Constrained Cross-Cumulant Based Multiuser Detection for Uplink CDMA

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***Abstract*— In this paper, a cross-cumulant based blind multiuser detection scheme is proposed for an uplink CDMA system. By imposing a code-aided constraint on each user's receiver and minimizing the cross cumulant of the output signals from all receivers, we show that the proposed approach is capable of detecting all users' signals in parallel without user ambiguity. The global convergence of the proposed algorithm is established. The resulted detectors are proven to be zero-forcing ones. Furthermore, the constraints are treated as unknown parameters and jointly optimized with the receivers to achieve performance gain. Adaptive implementation is discussed. Simulation results verify the satisfactory performance of the proposed approach in typical communication scenarios.**

I. Introduction

In the field of blind source separation for multiple input multiple output (MIMO) systems, considerable recent research has been focused on higher order statistics (HOS) based solutions, since second-order statistics (SOS) criteria in most cases do not suffice for the complete separation of the sources due to the existence of a unitary matrix ambiguity [1]. Those HOS approaches are based on either maximizing the cumulant [1], [2], [3], [4], [5], or minimizing the cross cumulant of the output signals [6], [7].

A CDMA system can be viewed as a special class of MIMO systems. When the HOS based techniques are applied to multiuser detection, they can be divided into two categories. One is to consider only the user of interest, such as in downlink CDMA systems. With the given spreading codes of the desired user, the detector is forced to satisfy a set of linear constraints such that the signal of the desired user is detected [8], [9], [10]. The other one is to design a bank of detectors to recover all users' signals, such as in base stations. In the second category, to ensure that each detector detects the signal of a distinct user, multistage successive cancellation (MSC) [2], [5] or Gram-Schmidt orthogonalization on the detectors [1], [3] must be implemented. However, the MSC approaches might suffer from error accumulation in the final solution, due to imperfect subtraction of the detected signals in previous stages, while the Gram-Schmidt orthogonalization performed at each step imposes some computational complexity. At the end of detection, the user ambiguity due to the position rotation is still unavoidable in those approaches.

In this paper, a code-constrained cross-cumulant based blind multiuser detection criterion is proposed to jointly detect all users' signals simultaneously for an uplink CDMA system. In the proposed scheme, each user's detector is linearly constrained by the user's spreading codes to some preselected vector. Then the total cross-cumulant of all detectors' output is minimized under the constraints. It is proven analytically that each user's receiver globally converges to that user's zero-forcing receiver, irrespective of Gaussian noise, and all users' signals can be detected in parallel without user ambiguity. To achieve performance gain, the constraints are proposed to be treated as unknown parameters and further optimized. Adaptive implementation is discussed for joint optimization of the detectors and constraints. Simulation results are finally presented to verify the effectiveness of the proposed scheme.

II. System Model

Consider a direct sequence (DS) CDMA system with J users. User j is assigned a periodic spreading sequence $c_j(k)$ ($k = 0, \ldots, P-1$). Let the chip sequence be transmitted through a multipath channel with unknown coefficients $g_j(n)$. Then the received chip-rate discrete-time signal $y(n)$ has the form [10]

$$y(n) = \sum_{j=1}^{J} \sum_{l=-\infty}^{\infty} w_j(l) h_j(n - \tau_j - lP) + v(n), \qquad (1)$$

where $h_j(n) = \sum_{m=-\infty}^{\infty} g_j(m) c_j(n-m)$, $w_j(n)$ is an equiproble ± 1 random variable, and $v(n)$ is assumed to be zero-mean additive white Gaussian noise (AWGN). Without loss of generality, we may assume that the delay $0 \le \tau_j < P$. We also assume the maximum channel order for all users is q (typically $q << P$ in many applications). If we collect $P+q$ samples of $y(n)$ in a vector and similarly define the noise vector $\widetilde{\boldsymbol{v}}(n)$, then the received signal $\widetilde{\boldsymbol{y}}(n)$ becomes

$$\begin{aligned}\widetilde{\boldsymbol{y}}(n) &= \sum_{j=1}^{J}[\widetilde{\boldsymbol{C}}_{j,1}\boldsymbol{g}_j w_j(n) + \widetilde{\boldsymbol{C}}_{j,2}\boldsymbol{g}_j w_j(n-1) \\ &+ \widetilde{\boldsymbol{C}}_{j,3}\boldsymbol{g}_j w_j(n+1)] + \widetilde{\boldsymbol{v}}(n), \end{aligned} \tag{2}$$

where $\boldsymbol{g}_j = [g_j(0),\ \ldots,\ g_j(q)]^T$ is the multipath channel vector of user j, and $\widetilde{\boldsymbol{C}}_{j,1}$, $\widetilde{\boldsymbol{C}}_{j,2}$, $\widetilde{\boldsymbol{C}}_{j,3}$ have following structures:

$$\widetilde{\boldsymbol{C}}_j = \begin{bmatrix} c_j(1) & & 0 \\ \vdots & \ddots & c_j(1) \\ c_j(P) & & \vdots \\ \boldsymbol{0} & \ddots & c_j(P) \end{bmatrix}, \tag{3}$$

$$\widetilde{\boldsymbol{C}}_{j,1} = \begin{bmatrix} \boldsymbol{0}_{\tau_j\times(q+1)} \\ \widetilde{\boldsymbol{C}}_j(1:P+q-\tau_j,:) \end{bmatrix}, \tag{4}$$

$$\widetilde{\boldsymbol{C}}_{j,2} = \begin{bmatrix} \widetilde{\boldsymbol{C}}_j(P+q-\tau_j:P+q,:) \\ \boldsymbol{0}_{(P+q-\tau_j-1)\times(q+1)} \end{bmatrix}, \tag{5}$$

$$\widetilde{\boldsymbol{C}}_{j,3} = \begin{bmatrix} \boldsymbol{0}_{(P+\tau_j)\times(q+1)} \\ \widetilde{\boldsymbol{C}}_j(1:q-\tau_j,:) \end{bmatrix}. \tag{6}$$

For convenience, we restrict our attention to a quasi-synchronous system, where $\tau_i << P$. Consider ν data points from $\widetilde{\boldsymbol{y}}(n)$ and formulate a partial data vector $\boldsymbol{y}(n) = [y(nP+P-\nu-1),\ \ldots,\ y(nP+P)]^T$ with $P-\nu = max(q+\tau_j)(j = 1,\ \ldots,\ J)$ to eliminate an intersymbol interference (ISI) effect, where the maximum chip delay is assumed known. A similar definition is applied to the new partial noise vector $\boldsymbol{v}(n)$. Then, from (2), we obtain the input/output relationship

$$\boldsymbol{y}(n) = \sum_{j=1}^{J}\boldsymbol{C}_j\boldsymbol{g}_j w_j(n) + \boldsymbol{v}(n) = \boldsymbol{H}\boldsymbol{w}(n) + \boldsymbol{v}(n), \tag{7}$$

where

$$\boldsymbol{C}_j = \widetilde{\boldsymbol{C}}_{j,1}(P-\nu+1:P,:),$$

$$\boldsymbol{w}(n) = [w-1(n),\ \ldots,\ w_j(n)]^T,$$

and

$$\boldsymbol{H} = [\boldsymbol{h}_1,\ \ldots,\ \boldsymbol{h}_J],\quad \boldsymbol{h}_j = \boldsymbol{C}_j\boldsymbol{g}_j. \tag{8}$$

III. Constrained Cross-Cumulant Based Multiuser Detection

Our objective is to design J linear detectors $\boldsymbol{F} = [\boldsymbol{f}_1,\ \ldots,\ \boldsymbol{f}_J]$ to blindly detect the J users' signals $[w_1(n),\ \ldots,\ w_J(n)]^T$ simultaneously, where the jth detector $\boldsymbol{f}_j$ is designed for the jth user. Based on (7), the output vector is found to be

$$\begin{aligned}\boldsymbol{z}(n) &\triangleq [z_1(n),\ \ldots,\ z_J(n)]^T \\ &= \boldsymbol{F}^H\boldsymbol{y}(n) = \boldsymbol{F}^H\boldsymbol{H}\boldsymbol{w}(n) + \boldsymbol{F}^H\boldsymbol{v}(n). \end{aligned} \tag{9}$$

In the absence of noise, if and only if $\boldsymbol{F}^H\boldsymbol{H} = \boldsymbol{\Gamma}$ where $\boldsymbol{\Gamma}$ is a permutated identity matrix, the output $\boldsymbol{z}(n)$ will be exactly the input sequence $\boldsymbol{w}(n)$ and the corresponding $\boldsymbol{F}$ is said to be a set of zero forcing (ZF) receivers. In the case of unknown channel matrix $\boldsymbol{H}$, to obtain optimal receivers whose output vector $\boldsymbol{z}(n)$ is a good approximation of $\boldsymbol{w}(n)$, we propose to minimize the sum of the cross-cumulant of the output signals, subject to a set of constraints on $\boldsymbol{F}$

$$\begin{aligned}&\min_{\boldsymbol{F}} \mathcal{J} = \sum_{i>j}|cum_{22}(z_i z_j)| \\ &\text{subject to}\ \ \boldsymbol{f}_i^H\boldsymbol{C}_i = \widehat{\boldsymbol{g}}_i^H,\quad i,j = 1,\ldots,J \end{aligned} \tag{10}$$

where

$$cum_{22}(z_i, z_j) = cum(z_i(n), z_i^*(n), z_j(n), z_j^*(n)),$$

and $\widehat{\boldsymbol{g}}_i(i = 1,\ \ldots,\ J)$ is a pre-selected constraint vector to ensure no cancellation of $w_i(n)$ in the ith output $z_i(n)$. Here we explicitly use the ith user's spreading sequence to constrain the ith receiver $\boldsymbol{f}_i$. In the next analysis, we will show that the constraints are essential for separating all users' signals and removing user ambiguity.

IV. Convergence Analysis

To facilitate the analysis, we simply consider BPSK signal sources and make the following assumptions:

- **AS1)** Each user's information sequence is non-Gaussian, mutually independent and temporally i.i.d. with unit power;
- **AS2)** Channel noise $\boldsymbol{v}_n$ is white Gaussian and independent of input signals;
- **AS3)** Matrix $\boldsymbol{H}$ has full column rank.

We first express (10) explicitly as a function of detectors. Based on (9) and the definitions of $\boldsymbol{F}$ and $\boldsymbol{H}$, the ith output is given by $z_i(n) = \sum_{l=1}^{J}\boldsymbol{f}_i^T\boldsymbol{h}_l w_l(n)$. Using the property of cumulant and applying (**AS1-AS2**), the cross-cumulant between $z_i(n)$ and $z_j(n)$ is derived as

$$\begin{aligned}cum_{22}(z_i, z_j) &= cum(z_i(n), z_i^*(n), z_j(n), z_j^*(n)) \\ &= \sum_{l=1}^{J}(\boldsymbol{f}_i^T\boldsymbol{h}_l)^2(\boldsymbol{f}_j^T\boldsymbol{h}_l)^2 cum_4(w_l(n))\end{aligned}$$

Applying the BPSK source assumption, the optimization in (10) becomes

$$\min_{\boldsymbol{F}} \mathcal{J}(\boldsymbol{F}) = 2\sum_{i>j}\sum_{l=1}^{J}(\boldsymbol{f}_i^T\boldsymbol{h}_l)^2(\boldsymbol{f}_j^T\boldsymbol{h}_l)^2 \quad \text{subject to} \quad \boldsymbol{f}_i^T\boldsymbol{C}_i = \widehat{\boldsymbol{g}}_i^T. \tag{11}$$

Directly applying the constraints on $\boldsymbol{f}_i$, the above constrained optimization can be transformed into the following unconstrained one

$$\begin{aligned}\min_{\boldsymbol{F}} \mathcal{J}(\boldsymbol{F}) &= 2\sum_{i>j}[(\widehat{\boldsymbol{g}}_i^T\boldsymbol{g}_i)^2(\boldsymbol{f}_j^T\boldsymbol{h}_i)^2 + (\widehat{\boldsymbol{g}}_j^T\boldsymbol{g}_j)^2(\boldsymbol{f}_i^T\boldsymbol{h}_j)^2 \\ &+ \sum_{l\neq i,j}(\boldsymbol{f}_i^T\boldsymbol{h}_l)^2(\boldsymbol{f}_j^T\boldsymbol{h}_l)^2]. \end{aligned} \tag{12}$$

We are now ready to derive the stationary points of the cost function (12). Taking derivative of (12) with respect to $\boldsymbol{f}_i$ ($i = 1, \ \ldots, \ J$), and rearranging terms according to $\boldsymbol{f}_i^T\boldsymbol{h}_j$, one can verify that the gradient is given by

$$\frac{\partial \mathcal{J}}{\partial \boldsymbol{f}_i} = 4\sum_{j\neq i}[(\widehat{\boldsymbol{g}_j}^T\boldsymbol{g}_j)^2 + \sum_{l\neq i,j}(\boldsymbol{f}_l^T\boldsymbol{h}_j)^2](\boldsymbol{f}_i^T\boldsymbol{h}_j)\boldsymbol{h}_j. \tag{13}$$

Zeroing the gradient and expressing it in a matrix form, we then have

$$\bar{\boldsymbol{H}}_i \begin{bmatrix} 4[(\widehat{\boldsymbol{g}}_1^T\boldsymbol{g}_1)^2 + \sum_{l\neq 1}(\boldsymbol{f}_l^T\boldsymbol{h}_1)^2]\boldsymbol{f}_i^T\boldsymbol{h}_1 \\ \vdots \\ 4[(\widehat{\boldsymbol{g}}_{i-1}^T\boldsymbol{g}_{i-1})^2 + \sum_{l\neq i-1}(\boldsymbol{f}_l^T\boldsymbol{h}_{i-1})^2]\boldsymbol{f}_i^T\boldsymbol{h}_{i-1} \\ 4[(\widehat{\boldsymbol{g}}_{i+1}^T\boldsymbol{g}_{i+1})^2 + \sum_{l\neq i+1}(\boldsymbol{f}_l^T\boldsymbol{h}_{i+1})^2]\boldsymbol{f}_i^T\boldsymbol{h}_{i+1} \\ \vdots \\ 4[(\widehat{\boldsymbol{g}}_J^T\boldsymbol{g}_J)^2 + \sum_{l\neq J}(\boldsymbol{f}_l^T\boldsymbol{h}_J)^2]\boldsymbol{f}_i^T\boldsymbol{h}_J \end{bmatrix} = \boldsymbol{0} \tag{14}$$

where $\bar{\boldsymbol{H}}_i$ denotes the remained matrix after removing the ith user's signature vector $\boldsymbol{h}_i$ from $\boldsymbol{H}$. According to (**AS3**), $\boldsymbol{H}$ has full column rank, therefore $\bar{\boldsymbol{H}}_i$ ($i = 1, \ldots, J$) has full column rank too. It then follows that the right column vector in (14) is a zero vector, which implies that

$$[(\widehat{\boldsymbol{g}}_j^T\boldsymbol{g}_j)^2 + \sum_{l\neq j}(\boldsymbol{f}_l^T\boldsymbol{h}_j)^2]\boldsymbol{f}_i^T\boldsymbol{h}_j = 0 \ \text{ for } i,j = 1,\ldots,J, \ j\neq i.$$

It is then straightforward to show that if the constraint vector $\widehat{\boldsymbol{g}}_j$ is properly pre-selected such that $(\widehat{\boldsymbol{g}}_j^T\boldsymbol{g}_j)^2 \neq 0$ for $j = 1, \ \ldots, \ J$, the following results can be obtained

$$\boldsymbol{f}_i^T\boldsymbol{h}_j = 0, \quad i,j = 1,\ldots,J \ \text{ and } j\neq i. \tag{15}$$

Let the optimal receivers corresponding to the stationary points be denoted as $\boldsymbol{F}^o = [\boldsymbol{f}_1^o, \ \ldots, \ \boldsymbol{f}_J^o]$. According to the above analysis, $\boldsymbol{f}_i^o$ ($i = 1,\ldots,J$) satisfies eq. (15). Further applying the code constraint on $\boldsymbol{f}_i^o$, we arrive at

$$(\boldsymbol{F}^o)^T\boldsymbol{H} = diag\{\widehat{\boldsymbol{g}}_1^T\boldsymbol{g}_1, \ \ldots \ \widehat{\boldsymbol{g}}_J^T\boldsymbol{g}_J\}. \tag{16}$$

Obviously, $\boldsymbol{F}^o$ is a set of ZF receivers. Moreover, eq. (16) suggests that $\mathcal{J}(\boldsymbol{F}^o) = 0$. Considering $\mathcal{J}(\boldsymbol{F}) \geq 0$, one can conclude that $\boldsymbol{F}^o$ is a minimum point. Using eq. (4.7), one can further obtain the optimal output, denoted as $z_i^o(n)$, of the ith optimal receiver

$$z_i^o(n) = (\boldsymbol{f}_i^o)^H * \boldsymbol{y}(n) = \widehat{\boldsymbol{g}}_i^T\boldsymbol{g}_i w_i(n) + \boldsymbol{f}_i^T\boldsymbol{v}(n), \tag{17}$$

where $\widehat{\boldsymbol{g}}_i^T\boldsymbol{g}_i$ is a scalar. Therefore, the separation of the multiuser signals with only scalar ambiguity is achieved in the absence of noise.

The constraints can be further optimized to achieve performance gain. Since at any minimum point, the output power of the ith receiver, according to (17), is given by $(\widehat{\boldsymbol{g}}_i^T\boldsymbol{g}_i)^2 + \sigma_v^2||\boldsymbol{f}_i^o||^2$. It is immediately observed that the signal power depends on the pre-selected constraint vector $\widehat{\boldsymbol{g}}_i$. To achieve performance gain, it is desirable to maximize only the signal power $(\widehat{\boldsymbol{g}}_i^T\boldsymbol{g}_i)^2$. Meanwhile, at any minimum point, based on (15) and the constraints on $\boldsymbol{F}^o$, the auto-cumulant of the ith receiver, irrespective of Gaussian noise, is given by

$$|cum_4(z_i^o)| = |cum_4(w_i)|(\widehat{\boldsymbol{g}}_i^T\boldsymbol{g}_i)^4$$

which is related to the square of the signal power of the ith receiver. Therefore, to maximize the signal power of the ith receiver after removing interference, we propose to maximize the cumulant $|cum_4(z_i^o)|$ at the minimum point. Noticing that the norm of the constraint will not affect the performance, a norm constraint is imposed on the constraint vector $\widehat{\boldsymbol{g}}_i$ to avoid infinite solutions. Therefore, the constraint vector is optimized as the following

$$\max_{\widehat{\boldsymbol{g}}_i} |cum_4(z_i^o)| \ \text{ subject to } \ ||\widehat{\boldsymbol{g}}_i|| = 1.$$

It is straightforward to show that $\widehat{\boldsymbol{g}}_i = \pm\frac{\boldsymbol{g}_i}{||\boldsymbol{g}_i||}$ is the optimal solution, which is also a perfect estimate of $\boldsymbol{g}_i$ with only a sclar ambiguity. Once the optimal constraints are obtained, they can be used back to (10) to obtain a new set of zero-forcing receivers. To conclude, by jointly optimizing the constraints, together with the detectors, the maximum signal power as well as minimal interference can be achieved for each detector, irrespective of Gaussian noise. As a by-product, a perfect channel estimate with only a scalar ambiguity for each user can be obtained, irrespective of Gaussian noise.

V. IMPLEMENTATION

Since the proposed optimization criterion is highly nonlinear, it is impossible to derive closed-form solutions for both receivers and constraints. We will seek adaptive solutions by jointly optimizing both receivers and constraints simultaneously for all users as the following,

$$\max_{\boldsymbol{g}_j} |cum_4(z_j)| \text{ and } \min_{\boldsymbol{f}_j} \sum_{i>j} |cum_{22}(z_i z_j)|$$
$$\text{subject to } \boldsymbol{f}_i^H \boldsymbol{C}_i = \widehat{\boldsymbol{g}}_i^H, \quad i,j = 1,\ldots,J. \tag{18}$$

Eq. (18) involves two different cost functions for optimizing $\boldsymbol{f}_j$ and $\boldsymbol{g}_j$, which are then connected by the constraint. It is also difficult to derive the adaptive solution for (18) directly. Instead, we first derive the adaptive solution for a generalized optimization criterion:

$$\max_{\boldsymbol{g}_j} \min_{\boldsymbol{F}} \beta_1 \sum_j |cum_4(z_j)| + \beta_2 \sum_{i>j} |cum_{22}(z_i z_j)|$$
$$\text{subject to } \boldsymbol{f}_i^H \boldsymbol{C}_i = \widehat{\boldsymbol{g}}_i^H, \quad i,j = 1,\ldots,J. \tag{19}$$

Then, the adaptive solution for $\boldsymbol{F}$ can be obtained by taking $\beta_1 = 0$ and $\beta_2 = 1$, and the solution for $\boldsymbol{g}_j$ can be obtained by taking $\beta_1 = 1$ and $\beta_2 = 0$.

Following similar procedures in [10], gradient based recursions for both receiver $\boldsymbol{f}_j$ and $\boldsymbol{g}_j$, $j = 1,\ldots,J$ for (19) can be obtained as,

$$\begin{aligned} \boldsymbol{f}_{j,n+1} &= \boldsymbol{\Pi}_{\boldsymbol{C}_j}^{\perp}\Big(\boldsymbol{f}_{j,n} - \mu_{f_j}\big(\beta_1 \nabla_{\boldsymbol{f}_j} |cum_4(z_j)| \\ &+ \beta_2 \sum_{i\neq j} \nabla_{\boldsymbol{f}_j} |cum_{2,2}(z_j, z_i)|\big)\Big) \\ &+ \boldsymbol{C}_j \boldsymbol{A}_j \boldsymbol{g}_{j,n} \end{aligned} \tag{20}$$

$$\begin{aligned} \boldsymbol{g}_{j,n+1} &= \boldsymbol{g}_{j,n} + \frac{\mu_g}{\mu_f}(\boldsymbol{I} - \frac{\boldsymbol{g}\boldsymbol{g}^H}{\boldsymbol{g}^H\boldsymbol{g}})\boldsymbol{A}\Big(\boldsymbol{g}_{j,n} - \boldsymbol{C}_j^H \boldsymbol{f}_{j,n} \\ &+ \mu_{f_j} \boldsymbol{C}_j^H \big(\beta_1 \nabla_{\boldsymbol{f}_j} |cum_4(z_j)| \\ &+ \beta_2 \sum_{i\neq j} \nabla_{\boldsymbol{f}_j} |cum_{2,2}(z_j, z_i)|\big)\Big) \end{aligned} \tag{21}$$

where μ_{f_j} and μ_{g_j} are step sizes,

$$\boldsymbol{A}_j = (\boldsymbol{C}_j^H \boldsymbol{C}_j)^{-1}, \quad \boldsymbol{\Pi}_{\boldsymbol{C}_j}^{\perp} = \boldsymbol{I} - \boldsymbol{C}_j \boldsymbol{A}_j \boldsymbol{C}_j^H$$

$$\begin{aligned} \nabla_{\boldsymbol{f}_j} |cum_4(z_j)| &= sign(cum_4(z_j))(2E\{|z_j|^2 z_j^* \boldsymbol{y}\} \\ &- 4E\{|z_j|^2\}E\{z_j^* \boldsymbol{y}\} \\ &- 4E\{z_j^2\}E\{z_j \boldsymbol{y}\}) \end{aligned} \tag{22}$$

and

$$\begin{aligned} \nabla_{\boldsymbol{f}_j} & |cum_{2,2}(z_j, z_i)| \\ = \; & sign(cum_{2,2}(z_j, z_i))(2E\{|z_i|^2 z_j^* \boldsymbol{y}\} \\ - \; & 4E\{|z_i|^2\}E\{z_j^* \boldsymbol{y}\} \\ - \; & 4E\{z_i^2\}E\{z_j \boldsymbol{y}\}). \end{aligned} \tag{23}$$

Then taking $\beta_1 = 0$, $\beta_2 = 1$ for optimizing $\boldsymbol{f}_j$ in (20), we immediately obtain the adaptive rule for the proposed receiver as the following

$$\begin{aligned} \boldsymbol{f}_{j,n+1} &= \boldsymbol{\Pi}_{\boldsymbol{C}_j}^{\perp}(\boldsymbol{f}_{j,n} - \mu_{f_j} \sum_{i\neq j} \nabla_{\boldsymbol{f}_j} |cum_{2,2}(z_j, z_i)|) \\ &+ \boldsymbol{C}_j \boldsymbol{A}_j \boldsymbol{g}_{j,n}. \end{aligned} \tag{24}$$

Taking $\beta_1 = 1$, $\beta_2 = 0$ for optimizing $\boldsymbol{g}_j$ according to (21), we update the constraints by

$$\begin{aligned} \boldsymbol{g}_{j,n+1} &= \boldsymbol{g}_{j,n} + \frac{\mu_g}{\mu_f}(\boldsymbol{I} - \frac{\boldsymbol{g}_{j,n}\boldsymbol{g}_{j,n}^H}{\boldsymbol{g}_{j,n}^H\boldsymbol{g}_{j,n}})\boldsymbol{A}_j(\boldsymbol{g}_{j,n} - \boldsymbol{C}_j^H \boldsymbol{f}_{j,n} \\ &+ \mu_{f_j} \boldsymbol{C}_j^H \nabla_{\boldsymbol{f}_j} |cum_4(z_j)|). \end{aligned} \tag{25}$$

VI. SIMULATION

In this section, we verify the effectiveness of the proposed approach. We consider a synchronous uplink CDMA system with spreading factor of 16. Each user in the system is assumed to experience a multipath fading channel with 4 paths. Each user's spreading sequence and channel vector are randomly generated. All simulation results below are based on 100 Monte Carlo runs.

We first verify the convergence of the proposed approach and compare it in two cases: (1) the constraint vectors are generated randomly and then fixed over the iterations for all users, (2) the constraint vectors are initially randomly generated and then jointly optimized with the receivers. Two active users and 15dB noise are assumed for the system. The average output SINR is plotted in Fig. 1. It is observed that the proposed approach converges for both users in both cases. However, with the constraint vectors jointly optimized with the receivers, the proposed approach can achieve significantly better performance, as shown by the much higher SINR levels. In all the next simulations, we focus on the proposed approach with constraint vectors jointly optimized with the receivers.

We then examine the performance of the proposed approach in the presence of different input SNRs from 0dB to 20dB at a step of 5dB. Two users are assumed for the system. As a reference, the ideal zero-forcing receiver build up on the perfect channel and code information is employed for comparison. Since the optimzed constraint vectors in the proposed approach can be viewed as channel estimation, the estimated ZF receivers built up on the estimated channels are also tested. Fig. 2 plots the output SINRs for each inspected SNR. It is found that the estimated ZF receivers are almost overlapped with the ideal ZF receivers, which suggests that the proposed optimized constraint vectors converge to the channels very well. The proposed receivers, which are adjusted adaptively at each step, show satisfactory performance with slight gap from the ideal ZF receivers. The small gap may be caused by the nontrivial step size. The sample size may also be a factor.

Next, we inspect the performance of the proposed approach by assuming different number of users in the system. Each subplot in Fig. 3 shows the output SINR for a system with 2 users, 3 users, and 4 users, respectively. It is observed that each user's output SINR converges to a satisfactory level. On the other hand, we have shown that the constraint vector for each user can be viewed as its channel estimate. We then plot channel estimation mean-square-error (MSE) for each user in Fig. 4. As expected, each user' constraint vector converges to its channel vector very well with a MSE below the level of 10^{-3}.

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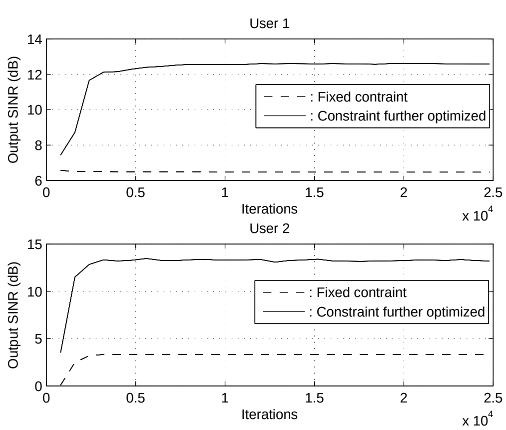

Fig. 1. SINR comparison for the approaches with constraint vector fixed and further optimized.

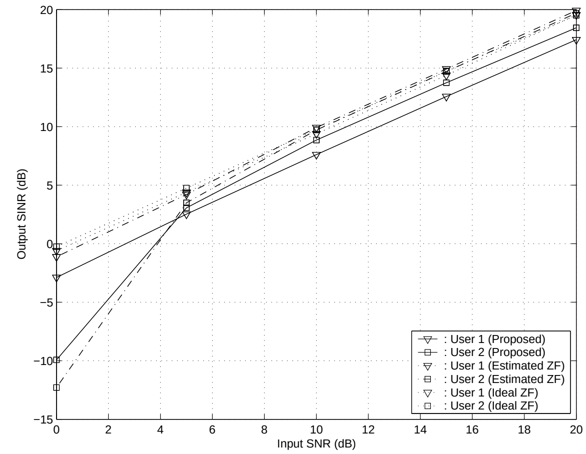

Fig. 2. Average output SINR for different input SNR.

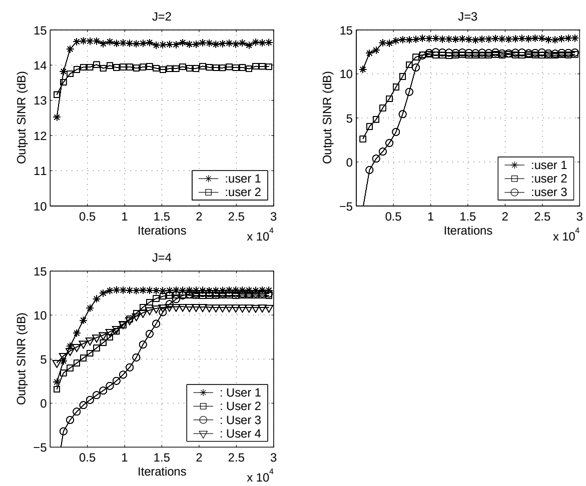

Fig. 3. Output SINR in different loadings.

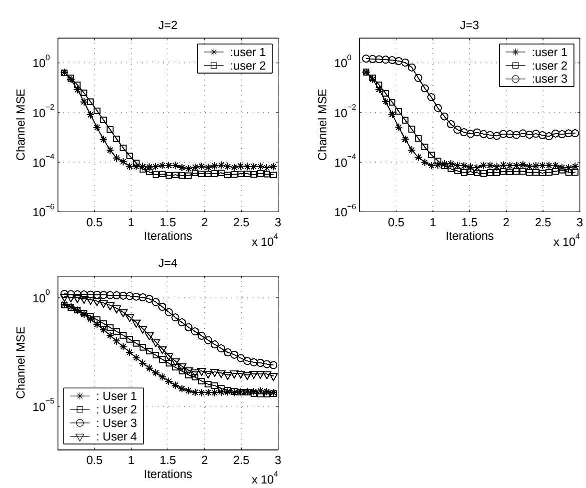

Fig. 4. Channel estimation mean-square-error in different loadings.