuracy of the subspace estimate, thus affect the performance of the estimator. In this section, we study the mean-square-error (MSE) of our channel estimator as a function of N. The analysis is based on the perturbation technique where applicability conditions are assumed valid ($N \to \infty$). Since channel parameters are jointly estimated with time delay, the performance is also determined by the delay estimation error. However, this joint problem will be simplified by assuming perfect timing in the following derivation (for example, [1]). Due to lack of space, we will only present major steps, references and results, while omitting the proof.

Our method is based on the noise subspace of estimated correlation matrix instead of the data matrix as used in [1], therefore the results therein are not directly applicable. However, similar procedures can be followed. In this sense, the following analysis is more relevant to [3]. An estimation error[1] $\delta\mathbf{R}$ is introduced in estimating $\mathbf{R}$ due to finite data samples [3]: $\widehat{\mathbf{R}} = \mathbf{R} + \delta\mathbf{R}$. In the current context, it is reasonable to assume that $\delta\mathbf{R}$ is Hermitian and is a small perturbation when N is large enough. This perturbation will cause an error in the estimate of noise subspace and finally transfer an error to the channel estimate.

For notational convenience, let

$$\mathbf{Z} = \mathbf{R} - \sigma_v^2\mathbf{I} = \mathbf{U}_s\boldsymbol{\Omega}\mathbf{U}_s^H, \;\; \boldsymbol{\Omega} = diag\{\lambda_i^2\}$$

$$\mathbf{X}_1 = \sum_{m=1}^{M_1}\sum_{j=-(Q_1+1)}^{Q-1}\mathbf{A}_{1,m,j}^H\mathbf{U}_n\mathbf{U}_n^H\mathbf{A}_{1,m,j} \tag{17}$$

It can be shown [5] that the first-order perturbation of the noise subspace is given by

$$\delta\mathbf{U}_n \approx -\mathbf{U}_s\boldsymbol{\Omega}^{-1}\mathbf{U}_s^H\delta\mathbf{R}\mathbf{U}_n = -\mathbf{Z}^{\dagger}\delta\mathbf{R}\mathbf{U}_n \tag{18}$$

with $(\cdot)^{\dagger}$ denotes pseudo-inverse.

Our channel estimate is the eigenvector of $\mathbf{X}_1$ in (17) corresponding to its unique null eigenvalue. It can be easily found that the first-order perturbation of $\mathbf{X}_1$ has the form

$$\delta\mathbf{X}_1 \approx \sum_{m=1}^{M_1}\sum_{j=-(Q_1+1)}^{Q-1}\mathbf{A}_{1,m,j}^H[\delta\mathbf{U}_n\mathbf{U}_n^H + \mathbf{U}_n\delta\mathbf{U}_n^H]\mathbf{A}_{1,m,j} \tag{19}$$

[1] We will use $\widehat{\cdot}$ to represent the perturbed version of the argument, and a composite term with δ before the argument to represent its perturbation next.

Assume the first-order perturbation of $\mathbf{g}_1$ is $\delta\mathbf{g}_1$. Then $\delta\mathbf{g}_1$ has the following form [1] [3]

$$\delta\mathbf{g}_1 \approx -\mathbf{X}_1^{\dagger}\delta\mathbf{X}_1\mathbf{g}_1 \tag{20}$$

After substituting (18) in (19) and then (19) in (20), and noticing that $\mathbf{A}_{1,m,j}\mathbf{g}_1$ is orthogonal to $\mathbf{U}_n$, we obtain

$$\delta\mathbf{g}_1 \approx \sum_{m=1}^{M_1}\sum_{j=-(Q_1+1)}^{Q-1}\mathbf{X}_1^{\dagger}\mathbf{A}_{1,m,j}^H\mathbf{U}_n\mathbf{U}_n^H\delta\mathbf{R}\mathbf{Z}^{\dagger}\mathbf{A}_{1,m,j}\mathbf{g}_1 \tag{21}$$

Eq. (21) shows that $\delta\mathbf{g}_1$ is directly related to $\delta\mathbf{R}$. Using the result about statistics for $\delta\mathbf{R}$ in [3], it can be shown that the MSE of channel estimate satisfies

$$E\{\|\delta\mathbf{g}_1\|^2\} \approx \frac{\sigma_v^2}{N}\sum_{m_1,m_2}\sum_{j_1,j_2}\beta_{m_1,m_2,j_1,j_2}trace\{\mathbf{B}_{m_1,m_2,j_1,j_2}\} \tag{22}$$

where $trace$ is a trace operator for a matrix,

$$\beta_{m_1,m_2,j_1,j_2} = \mathbf{g}_1^H\mathbf{A}_{1,m_2,j_2}^H\mathbf{U}_s\boldsymbol{\Delta}^{-1}\mathbf{U}_s^H\mathbf{A}_{1,m_1,j_1}\mathbf{g}_1$$

$$\boldsymbol{\Delta} = diag\{\frac{\lambda_i^4}{\lambda_i^2 + \sigma_v^2}\}$$

$$\mathbf{B}_{m_1,m_2,j_1,j_2} = \mathbf{X}_1^{\dagger}\mathbf{A}_{1,m_1,j_1}^H\mathbf{U}_n\mathbf{U}_n^H\mathbf{A}_{1,m_2,j_2}\mathbf{X}_1^{\dagger}$$

In is clear that the MSE is proportional to $\frac{\sigma_v^2}{N}$ which is similar to the results obtained in [1]. The MSE is inversely proportional to the power of the transmitted signal approximately due to the term $\boldsymbol{\Delta}$.

5. Simulations

First we test the proposed generalized subspace method (16) by simulating a variable rate system with 5 users. Each user has i.i.d. binary bits to transmit. Hadamard codes of length $P = 8$ are chosen for each of the five precoders. Different information rates are obtained by choosing downsampling factors (M_k) for different filterbanks as $[2,1,3,1,1]$ respectively. Channels are randomly generated with unit norm for different users and have orders (q_k) $[5,11,19,19,18]$. Q can be chosen as 4 and finally $\nu = 32$. To satisfy (12) for estimation of channel 1, we use 3 antennas ($L = 3$) at the receiver. Delays are $[1,3,2,0,2]$. The SNR is set to be $20dB$. With these parameters, we test the effect of the input data length (in blocks) on the MSE of estimation for channel 1. We compare the proposed method (16) with conventional subspace method (11) based on the average results from 50 realizations ($j = 0$ and $m = 1$ is chosen for the conventional subspace method to have a full signature). The channel estimation error versus the number of blocks of the input is plotted in Fig. 3. The dashed-dotted line represents the conventional subspace method, the solid line for the proposed, and the dashed line is from the theoretical result (22). It can be observed that the experimental result coincides with our performance analysis. If only partial information related to the channel vector is used as in the subspace method, the error is much larger than the proposed method. Together with channel estimation, delays are also estimated and shown for four users in Fig. 4. It is seen that for user 1 to 3, estimates are reliable, while for user 4, it does not give a proper estimate. This is not surprising since the similar condition for user 4 as (12) is not satisfied. However, in this case even (12) is violated for user 3, the delay estimate is still correct. It is also observed in our experiments that our channel estimate for user 3 is reliable, while the subspace method fails. We also test the code effect on the performance by choosing different coefficients for the precoders such as Hadamard codes, real random codes in the range $(0, 1)$ and binary random codes from $\{\pm 1\}$. The MSEs are plotted in Fig. 5. It can be seen that with severe multipath distortions, the random codes give better performance. However, the choice of the precoders based on some criteria is still under investigation.

References

[1] E. Aktas and U. Mitra, "Complexity Reduction in Subspace-Based Blind Channel Identification for DS/CDMA Systems", *IEEE Trans. on Comm.*, vol. 48, no. 8, pp. 1392-1404, August 2000.

[2] S. E. Bensley and B. Aazhang, "Subspace-based channel estimation for code division multiple access communication Systems", *IEEE Trans. Commun.*, pp. 1009-1020, August 1996.

[3] B. Friedlander and A. Weiss, "On the Second-Order Statistics of the Eigenvectors of Sample Covariance Matrices", *IEEE Trans. on Signal Processing*, vol. 46, no. 11, pp. 3136-3139, November 1998.

[4] M.J. Juntti, "System concept comparisons for multirate CDMA with multiuser detection", *Proc. of 48th IEEE Vehicular Technology Conference*, vol.1, pp.36-40, Ottawa, Canada, May 1998.

[5] F. Li, H. Liu and R. Vaccaro, "Performance Analysis for DOA Estimation Algorithms: Unification, Simplification, and Observations", *IEEE Trans. on Aerospace and Electronic Systems*, vol. 29, no. 4, pp. 1170-1184, October 1993.

[6] E. Moulines, P. Duhamel, J.-F. Cardoso, and S. Mayrargue, "Subspace Methods for the Blind Identification of Multichannel FIR Filters," *IEEE Trans. on Signal Processing*, vol. 43, no. 2, pp. 516-525, February 1995.

[7] A. Scaglione, G. B. Giannakis, and S. Barbarossa, "Redundant Filterbank Precoders and Equalizers, Part II: Blind Channel Estimation, Synchronization, and Direct Equalization," IEEE Transactions on Signal Processing, vol. 47, no. 7, pp. 2007-2022, July 1999.

[8] A. Scaglione, G. B. Giannakis, and S. Barbarossa, "Redundant Filterbank Precoders and Equalizers, Part I: Unification and Optimal Designs," IEEE Transactions on Signal Processing, vol. 47, no. 7, pp. 1988-2006, July 1999.

[9] M. K. Tsatsanis and Z. Xu, "Performance Analysis of Minimum Variance CDMA Receivers", *IEEE Trans. on Signal Processing*, vol. 46, no. 11, pp. 3014-3022, November 1998.

[10] A. J. Van Der Veen, A.-J. "Blind subspace-based uplink receiver algorithm for wideband CDMA", *Proc. IEEE VTS 50th Vehicular Technology Conference*, vol. 1, pp.176-180, Amsterdam, Netherlands, Sept. 19-22, 1999.

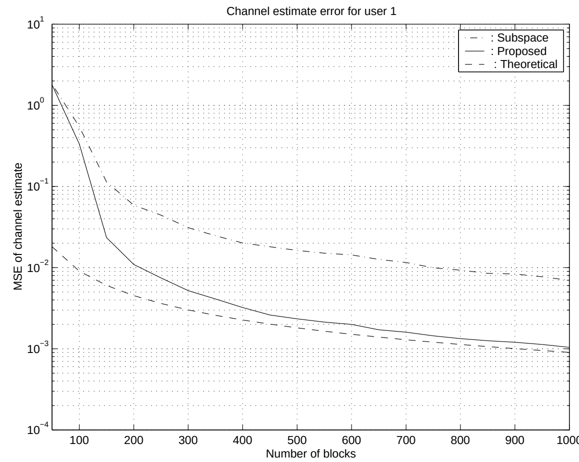

Fig. 3. Data length effect on channel estimation error.

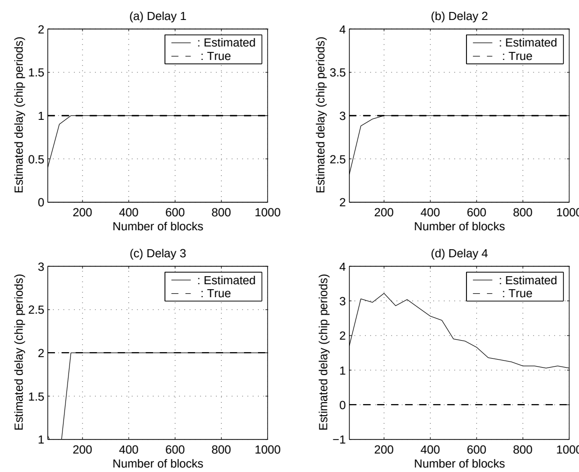

Fig. 4. Data length effect on delay estimation.

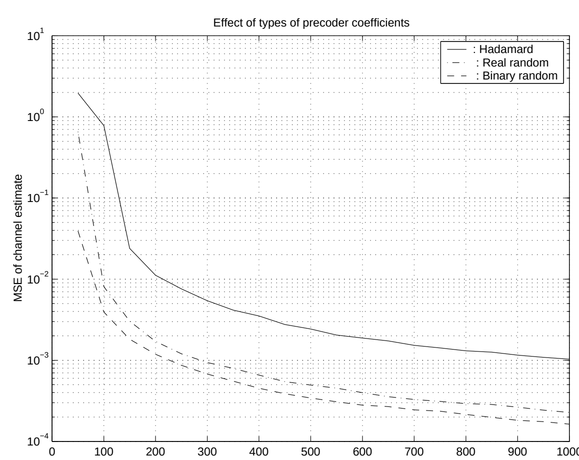

Fig. 5. Effects of types of precoder coefficients on channel estimation error.