PERFORMANCE STUDY OF A HIGH-RATE MULTIUSER TRANSMITTED REFERENCE ULTRA-WIDEBAND TRANSCEIVER

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ABSTRACT

Inheriting all merits of a multiuser transmitted reference (MTR) ultra-wideband (UWB) transceiver, a biorthogonal MTR (BO-MTR) transceiver for high data-rate communication has been recently reported. It utilizes 4-ary biorthogonal signaling to carry two bits of each user's information in amplitude and position of the data pulse simultaneously. Assisted by a simple coding scheme, both bits are independently decoded using low complexity correlators whose templates are obtained from aggregated reference signals spanning multiple symbol intervals. This paper further studies its performance in different scenarios. In a multipath-free fading scenario, the receiver's symbol error rate (SER) performance is shown to be optimal while no complex joint detection of two bits is necessary, and the bit error rate (BER) performance of each detector is balanced. For multipath channels, when channel parameters are either deterministic and estimated with errors, or random in Rician distribution, the Chernoff bounds are derived for BER of each detector. A much tighter bound is also provided according to the property of the complementary Gaussian distribution function.[1]

INTRODUCTION

An ultra-wideband (UWB) signal suffers from severe multipath fading. Transmitted reference (TR) techniques have demonstrated exceptional demodulation capabilities in an unknown UWB channel [1], [2], [3], [4]. Each time, a pair of time separated reference-data pulses is transmitted using binary phase shift keying (BPSK), or pulse amplitude modulation (PAM), or pulse position modulation (PPM). The time delay of the data pulse with respect to the reference pulse is set larger than the channel delay spread to avoid inter-pulse interference (IPI) at the receiver. Then a template is extracted from the reference signal to demodulate the data signal via a low complexity correlator [1], [2]. However, the minimum spacing of the two pulses inevitably sacrifices data rate for performance, especially when the channel delay spread is very large. Meanwhile, the channel is used for transmission of user's information only about half the time, thus incurring about 50% rate penalty. Moreover, the template may be very noisy, making the conventional TR demodulation technique ineffective. If smaller spacing of the two pulses is anticipated in order to increase transmission rate, then IPI severely contaminates the template and consequently yields poorer detection performance.

In order to improve the quality of the template and consequently detection performance, some works propose a sample averaging technique using all received reference signals [5], [6], [7]. Aware of the data rate limitation imposed by the pulse separation, the pulse separation can be reduced to increase data rate and corresponding IPI suppression techniques have to be employed [8], [9]. Alternatively, M-ary biorthogonal (BO) signaling by joint PAM and PPM is able to at least double the data rate if the data pulse is allowed to carry information in both its amplitude and position [10], [11]. Similarly in the category of hybrid data modulation, those two bits can be distributed to two consecutive pulses in PAM and PPM respectively and then a mutual reference idea is exploited to demodulate data [12]. In this setup, no clear concept of reference and data pulses exists, but merits of the TR scheme are still preserved. Meanwhile, some benefits of smoothed signal spectrum are achieved. Corresponding transceiver structures are suggested in [13]. To support multiple access, pseudo-random (PN) sequences and time hopping sequences are

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combined. Thus spreading and pulsed transmission help to protect user signal from interference by a multiuser transmitted reference (MTR) transceiver [14] or a BO-MTR scheme [15] to achieve much higher data rate.

This paper continues the investigation of the performance analysis of the high-rate BO-MTR transceiver proposed in [15]. First, under multipath-free channels, the single-user bit error rate (BER) detection performance for each of the two bits is derived as a function of the autocorrelation of the pulse. It can serve as a benchmark for evaluation of the receiver in other scenarios. It is observed that the bit detection performance is balanced if biorthogonal signaling is employed. Not only does each detector enjoy low complexity, but also the symbol detection performance in terms of symbol error rate (SER) is optimal, same for the optimal detector for 4-ary biorthogonal signals. Symbol detection performance is also compared with 4-ary PAM and 4-ary PPM signals [16]. Results demonstrate an excellent power and spectral tradeoff. Second, when a multipath channel is estimated with an error using a sample averaging technique, the Chernoff BER bound is derived under the channel estimation penalty. The bound is tightened according to the property of the complementary Gaussian distribution function (Q-function). If a random channel such as Rayleigh or Rician distributed multipath channel is considered [17], [18], the Chernoff bound is also derived.

A HIGH-RATE BO-MTR UWB TRANSCEIVER

In the high-rate BO-MTR UWB scheme [15], the first pulse is a reference pulse, and the second pulse is modulated by two information bits $I_{k,n}^{(1)}$ and $I_{k,n}^{(2)}$ for user k in its amplitude and delay using PAM and PPM respectively. Consider binary information $I_{k,n}^{(1)}$, $I_{k,n}^{(2)}$ $\in \{\pm 1\}$. The transmitted signal of power $\mathcal{P}_k$ due to user k in a K-user system can be described by

$$s_k(t) = \sqrt{\frac{\mathcal{P}_k}{2}} \sum_n [A_{k,n} w(t - nT_f) + I_{k,\lfloor n/N_f \rfloor}^{(1)} B_{k,n} w(t - nT_f - \tau_k - b_{k,\lfloor n/N_f \rfloor}\sigma_d)], \quad (1)$$

where $w(t)$ is the monopulse, T_f is frame duration, $A_{k,n}$ and $B_{k,n}$ are two binary PN sequences taking $\{\pm 1\}$, $\tau_k = c_k T_c$ is the nominal delay of the data pulse in multiples of T_c and controlled by a hopping code c_k, $b_{k,n} \triangleq \frac{1}{2}|I_{k,n}^{(1)} - I_{k,n}^{(2)}|$ is a coded information bit, σ_d is a PPM modulation parameter.

After propagating through a multipath channel $\theta_k(t)$, each pulse is distorted by the transmitter antenna, channel, receiver antenna and bandpass filter $g(t)$, so that the effective channel is $h_k(t) = \sqrt{\frac{\mathcal{P}_k}{2}} w(t) \star \theta_k(t) \star g(t)$. Assume the receiver is synchronized to the transmitter. Then the received signal takes the following form

$$\begin{aligned} r(t) &= \sum_{k,n} [A_{k,n} h_k(t - nT_f) \\ &+ I_{k,\lfloor n/N_f \rfloor}^{(1)} B_{k,n} h_k(t - nT_f - \tau_k - b_{k,\lfloor n/N_f \rfloor}\sigma_d)] \\ &+ v(t), \end{aligned} \quad (2)$$

where $v(t) = n(t) \star g(t)$ is the output of the filter $g(t)$ whose input is additive white Gaussian noise (AWGN) $n(t)$ with double-sided power spectral density $\frac{N_0}{2}$.

Suppose $h_k(t)$ has support in $(0, T_h)$ where $T_h \gg \tau_k$ and $T_h < T_f$. Therefore, no IPI occurs at the receiver. The goal of receiver design is to detect $I_{k,n}^{(1)}$ and $I_{k,n}^{(2)}$. For simplicity, term the former as the PAM bit and the latter as the PPM bit. In [15], the template $U_k(t) = h_k(t) + \nu h_k(t - \sigma_d)$ is proposed to independently detect the PAM bit and PPM bit by setting $\nu = 1$ and $\nu = -1$ respectively. Each template requires acquisition of the waveform $h_k(t)$ from the received noisy signal. This waveform repeats from frame to frame, but is corrupted by various interferences. Rather than directly using the noisy received signal, we use a smoothing technique to obtain a cleaner waveform from aggregated reference signals.

Let's consider the received signal $r(t)$ in N_s symbol intervals, and partition it into segments each of frame duration. Totally, there are $N_p \triangleq N_f N_s$ segments. The m'-th ($m' = 1, \cdots, N_p$) segment is denoted by $r_{m'}(t) \triangleq r(t + m'T_f)$ for $t \in [0, T_f)$, and $r_{m'}(t) \triangleq 0$ elsewhere. Similarly, define $v_{m'}(t)$ for the noise. Then, the waveform is estimated as follows

$$\widehat{h}_k(t) = \frac{1}{N_p} \sum_{m'} A_{k,m'} r_{m'}(t). \quad (3)$$

The n-th symbol interval is comprised of N_f frame intervals ($m = nN_f, \cdots, (n+1)N_f - 1$). For each frame segment, a generic signal $\widetilde{r}_{k,m}(t)$ is obtained from $r_m(t)$ by subtracting estimated reference signal, that is then shifted and multiplied by $B_{k,m}$

$$\begin{aligned} \widetilde{r}_{k,m}(t) &= r_m(t) - A_{k,m} \widehat{h}_k(t), \\ \bar{r}_{k,m}(t) &= B_{k,m} \widetilde{r}_{k,m}(t + c_k T_c). \end{aligned} \quad (4)$$

Then detection of both PAM and PPM bits can be performed as [15]

$$\widehat{I}_{k,n} = \text{sign}\Big(\frac{1}{N_f} \sum_m \int_0^{T_f} \widehat{U}_k(t)|_{\nu=\pm 1} \ \bar{r}_{k,m}(t) dt \Big). \quad (5)$$

Modulation parameter $\sigma_d > 0$ affects the autocorrelation of different waveforms and consequently detection performance. Its effect will be analyzed later.

PERFORMANCE STUDY

This section studies BER and SER performance of the BO-MTR UWB system in terms of single user performance, BER bound with template estimation penalty, and BER bound in a multipath fading scenario.

SINGLE USER PERFORMANCE BOUND

Consider a single user scenario with perfect template, free of multipath and reference pulse. The performance can serve as a benchmark for other scenarios. Assume the symbol energy is $\mathcal{E}_{TX}$. Two information bits $I^{(1)}$ and $I^{(2)}$ are transmitted using PAM for $I^{(1)}$ and PPM for coded bit $b = \frac{1}{2}|I^{(1)} - I^{(2)}|$ as

$$s(t) = I^{(1)}\sqrt{\mathcal{E}_{TX}/N_f}\sum_{m=0}^{N_f-1} w(t - mT_f - b\sigma_d).$$

The N_f received signal segments become

$$r_m(t) = I^{(1)}\sqrt{\mathcal{E}_{RX}/N_f}w(t - b\sigma_d) + n_m(t).$$

Using the template $U(t) = h(t) + \nu h(t - \sigma_d)$, the correlator's output becomes

$$Y = I^{(1)}\sqrt{\mathcal{E}_{RX}/N_f}[R_{b\sigma_d} + \nu R_{(1-b)\sigma_d}] + \int U(t)\bar{n}(t)dt,$$

where R_τ is the deterministic autocorrelation of pulse $w(t)$ at lag τ, $\bar{n}(t) = \frac{1}{N_f}\sum_m n_m(t)$, and $n_m(t)$ are independent samples with power spectrum density $\sigma_v^2 = N_0/2$. It can be found that the noise in Y has variance $2(R_0 + \nu R_{\sigma_d})\sigma_v^2/N_f$.

For the PAM bit $I^{(1)}$, $\nu = 1$, we have

$$Y = I^{(1)}\sqrt{\mathcal{E}_{RX}/N_f}[R_0 + R_{\sigma_d}] + \int U(t)\bar{n}(t)dt,$$

regardless of $b = 0$ or 1. Let $\mathcal{E}_{RX} = 2E_b$. The BER for detecting the PAM bit is

$$P_{b,PAM} = Q\Big(\sqrt{\frac{E_b(R_0 + R_{\sigma_d})}{\sigma_v^2}}\Big).$$

where $Q(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}}e^{-t^2/2}dt$ is the Q-function. If we denote $\gamma_b = E_bR_0/N_0$ and $\delta = R_{\sigma_d}/R_0$, then it becomes

$$P_{b,PAM} = Q\Big(\sqrt{2\gamma_b(1+\delta)}\Big). \tag{6}$$

For the PPM bit $I^{(2)}$, $\nu = -1$, we have

$$Y = I^{(1)}\sqrt{\mathcal{E}_{RX}/N_f}[R_{b\sigma_d} - R_{(1-b)\sigma_d}] + \int U(t)\bar{n}(t)dt.$$

Considering all four possible pairs of $(I^{(1)}, I^{(2)})$ as $(1,1)$, $(1,-1)$, $(-1,1)$, and $(-1,-1)$, it can be written as

$$Y = I^{(2)}\sqrt{\mathcal{E}_{RX}/N_f}[R_0 - R_{\sigma_d}] + \int U(t)\bar{n}(t)dt.$$

Using $\mathcal{E}_{RX} = 2E_b$ and the definition of δ, the BER for detecting the PPM bit is

$$P_{b,PPM} = Q\Big(\sqrt{2\gamma_b(1-\delta)}\Big). \tag{7}$$

Detection performance for either PAM bit or PPM bit is a function of δ or equivalently the PPM modulation parameter σ_d. If σ_d is chosen such that $R_{\sigma_d} = 0$ for biorthogonal signaling, then the detection performance is balanced. If σ_d is chosen such that $R_{\sigma_d} > 0$, then it is more favorable to the PAM bit. In an extreme case, $\sigma_d = 0$, then $R_{\sigma_d} = R_0$, and $\delta = 1$. A $3dB$ gain is achieved for the PAM bit detection by significantly sacrificing the PPM bit detection performance. If σ_d is chosen such that $R_{\sigma_d} < 0$, then it is more favorable to PPM bit. This situation is similar to pure PPM signaling.

Since both BERs are functions of R_{σ_d} or equivalently δ, a question arises as of its choice. Let's consider the SER

$$P_s = 1 - (1 - P_{b,PAM})(1 - P_{b,PPM}).$$

Invoking (6) and (7), it can be expanded as

$$\begin{aligned} P_s &= Q\Big(\sqrt{2\gamma_b(1+\delta)}\Big) + Q\Big(\sqrt{2\gamma_b(1-\delta)}\Big) \\ &\quad - Q\Big(\sqrt{2\gamma_b(1+\delta)}\Big)Q\Big(\sqrt{2\gamma_b(1-\delta)}\Big). \end{aligned} \tag{8}$$

We would like to choose δ such that the SER is minimized. It is a nonlinear function of δ. Recall the Leibnitz differentiation rule

$$\begin{aligned} \frac{dF_z(z)}{dz} &= \frac{db_z(z)}{dz}f(b_z(z), z) - \frac{da_z(z)}{dz}f(a_z(z), z) \\ &\quad + \int_{a(z)}^{b(z)} \frac{\partial f(x,z)}{\partial z}dx, \end{aligned} \tag{9}$$

For differentiating the function

$$F_z(z) = \int_{a(z)}^{b(z)} f(x,z)dx,$$

we obtain

$$\frac{dQ(x)}{dx} = -\frac{1}{\sqrt{2\pi}}e^{-x^2/2}.$$

Then taking the derivative of P_s with respective to δ term by term and setting it equal to zero after ignoring an irrelevant factor $e^{-\gamma_b}\sqrt{\gamma_b/4\pi}$, we obtain

$$-\frac{e^{-\gamma_b\delta}}{\sqrt{1+\delta}}+\frac{e^{\gamma_b\delta}}{\sqrt{1-\delta}}+\frac{e^{-\gamma_b\delta}}{\sqrt{1+\delta}}Q\Big(\sqrt{2\gamma_b(1-\delta)}\Big) -\frac{e^{\gamma_b\delta}}{\sqrt{1-\delta}}Q\Big(\sqrt{2\gamma_b(1+\delta)}\Big)=0. \quad (10)$$

Multiplying by $e^{\gamma_b\delta}\sqrt{1+\delta}$ on both sides, it becomes

$$e^{2\gamma_b\delta}\sqrt{\frac{1+\delta}{1-\delta}}=\frac{1-Q\Big(\sqrt{2\gamma_b(1-\delta)}\Big)}{1-Q\Big(\sqrt{2\gamma_b(1+\delta)}\Big)}. \quad (11)$$

The solution to this equation can be found by the following observation. If $\delta>0$, the left hand side is greater than 1, while the right hand side is less than 1. Similarly, (11) is not satisfied for the case of $\delta<0$. The only solution to the equation is $\delta=0$. We thus conclude that when biorthogonal signaling is applied, the SER is minimized and the symbol detection performance is optimal. Under this condition, the proposed receiver is regarded as a 4-ary BO-MTR receiver. According to (6) and (7), the individual bit detection performance is balanced, each of which is $Q(\sqrt{2\gamma_b})$. Using (8), the minimum SER becomes

$$P_{s,min}=2Q(\sqrt{2\gamma_b})-Q^2(\sqrt{2\gamma_b})>Q(\sqrt{2\gamma_b}). \quad (12)$$

It can be approximated by $2Q(\sqrt{2\gamma_b})$ for large γ_b.

It is interesting to compare the minimum achievable SER with detection performance of M-ary PAM, M-ary PPM and M-ary biorthogonal signals when $M=4$. Let $\gamma_s=\gamma_b\log_2 M$. Analytical SER expressions for different signals can be found from [16]. Figure 1 shows SERs for different signaling formats. Clearly, the SER achieved by the BO-MTR receiver has the same performance as the optimal receiver for 4-ary biorthogonal signals (the two corresponding curves overlap). It is much better than 4-ary PAM and slightly better than 4-ary PPM. Therefore, in terms of detection performance, biorthogonal signaling is preferable. It should be mentioned that when the reference signal is transmitted together with data signal with equal power, the signal energy incurs a 3dB penalty and detection performance degrades.

A comparison can also be made among different signaling formats from the perspective of the normalized data rate, i.e., the number of bits per second per hertz of signal bandwidth [16]. For M-ary PAM signaling, it is $2\log_2 M>1$; for M-ary PPM signaling, it is $2\log_2 M/M<1$; for M-ary biorthogonal signaling, it is $4\log_2 M/M$. Thus PAM is appropriate for bandwidth-limited channels at a price of increased signal energy per bit. PPM is suitable for power-limited channels but is bandwidth inefficient. Biorthogonal modulation achieves a compromise for a reasonable tradeoff between power and bandwidth efficiency.

PERFORMANCE BOUND WITH CHANNEL ESTIMATION PENALTY

The template is estimated based on finite noisy observations corrupted by multiple access interference (MAI). The estimation penalty causes degraded detection performance. In [15], BER expressions for detection of PAM and PPM bits are derived based on the signal to interference plus noise ratio (SINR) and Gaussian approximation to the interference signal. We will derive the Chernoff bound in this situation.

The decision variable given by (5) is based on

$$\bar{r}_{k,m}(t)=I_{k,n}^{(1)}h_k(t-b_{k,n}\alpha T_c)+u_{k,m}(t), \quad (13)$$

where $\sigma_d=\alpha T_c$, $u_{k,m}(t)$ represents waveform estimation error, MAI, plus noise,

$$u_{k,m}(t)=-A_{k,m}B_{k,m}\delta h_k(t+c_kT_c) +B_{k,m}v_m(t+c_kT_c)+\sum_{l\neq k}[A_{l,m}B_{k,m}h_l(t+c_kT_c) +B_{l,m}B_{k,m}I_{l,n}^{(1)}h_l(t+c_kT_c-c_lT_c-b_{l,n}^{(2)}\alpha T_c)]. \quad (14)$$

Consider the PAM bit first. Suppose $I_{k,n}^{(1)}=-1$. Then $b_{k,n}=|I_{k,n}^{(2)}+1|/2$ takes 0 and 1 with equal probability. The decision variable is given by

$$y_n=\int_0^{T_f}\widehat{U}_k(t)[-h_k(t-b_{k,n}\sigma_d)+\frac{1}{N_f}\sum_m u_{k,m}(t)]dt.$$

The probability of detection error satisfies

$$P_E=Pr(y_n>0|-1)<\min_{\rho>0}\frac{1}{2}E\Big[\exp(\rho y_n)\Big]. \quad (15)$$

Since y_n is a function of multiple random variables, carrying out expectation on $b_{k,n}$ first, we obtain

$$E\Big[\exp(\rho y_n)\Big]=e^{-\rho(R_0+\nu R_{\sigma_d})}E\{e^{\rho z_n}\}, \quad (16)$$

where

$$R_0=\int_0^{T_f}\Big[h_k(t)\Big]^2dt=\mathcal{E}_{k,k,0,0}, \quad (17)$$

$$R_{\sigma_d}=\int_0^{T_f}\Big[h_k(t)h_k(t-\sigma_d)\Big]dt=\mathcal{E}_{k,k,0,\alpha}, \quad (18)$$

$$z_n=-\int_0^{T_f}\delta U_k(t)h_k(t-b_{k,n}\alpha T_c)dt +\int_0^{T_f}\widehat{U}_k(t)\frac{1}{N_f}\sum_m u_{k,m}(t)dt. \quad (19)$$

For the interference plus noise term z_n, we assume it is Gaussian with zero mean whose variance ϵ_n is given by eq. (20) in [15] as a function of sample size N_p and other system parameters such as N_f, noise power, PPM modulation parameter σ_d, cross-correlation of users' channels, hopping codes. Then [18]

$$E[e^{\rho z_n}] = e^{\frac{1}{2}\rho^2 \epsilon_n}. \quad (20)$$

Using (16) and (20), (15) becomes

$$P_E = \min_{\rho>0} \frac{1}{2} \exp\Big[-\rho(R_0 + \nu R_{\sigma_d}) + \frac{1}{2}\rho^2 \epsilon_n \Big]. \quad (21)$$

The exponent is a quadratic function of ρ and reaches its minimum as follows when $\rho = (R_0 + \nu R_{\sigma_d})/\epsilon_n$

$$P_E < \frac{1}{2} e^{-\frac{1}{2}\frac{(R_0+\nu R_{\sigma_d})^2}{\epsilon_n}} = \frac{1}{2} e^{-\frac{1}{2} SINR}. \quad (22)$$

If P_E is approximated by $Q(\sqrt{SINR})$ as in [15], the above result is not surprising since

$$Q(x) \leq \frac{1}{2} e^{-x^2/2}.$$

The Chernoff bound (22) is loose and can be tightened as

$$P_E < \frac{1}{\sqrt{2\pi}\sqrt{SINR}} e^{-\frac{1}{2}SINR}, \quad (23)$$

due to the property of Q-function. The above result is also applicable to the PPM bit. Set $\nu = -1$ and replace ϵ_n by the corresponding expression.

PERFORMANCE BOUND FOR FADING CHANNELS

Consider multipath fading channel for user k

$$\theta_k(t) = \sum_{q=1}^{L_k} h_{k,q} \delta(t - \tau_{k,q}),$$

where $h_{k,q}$ are independent Rician random variables. Define $\gamma_{k,q} = h_{k,q}^2$, and $\sigma_{k,q}^2 = E\{\gamma_{k,q}\}$. Then $\gamma_{k,q}$ is non-central chi-squared with probability density function (pdf)

$$p(\gamma_{k,q}) = \frac{1}{\sigma_{k,q}^2} e^{-\frac{\gamma_{k,q}+\zeta}{\sigma_{k,q}^2}} \varphi_0(\frac{2\sqrt{\gamma_{k,q}\zeta}}{\sigma_{k,q}^2}),$$

where ζ is the Rician parameter, φ_0 is the modified Bessel function of first kind and zero-th order. It reduces to a Rayleigh random variable if $\zeta = 0$. The waveform takes the form of $h_k(t) = \sqrt{\frac{\mathcal{P}_k}{2}} w(t) \star \theta_k(t) \star g(t) = \sqrt{\frac{\mathcal{P}_k}{2}} \sum_q h_{k,q} s(t - \tau_{k,q})$, $s(t) = w(t) \star g(t)$. For simplicity, we assume paths are resolvable after matched filtering, i.e., $s(t)$ does not overlap with its delayed copies.

Consider the PAM bit first. Compared with the case of channel estimation errors, now we also have to take expectations with respect to the random channel in order to obtain an average performance bound. Write $y_n = z_s + z_n$. Then probability of detection error is upper bounded by the Chernoff bound

$$P_E < \min_{\rho>0} \frac{1}{2} E\Big[\exp(\rho z_s)\Big] E\Big[\exp(\rho z_n)\Big]. \quad (24)$$

Each term can be evaluated as follows.

A. Signal part

Using (16), we can find

$$E\Big[\exp(\rho z_s)\Big] = E_{h_k}\Big\{\exp[-\rho(R_0 + \nu R_{\sigma_d})]\Big\}, \quad (25)$$

where R_0 and R_{σ_d} are the autocorrelation of the waveform as defined in (17) and (18). Using the expression for $h_k(t)$ and noticing multiple pulses do not overlap, we obtain

$$R_0 + \nu R_{\sigma_d} = \sum_q h_{k,q}^2 a_{k,q},$$

where

$$a_{k,q} = \frac{\mathcal{P}_k}{2} \int_0^{T_f} \Big[s^2(t-\tau_{k,q}) + \nu s(t-\tau_{k,q}) s(t-\tau_{k,q}-\sigma_d)\Big] dt.$$

Therefore,

$$E\Big[\exp(\rho z_s)\Big] = \prod_{q=1}^{L_k} E\Big[\exp(-\rho a_{k,q}\gamma_{k,q})\Big]. \quad (26)$$

For Rician random variable $\gamma_{k,q}$, it can be found that

$$E\Big[\exp(-\rho a_{k,q}\gamma_{k,q})\Big] = \frac{1}{1+\rho a_{k,q}\sigma_{k,q}^2} e^{-\frac{\rho a_{k,q}\zeta}{1+\rho a_{k,q}\sigma_{k,q}^2}}. \quad (27)$$

For convenience of optimization of (24), we will further bound each term on the right hand side above in an exponential form with the exponent as a linear function of ρ. A positive μ_1 and μ_2 can be found to upper bound $\frac{1}{1+x}$ and lower bound $\frac{x}{1+x}$ tightly in the range $0 < x \ll 1$ as

$$\frac{1}{1+x} < e^{-\mu_1 x}, \quad \frac{x}{1+x} > \mu_2 x.$$

Thus for low SNR regions

$$\frac{1}{1+\rho a_{k,q}\sigma_{k,q}^2} < e^{-\mu_1 \rho a_{k,q}\sigma_{k,q}^2}, \quad (28)$$

$$\frac{-\rho a_{k,q,q}\zeta}{1+\rho a_{k,q}\sigma_{k,q}^2} = -\frac{\zeta}{\sigma_{k,q}^2} \frac{\rho a_{k,q}\sigma_{k,q}^2}{1+\rho a_{k,q}\sigma_{k,q}^2} < -\rho a_{k,q}\mu_2\zeta. \quad (29)$$

Applying (28) and (29) to (27), we obtain

$$E\Big[\exp(-\rho a_{k,q}\gamma_{k,q})\Big] < \exp\Big[-\rho a_{k,q}(\mu_1\sigma_{k,q}^2+\mu_2\zeta)\Big].$$

Then (26) becomes

$$E\Big[\exp(\rho z_s)\Big] < e^{-\rho\xi}, \quad \xi=\sum_{q=1}^{L_k} a_{k,q}(\mu_1\sigma_{k,q}^2+\mu_2\zeta). \quad (30)$$

B. Interference plus noise part

Assume z_n is Gaussian with zero mean and variance $\sigma_{z_n}^2$. Then

$$E[e^{\rho z_n}] = e^{\frac{1}{2}\rho^2\sigma_{z_n}^2}. \quad (31)$$

Its power $\sigma_{z_n}^2$ can be obtained as follows. Taking the expected value of the power for non-fading channels when $N_p \to \infty$ in [15], we obtain

$$\begin{aligned}\sigma_{z_n}^2 &= \frac{\sigma_v^2}{N_f}E[\mathcal{Q}_{k,0}] + \frac{1}{N_f}\sum_{l\neq k}E[\mathcal{F}_{l,k,-c_k,0}^2] \\ &+ \frac{1}{2N_f}\sum_{i,l\neq k}E[\mathcal{F}_{l,k,c_l-c_k+i\alpha,0}^2], \quad (32)\end{aligned}$$

where $\sigma_v^2 \triangleq \frac{N_0}{2}\mathcal{B}$, $\phi(t)\triangleq \mathrm{sinc}(\pi\mathcal{B}t)$, B is the bandwidth of the filter. According to the definitions of $\mathcal{Q}_{k,d}$ and $\mathcal{F}_{l_1,l_2,d_1,d_2}$ therein, the term $E[U_k(t)U_k(\tau)]$ is needed for evaluation of the terms on the right hand side. Substituting the expression of $h_k(t)$, we have

$$E[U_k(t)U_k(\tau)] = \frac{\mathcal{P}_k}{2}\sum_q \sigma_{k,q}^2\beta_{k,q}(t)\beta_{k,q}(\tau), \quad (33)$$

where $\beta_{k,q}(t) = s(t-\tau_{k,q})+\nu s(t-\sigma_d-\tau_{k,q})$. Therefore

$$E[\mathcal{Q}_{k,0}] = \sum_q \sigma_{k,q}^2\iint_0^{T_f}\phi(t-\tau)\beta_{k,q}(t)\beta_{k,q}(\tau)dt\,d\tau. \quad (34)$$

Similarly, $E[\mathcal{F}_{l,k,-c_k,0}^2]$ is simplified as

$$\begin{aligned}&E[\mathcal{F}_{l,k,-c_k,0}^2] = \\ &\iint_0^{T_f}E[h_l(t+c_kT_c)h_l(\tau+c_kT_c)]E[U_k(t)U_k(\tau)]dt\,d\tau \\ &= \frac{\mathcal{P}_k\mathcal{P}_l}{4}\sum_{q_1,q_2}\sigma_{k,q_1}^2\sigma_{l,q_2}^2\iint_0^{T_f}d_{k,l,q_1,q_2}(t)d_{k,l,q_1,q_2}(\tau)dt\,d\tau,\end{aligned}$$

where

$$d_{k,l,q_1,q_2}(t) = \beta_{k,q_1}(t)s(t-\tau_{l,q_2}+c_kT_c).$$

Substituting c_k above by $c_k - c_l - i\alpha$, we obtain $E[\mathcal{F}_{l,k,c_l-c_k+i\alpha,0}^2]$. Substituting these results into (32), we obtain $\sigma_{z_n}^2$.

C. Chernoff bound

Substituting (31) and (30), (24) becomes

$$P_E < \frac{1}{2}\min_{\rho>0} e^{\frac{1}{2}\sigma_{z_n}^2\rho^2-\xi\rho}. \quad (35)$$

The right hand side reaches its minimum when $\rho = \xi/\sigma_{z_n}^2$

$$P_E < \frac{1}{2}e^{-\frac{1}{2}\frac{\xi^2}{\sigma_{z_n}^2}} = \frac{1}{2}e^{-\frac{1}{2}SINR}, \quad (36)$$

where SINR is defined by regarding ξ^2 as the effective signal power. The bound has a form similar to that in (22). A tighter bound can follow (23) as well. After setting $\nu=-1$, all results for the PAM bit are directly applicable to the PPM bit.

We note that the approach adopted here is slightly different from [18]. Notice the following holds in general for function $F(\rho,x)$ of random variable x and deterministic variable ρ

$$\min_\rho E_x\Big[F(\rho,x)\Big] \neq E_x\Big[\min_\rho F(\rho,x)\Big].$$

Our approach directly considers $\min_\rho E_x\Big[F(\rho,x)\Big]$ rather than $E_x\Big[\min_\rho F(\rho,x)\Big]$ in [18]. It is thus not surprising that SINR involves parameters μ_1 and μ_2.

SIMULATION

We verify analysis by computer simulation. The second derivative of Gaussian pulse with $D_g = 0.7ns$ and $\tau_m = 0.2877ns$ is adopted as the transmitted pulse [15]. The PPM modulation parameter σ_d is chosen as $0.27ns$ to achieve 4-ary biorthogonal signaling. Except when stated otherwise, the following typical parameters are set: $N_s = 500$ symbols, $N_f = 2$ frames per symbol, $T_c = 1ns$, $K=4$ users. Each TH code c_k is chosen randomly from a set $\{D,\cdots,D_{max}\}$ in each of 100 independent channel realizations. $D=3$, $D_{max}=D+K$. T_f is set to be slightly larger than the maximum channel delay spread. Based on the IEEE UWB CM1 channel model, Figure 2 shows the BER performance for PAM and PPM bits when channel estimation has a penalty from noisy observations. In each subplot, the experimental curve is from simulation, analytical curve from approximation by Q-function, loose bound and tight bound are based on (22) and (23) respectively, true channel curve is under an assumption of perfect channel information, and conventional curve is based on instantaneous noisy template. We can observe that the tight bound is reliable in predicting the receiver performance for a large range of E_b/N_0. BER performance is also tested in a Rayleigh fading scenario. Each channel has five paths with equal

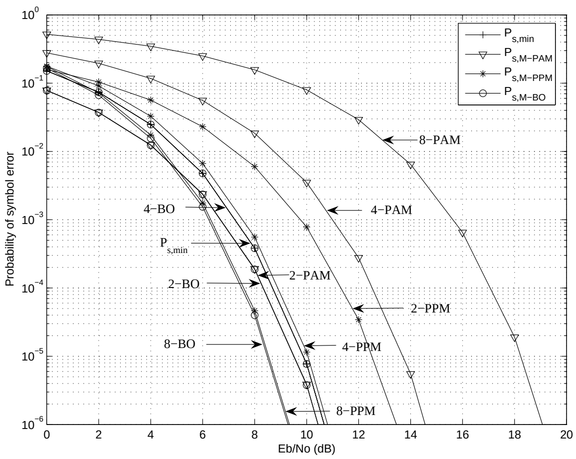

Fig. 1. Comparison of SERs for different signaling schemes.

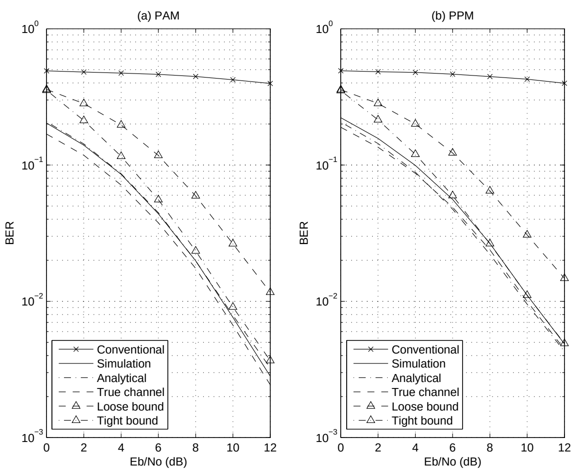

Fig. 2. BER bounds with channel estimation penalty.

power. The path differentials are random from 0 to $20ns$. Figure 3 demonstrates loose and tight BER bounds for the number of users increasing from 1 to 4. PPM signals are less sensitive to the number of interfering users in fading channels than PAM signals. [2]

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[2]The views and conclusions contained in this document are those of the authors and should not be interpreted as representing the official policies, either expressed or implied, of the Army Research Laboratory or the U.S. Government.

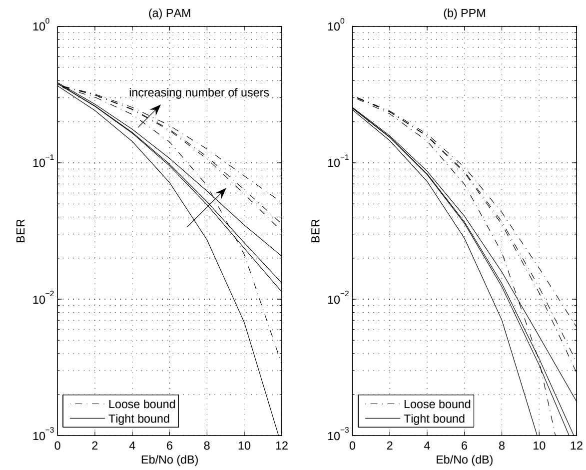

Fig. 3. BER bounds with independent multipath Rayleigh fading.

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