

Transmitted Reference Schemes for Wireless Optical Communications

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Abstract— On-off keying (OOK) and pulse position modulation (PPM) are widely used modulation formats in wireless optical communications. Performance of OOK depends on the photon count threshold that should be adapted to channel fading, while performance of PPM degrades with pulse distortion caused by channel. Transmitted reference scheme, widely adopted in radio frequency communications, can help acquire channel information used for demodulation. This scheme is introduced to both OOK and PPM optical systems in this paper. The average signal and noise photon counts are estimated from reference sequences for OOK from which the optimal threshold is obtained. Since PPM symbols carry information in the pulse position in a non-linear form, Fourier transform translates time shift into a linear phase shift. Based on the linear regression technique, the parameter in the phase equation that is related to the information of pulse position is estimated. A noise model in the frequency domain is also presented.

I. INTRODUCTION

Optical communications have received considerable attention because of the large modulation bandwidth and the ability to concentrate power in extremely narrow beams [1]. The former feature may potentially support high data rate services, while the latter helps enhance the signal quality and improve detection performance. Two widely used modulation schemes over wireless optical channels are on-off keying (OOK) and pulse-position modulation [2]. OOK is the easiest way to map binary codes into transmitted symbols. But as nonreturn-to-zero (NRZ) pulses are used, OOK systems may suffer a synchronization loss. On the contrary, in PPM systems, the presence of a pulse in each symbol frame facilitates the recovery of the clock regardless of the unknown data sequence. Another advantage is its relative insensitivity to the power level of the signal and background noise [3].

In a wireless optical system, an optical channel usually changes from symbol to symbol due to environment dynamics and the random photon counting process in a photodetector. An optical transceiver needs to combat such adverse effects effectively. Transmitted reference (TR) modulation has demonstrated its unique capability of mitigating unknown distortion

of a radio frequency channel, either narrowband [4], or ultra-wideband (UWB) [5], [6], [7]. A TR system transmits a pulse pair for each data symbol. The first pulse without data modulation is called reference pulse, and the second pulse is a data-modulated pulse. Because the two pulses undergo the same multipath channel, the reference pulse in the doublet can be used to detect the information carried on the second pulse. Accordingly, a receiver takes a very simple form as a correlator followed by a decision-making device.

In this paper, we apply this TR transmission scheme to wireless optical system designs to improve system performance in an unknown wireless channel. In an OOK system, the detector performance depends on the optimal photon count threshold [1] to decide whether the pulse is on or off. That threshold is determined by the signal count and noise count. These two quantities can be obtained from the interval of reference signal transmission. For a PPM system, information is carried on the delay of a laser pulse. This nonlinear modulation poses some difficulties in receiver design. However, a time delay is easily translated into a linear phase shift in the frequency domain by Fourier transform. Therefore, the problem is equivalent to phase detection without *a priori* channel knowledge, as practiced for PPM based UWB signal detection in the frequency domain [8]. After a logarithm operation on frequency domain signals corresponding to both reference and data intervals and constructing a difference signal, the information-carrying phase is extracted irrespective of the unknown channel effect. At a particular frequency, that phase linearly depends on frequency, PPM modulation interval and input data. Considering multiple frequency points, a linear regression technique from parameter estimation can be applied to estimate input [9].

The paper is organized as follows. In Section II, we present a description of an OOK signal model and derive an analytical expression for the detector bit error rate (BER). We also show how the performance of the detector improves with the increased number of reference samples from the TR modulation. In Section III, we propose a frequency domain linear regression detection method for PPM based input and perform analysis of noise distributions at different frequencies. Using that information, we then suggest an improved optimization criterion. We finally provide some simulation results in Section

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IV and draw some conclusions in Section V.

II. OOK SIGNAL DECODING

A. OOK Signal Model

In an OOK system, the pulse is turned on or off when bit 1 or 0 is to transmit. The receiver and decoder diagram is shown in Fig. 1 [1]. The photon counts in one symbol interval for 1

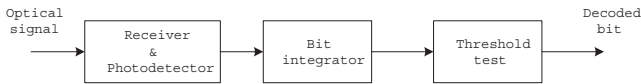


Fig. 1. An OOK receiver and decoder.

and 0 are Poisson random variables with parameters $K_s + K_b$ and K_b respectively, where K_s is the photon count due to the signal pulse, and K_b due to background noise. They are related to signal power P_s , noise power P_b and symbol period T_s by $K_s = \alpha P_s T_s$ and $K_b = \alpha P_b T_s$, where $\alpha = \eta/(hf)$ is a proportionality constant related to detector quantum efficiency η , Planck's constant h and the frequency f of the optical field [1]. Then the distribution of photon count m is given by [10]

$$P(m = k|d) = \frac{(K_s d + K_b)^k}{k!} e^{-(K_s d + K_b)} \quad (1)$$

where $d = 0, 1$ represents the transmitted data. Setting a threshold m_T , the decoder decodes the symbol as 1 if $m \geq m_T$ and 0 if $m < m_T$. Then the probability of decoding error based on m_T is

$$P_e(m_T) = \frac{1}{2} \sum_{k=0}^{m_T-1} P(m = k|1) + \frac{1}{2} \sum_{k=m_T}^{\infty} P(m = k|0). \quad (2)$$

An optimal threshold can be found by minimizing $P_e(m_T)$, or equivalently considering inequalities

$$P_e(m_T) \leq P_e(m_T \pm 1). \quad (3)$$

The solution is given by [1]

$$m_T = \left\lfloor \frac{K_s}{\ln(1 + \frac{K_s}{K_b})} \right\rfloor, \quad (4)$$

where $\lfloor \cdot \rfloor$ is an integer floor operator. Unlike a radio frequency system, this threshold depends on not only the signal to noise ratio K_s/K_b , but also K_s . This feature is due to asymmetry of the distribution of a Poisson random variable and the unipolar modulation.

B. TR Assisted Threshold Determination

According to eq. (4), the optimal decoder needs the knowledge of both K_s and K_b . As fading exists in most of wireless optical communication systems, the received signal power may change with time. For this reason, the optimal threshold needs to be adapted to channel variation. This can be assisted by the TR scheme. Both signal power and noise power can be estimated from received signals corresponding to reference signal intervals.

Assume that the photon count in one symbol duration due to the background noise has been observed to be K_b and is a constant for a relatively long period irrespective of channel conditions. This parameter is irrespective of user signal and can be obtained before any signal transmission. Then we need to estimate K_s from the photon count due to the user signal, or equivalently to estimate the Poisson distribution parameter $K_s + K_b$ from the noisy observations. Before data transmission, the transmitter transmits n pulses consecutively as reference signals. Denote the photon count in n pulse durations as $C = \sum_{j=1}^n c_j$ where c_j is a Poisson random variable with parameter $K_s + K_b$. Noticing the mean and variance of sample mean C/n take the following forms

$$\begin{aligned} E(C/n) &= K_s + K_b, \\ \lim_{n \rightarrow \infty} \text{Var}\left(\frac{C}{n}\right) &= \lim_{n \rightarrow \infty} \frac{K_s + K_b}{n} = 0, \end{aligned} \quad (5)$$

we can obtain our consistent estimator for K_s as $\hat{K}_s = \lfloor C/n \rfloor - K_b$. From eq. (3), the estimated optimal threshold when K_b is constant becomes

$$\hat{m}_T = \left\lfloor \frac{\hat{K}_s}{\ln(1 + \frac{\hat{K}_s}{K_b})} \right\rfloor. \quad (6)$$

C. Detection Performance

According to eq. (2), the error probability is a function of the chosen threshold, which in turn depends on \hat{K}_s . It can be observed that \hat{K}_s is a sample average of n independent Poisson random variables with parameter K_s . It can be found that if x is a linear combination of n independent Poisson random variables with parameter λ as $x = \sum_{l=1}^n a_l x_l$ where a_l are weights, then the characteristic function of x has a form [10]

$$x = \sum_{l=1}^n a_l x_l \rightarrow \phi_x(\omega) = e^{-\lambda(n - \sum_{l=1}^n e^{j a_l \omega})}. \quad (7)$$

If all a_l equals one, then x is still Poisson with parameter $n\lambda$. However if all a_l equals $1/n$, then the distribution $P(\hat{K}_s = u)$ of \hat{K}_s has to be found from the inverse Fourier transform of (7), same as the case with arbitrary weights.

According to the definition of expected value [10], the average error probability depends on the distribution $P(\hat{K}_s = l)$ as follows

$$P_e = E\{P_e(\hat{m}_T)\} = \sum_{l=0}^{+\infty} P_e(\hat{m}_T | \hat{K}_s = l) P(\hat{K}_s = l). \quad (8)$$

Using (2) and (6), it becomes

$$\begin{aligned} P_e &= \frac{1}{2} \sum_{l=0}^{+\infty} \left[\sum_{k=0}^{m_l-1} \frac{(K_s + K_b)^k}{k!} e^{-(K_s + K_b)} \right. \\ &\quad \left. + \sum_{k=m_l}^{\infty} \frac{K_b^k}{k!} e^{-K_b} \right] P(\hat{K}_s = l), \end{aligned} \quad (9)$$

where

$$m_l = \left\lfloor \frac{l}{\ln(1 + \frac{l}{K_b})} \right\rfloor.$$

If \hat{K}_s can be approximated as a Poisson random variable with parameter K_s , then it can be simplified as

$$P_e \approx \frac{e^{-(K_s+K_b)}}{2} \sum_{l=0}^{+\infty} \left[\sum_{k=0}^{m_l-1} \frac{(K_s+K_b)^k}{k!} e^{-K_s} + \sum_{k=m_l}^{+\infty} \frac{K_b^k}{k!} \right] \frac{K_s^l}{l!}. \quad (10)$$

This analytical result can be evaluated based on K_s and K_b .

The general analytical result by (9) depends on sample size n due to the distribution of \hat{K}_s . That relation is highly complex. In order to gain some insight into its dependence on n , we can make some approximations to further simplify it. Applying Taylor expansion on $P_e(\hat{m}_T)$ at m_T (equivalently K_s) up to the second order and taking the expected value, we obtain [10]

$$P_e \approx P_e(m_T) + \frac{1}{2} P_e''(m_T) \text{Var}(\hat{K}_s),$$

where the first order term vanishes because the mean of \hat{K}_s is K_s , $P_e''(m_T)$ is the second order derivative of $P_e(x)$ at m_T after approximating m_T as a continuous variable. Using (5), it becomes

$$P_e \approx P_e(m_T) + \frac{K_s}{2n} P_e''(m_T). \quad (11)$$

Since m_T is the minimum point of $P_e(\hat{m}_T)$, $P_e''(m_T)$ is positive. So P_e monotonically decreases as the length n of the reference sequence increases. If the channel remains static in a relatively long time, then the threshold can be determined by transmitting only one reference pulse and is used to decode a subsequent long data sequence. Thus it reduces transmission overhead. In a case of fading however, threshold needs to be updated frequently. Assume the channel has slow fading, and coherence time is LT_s . One can consider to transmit reference sequence in the first n symbol durations and data sequence in the remaining $L-n$ durations. Increasing n can decrease the bit error rate according to (11) but increase the overhead of transmission. So tradeoff must be made between the transmission efficiency and the bit error rate. The effect of n will be studied in Section IV.

III. PPM SIGNAL DECODING

A. PPM Signal Model

Another widely used modulation is PPM. In a PPM system, each symbol duration T_s is divided into Q time slots of duration $T_c = T_s/Q$. The pulse is on for one time slot and the position of the pulse carries information [1], as showed in Fig. 2. Since the total number of slots is Q , each symbol carries information of $\log_2 Q$ bits. Compared to OOK systems, it is easy to recover the information of the timing of the symbol [11]. If a pulse is distorted by channel, the received pulse may spread out of the pulse duration, causing a difficulty in determining the pulse position. In order to combat the unknown channel distortion, we consider TR scheme for PPM systems as well. The transmitter transmits a reference sequence before a data sequence. By comparing the distorted signals from the reference sequence and data sequence, we can acquire the position information of the data symbol and then

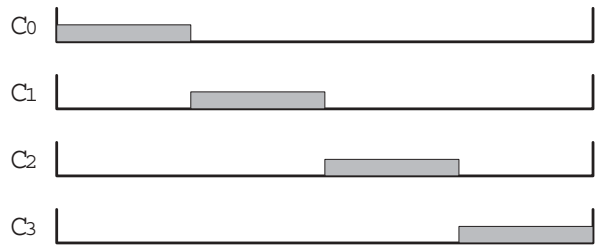


Fig. 2. PPM symbols with $Q = 4$.

successfully demodulate. Denote the received reference signal by $s_{ref}(t)$. Then the data signal in the i th symbol interval becomes $s_{ref}(t - a_i T_c - D)$ where $a_i \in \{0, 1, \dots, M-1\}$ is a position index mapped from input data sequence, D is the nominal time separation of reference and data pulses. Its effect can be absorbed into a_i . In our later discussion, we may ignore D without loss of generality. The transmitted signal can be written as

$$s(t) = \sum_{i=1}^{\infty} w(t - (i-1)T_s), \quad w(t) = s_{ref}(t) - s_{ref}(t - a_i T_c).$$

Assume the channel impulse response is $h(t)$. Without loss of generality, consider the first symbol interval. Then the received signal due to reference sequence is $r_{ref}(t) = s_{ref}(t) * h(t)$ and that due to data sequence is

$$r(t) = r_{ref}(t - a_1 T_c). \quad (12)$$

The information is embedded in the waveform delay that can be translated into a linear phase shift after Fourier transform. By comparing the phases of reference signal and data signal, we will recover a_1 .

B. Frequency Domain Regression Approach

Taking Fourier transformation on both sides of (12), we obtain the frequency domain relation between the reference signal and data signal

$$R(\omega) = R_{ref}(\omega) e^{-j\omega a_1 T_c}.$$

To easily extract phase, we take logarithm of the ratio and obtain

$$j\omega a_1 = \frac{1}{T_c} \ln \frac{R_{ref}(\omega)}{R(\omega)}, \quad (13)$$

which is a linear equation of unknown parameter a_1 . The imaginary part of difference signal $\ln R_{ref}(\omega) - \ln R(\omega)$ can thus be used to estimate a_1 . To improve estimation accuracy, we consider samples of this difference signal at N frequency points

$$\omega = [\omega_1, \dots, \omega_N]^T$$

and apply the linear regression technique [9]. Define

$$\phi = \text{Im} \begin{bmatrix} \phi(\omega_1) \\ \vdots \\ \phi(\omega_N) \end{bmatrix}, \quad \phi(\omega_k) = \frac{1}{T_c} \ln \frac{R_{ref}(\omega_k)}{R(\omega_k)}.$$

The regression equation in a vector form becomes

$$a_1 \boldsymbol{\omega} = \boldsymbol{\phi}. \quad (14)$$

Input a_1 can be estimated as

$$a_1 = (\boldsymbol{\omega}^T \boldsymbol{\omega})^{-1} \boldsymbol{\omega}^T \boldsymbol{\phi}. \quad (15)$$

The decision rule reduces to choose an integer in $\{0, 1, 2, \dots, M-1\}$ that is the closest to a_1 , an index of the transmitted symbol. If fast Fourier transform (FFT) is used, we let $N = 2^m$ and $\omega_k = \frac{2k\pi}{N}$.

C. Removal of Phase Ambiguity

From complex analysis, the operation of logarithm on a complex number is not a one to one mapping. As $e^{j2k\pi} = 1$, result of $\ln e^x$ is $x + j2k\pi$. So there is a $2k\pi$ phase wrapping at some frequencies as ω_k (or k) increases. Therefore, this ambiguity needs to be removed. In order to constrain the phase to be within $(-\pi, \pi)$, we take phase modulo 2π and keep the residual in the above estimation.

D. Signal Plus Noise Model in the Frequency Domain

Our previous discussion is under a noise-free assumption. Consider the case that background noise exists. The vector $\boldsymbol{\phi}$ and $\boldsymbol{\omega}$ are not in a strict linear relation. In this section, we will derive a noise model in the frequency domain. The received signal can be expressed as

$$r(t) = r_{ref}(t - \tau) + n(t),$$

where τ is an information bearing delay. After the transform, we have

$$\ln R_k = \ln[e^{-j\omega_k \tau} R_{refk} + N_k],$$

or

$$\ln R_k - \ln R_{refk} = -j\omega_k \tau + \ln\left[1 + \frac{e^{j\omega_k \tau} N_k}{R_{refk}}\right], \quad (16)$$

where $R_k = R(\omega_k)$, $R_{refk} = R_{ref}(\omega_k)$ and N_k is the k th frequency component of all noise counts n_i

$$N_k = \sum_{i=0}^{N-1} n_i e^{-j \frac{2ik\pi}{N}}. \quad (17)$$

In order to analyze the noise effect, assume high SNR and approximate $\ln(1+x)$ as x when $|x| \ll 1$. Eq. (16) can be approximated as

$$\ln R_k - \ln R_{refk} \approx -j\omega_k \tau + \frac{e^{j\omega_k \tau} N_k}{R_{refk}},$$

where the second item on the right hand side is due to the background noise n_i . Denote $\frac{e^{j\omega_k \tau} N_k}{R_{refk}}$ explicitly in its real part and imaginary part as $x_k + jy_k$. We are interested in y_k which affects our estimate from the imaginary part of $\phi(\omega_k)$. This term depends on $\frac{e^{j\omega_k \tau}}{R_{refk}}$ and noise. The former is information dependent. Without loss of generality, denote

$$\frac{e^{j\omega_k \tau}}{R_{refk}} = \rho_k e^{j(\frac{2l_k\pi}{N} + \phi_k)}, \quad (18)$$

where $0 \leq \phi_k < \frac{2\pi}{N}$, $0 \leq l_k \leq N-1$, $\rho_k = 1/|R_{refk}| = 1/r_k$. Using (17), we can express y_k as

$$y_k = \text{Im} \sum_{i=0}^{N-1} \rho_k n_{m_i} e^{j(\frac{2ik\pi}{N} + \phi_k)} = \sum_{i=0}^{N-1} \rho_k n_{m_i} \sin\left(\frac{2i\pi}{N} + \phi_k\right). \quad (19)$$

The noise index has changed from i to m_i due to the effect of phase in (18) and corresponding change of mapping. It can be easily verified that a complete cycle from 0 to $N-1$ is still experienced. Since y_k is a linear combination of Poisson random variables n_{m_i} of parameter K_b/N , the distribution of can be found from (7). The mean and variance are of most important. It is easily found that $E\{y_k\} = 0$ and $\text{Var}(y_k) = 1/2\rho_k^2 K_b = 1/2K_b r_k^{-2}$. As we can see, the noise distributions or statistics depend on frequency index k . Instead of least-squares (LS) criterion to minimize $\|\boldsymbol{\phi} - a_1 \boldsymbol{\omega}\|^2$, we may consider weighted LS with a weighting matrix $\boldsymbol{\Lambda}$ to account for the noise effect

$$\min(\boldsymbol{\phi} - a_1 \boldsymbol{\omega})^T \boldsymbol{\Lambda} (\boldsymbol{\phi} - a_1 \boldsymbol{\omega}). \quad (20)$$

Accordingly, (15) changes to

$$a_1 = (\boldsymbol{\omega}^T \boldsymbol{\Lambda} \boldsymbol{\omega})^{-1} \boldsymbol{\omega}^T \boldsymbol{\Lambda} \boldsymbol{\phi}. \quad (21)$$

Weighting matrix $\boldsymbol{\Lambda}$ can be either optimized jointly with a_1 , or pre-selected. The rule of thumb is to apply some combining techniques such as the maximum ratio combining. Considering the power of noise sample at frequency ω_k is proportional to r_k^{-2} , one possibility is to adopt the following matrix $\boldsymbol{\Lambda} = \text{diag}(r_1^2, \dots, r_N^2)$, proportional to signal power.

IV. SIMULATION RESULTS

We test both transmitted reference OOK and PPM system performance. The effects of K_s and K_b and the length n of the reference sequence are shown in Figs. 3 and 4. It is observed that BER performance improves as n increases. When $n = 4$, the performance is very close to the ideal threshold that assumes perfectly known K_s instead of being estimated. When K_b decreases from 50 (Fig. 3) to 20 (Fig. 4), the BER performance improves significantly. That means it is sensitive to background noise..

The BER of PPM with transmitted reference is shown in Fig.5. The K_s and K_b are total photon counts of all samples within a chip. Again we observe that the BER decreases rapidly as K_b decreases.

V. CONCLUSION

In this paper, the transmitted reference scheme for OOK and PPM wireless optical communications is presented. In OOK systems, the detection threshold is estimated using reference signals. The BER performance is analytically studied, and its dependence on the length of reference sequence is illustrated. In PPM systems, a frequency domain linear regression techniques is applied to estimate input from the phase difference of the received data signal and reference signal. The perturbation in frequency domain due to the background noise is modeled,

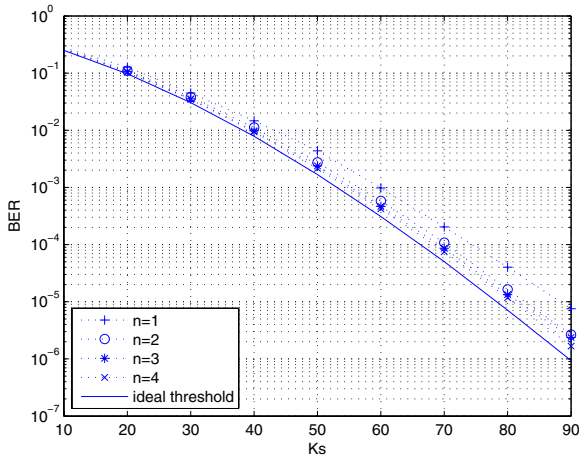


Fig. 3. BER of OOK with different length n of reference sequence, $K_b = 50$.

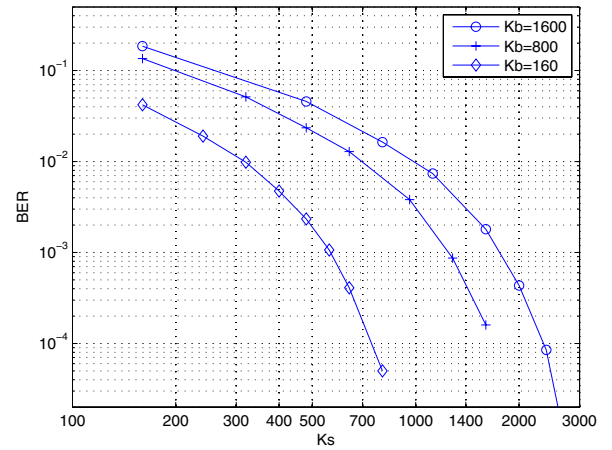


Fig. 5. BER of PPM with different K_b .

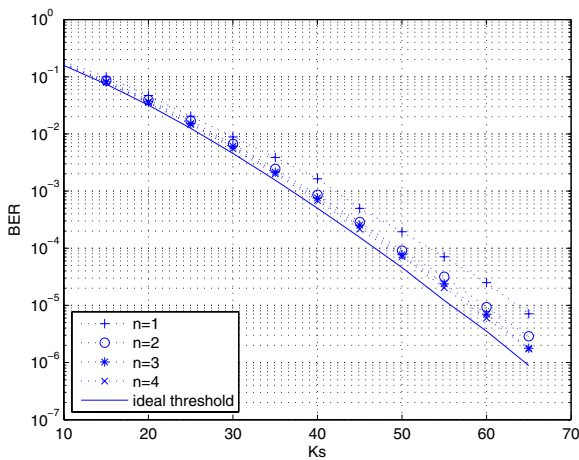


Fig. 4. BER of OOK with different length n of reference sequence, $K_b = 20$.

whose statistics can be used to improve the detection performance using a weighting matrix in the cost function. Future research in fading channels will be conducted.

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