Time Delay Estimation Bounds in Wideband Random Channels

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***Abstract*— Improved Ziv-Zakai bounds (ZZBs) on time delay estimation (TDE) in wideband random channels are developed. The mean-square error (MSE) bounds are independent of estimator bias and do not require regularity conditions. They assume a known transmitted signal with uniform prior on the delay in $[0, T]$, and a tapped delay line random channel model. The taps are modeled as Gaussian, including non-zero mean and possible correlation between taps. The channel model may be tuned to a variety of environments through the choice of mean and covariance, and models wideband and ultra-wideband line of sight (LOS) and non-LOS (NLOS) cases. The bounds are derived for a given channel realization first, and then averaged over the random channel model. They reflect the impact of the random channel, bounding average MSE performance assuming perfect channel estimates are available to the receiver. Examples illustrate the impact of the key system, channel, and signal parameters.**

***Index Terms*— Ziv-Zakai bound, time delay estimation, ultra-wideband, performance analysis.**

I. Introduction

Time delay estimation (TDE) is a basic problem in many signal processing scenarios, such as radar, communications, and geolocation, and TDE performance analysis is needed to find limits in these applications. Ultra-wideband (UWB) signals have desirable properties for radar and indoor ranging, enabling building penetration and fine time resolution [1]. Timing for ranging may also be employed in sensor networks for node geolocation [2], [3]. TDE is often analyzed in an additive white Gaussian noise (AWGN) channel, or for narrowband fading channels; e.g., see the survey in [4]. The impact of multiple narrowband fading diversity channels on TDE Cramér-Rao bounds (CRBs) is considered in [5]. In general, the resulting CRBs provide a local bound on TDE, but are not tight for all signal-to-noise ratios (SNRs), and do not account for random channel effects.

In this paper we develop mean-square error (MSE) bounds on TDE. These bounds can then be applied in the above applications. We adopt the TDE performance bounding approach of Ziv and Zakai to the wideband convolutive random channel case. The basic methodology was introduced in [6], and later an improved version was developed [7] that avoided the use of the Chebyshev inequality and yields a tighter bound over both low and high SNR regimes. The ZZB is applied to TDE for UWB signals in additive white Gaussian noise in [8].

Hereafter, we refer to the improved version [7] as the Ziv-Zakai bound (ZZB). This approach yields an MSE bound on time delay that is independent of estimation bias, and assumes a uniform prior on the TD parameter. The random convolutive channel model is general and accommodates wideband and ultra-wideband cases. The resulting ZZB reveals the impact of fading as well as randomness that arises over channel realizations that occur due to movement of the scatters or the terminals. The ZZB is derived for a given channel realization, and then averaged over the random channel model. The derivation incorporates a probability of error expression that is based on a receiver that has knowledge of the channel realization.

II. Signal and Channel Models

A. Channel Model

The channel is modeled as a tapped delay line with tap spacing T_t seconds, given by

$$g(t) = \sum_{l=0}^{L-1} \alpha_l \delta(t - lT_t), \tag{1}$$

where L is the total number of taps, and α_l is the gain for the $(l+1)$-th tap modeled as a Gaussian random variable with mean m_l and variance γ_l. We model the α_l's as jointly Gaussian with distribution $\mathcal{N}(\boldsymbol{m}, \boldsymbol{V})$ where $\boldsymbol{m}$ is the mean vector and $\boldsymbol{V}$ is the covariance matrix. Block fading is assumed, so that each realization of $g(t)$ is constant over a block of time, and then an independent realization is drawn for the next block. (Note that, while the channel has equally-spaced delays in time, the signal model below in (3) has a continuous time delay t_0 that we wish to estimate.) We easily obtain the narrowband flat fading case by setting $L = 1$.

This channel model (1) is well known in the wideband case [9]. From a statistical model selection viewpoint, using the Akaike information criterion (AIC), there is strong evidence to support this model and the Gaussian assumption in the UWB case as well (see [10] for a comprehensive discussion). The propagation environment is tuned by selection of $\boldsymbol{m}$ and $\boldsymbol{V}$,

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and the power delay profile is incorporated into the variance of each tap.

B. Signal Model

The transmitted signal $s(t)$ is assumed known to the receiver and given by

$$s(t) = \sqrt{E_{\text{tx}}} p(t). \quad (2)$$

Here E_{tx} is the transmission energy, and $p(t)$ has unit energy and finite duration T_p. In the case of a bandlimited signal, which is inherently not time limited, we assume it can be truncated in time to duration T_p with some small approximation error.

The signal is transmitted through the multipath channel $g(t)$, with propagation time t_0, so that the received signal with energy E_{rx} is described by

$$y(t) = \sqrt{E_{\text{rx}}} h(t - t_0) + v(t). \quad (3)$$

For ease of notation we assume $t_0 \in [0, T]$, and assume T is known *a priori*. In (3), $h(t)$ is the overall channel response of duration $T_h = (L-1)T_t + T_p$, given by

$$h(t) = p(t) * g(t) = \sum_{l=0}^{L-1} \alpha_l p(t - lT_t), \quad (4)$$

and $v(t)$ is additive white Gaussian noise (AWGN) with double sided spectral density $N_0/2$.

III. Time Delay Estimation Bound

The development of the ZZB links estimation of t_0 with a hypothesis testing problem that discriminates a signal at two possible delays [7]. For a received signal at one of the two possible delays $h(t-a)$ or $h(t-a-\Delta)$, where $\Delta > 0$ and $a, a+\Delta \in [0, T]$, a binary decision problem based on a delay estimate $\widehat{t_0}$ is described as follows

$$\begin{aligned} &\text{Decide } H_0: \quad t_0 = a \qquad \text{if } |\widehat{t_0} - a| < |\widehat{t_0} - a - \Delta|, \\ &\text{Decide } H_1: \quad t_0 = a + \Delta \quad \text{if } |\widehat{t_0} - a| > |\widehat{t_0} - a - \Delta|. \end{aligned} \quad (5)$$

Denote the estimation error by $\epsilon = \widehat{t_0} - t_0$, and let $P_e(a, a+\Delta)$ be the minimal probability of error of some detection scheme in making the above decision. If the two hypothesized delays are equally likely to occur, then the estimation MSE is lower bounded by [7]

$$\overline{\epsilon^2} \geq \frac{1}{T} \int_0^T \Delta \int_0^{T-\Delta} P_e(a, a+\Delta) d\Delta. \quad (6)$$

In particular, if $P_e(a, a+\Delta)$ is independent of a and denoted by $P_e(\Delta)$, then

$$\overline{\epsilon^2} \geq \frac{1}{T} \int_0^T \Delta (T - \Delta) P_e(\Delta) d\Delta. \quad (7)$$

Evaluation of the bound (7) relies on finding the probability of error $P_e(\Delta)$. For our case, consider the received signal

$$z(t) = \sqrt{E_{\text{rx}}} h(t - b\Delta) + v(t), \quad (8)$$

where b takes values of 0 or 1 corresponding to the two hypotheses, and Δ is the relative delay in the hypothesis test. Here, $P_e(\Delta)$ is equivalent to the error probability of a binary pulse position modulation (PPM) communications scheme, as a function of the relative delay Δ. Thus, we can appeal to binary PPM error results, but with the added complication of the random channel. We proceed as follows. First, we find $P_e(\Delta)$ conditioned on the channel realization, and second, we find the average probability of error $\overline{P_e}(\Delta)$, averaged over channel realizations. The ZZB is found by using $\overline{P_e}(\Delta)$ in (7). As further explained below, this leads to a ZZB that averages over the random channel while assuming perfect channel knowledge at the receiver, which is sometimes referred to as a perfect measurement based lower bound (e.g., see Van Trees [12]).

A. Probability of Error for a Given Channel Realization

When $h(t)$ is known to the receiver, the optimal binary PPM detector forms a correlation template $m(t) = h(t) - h(t - \Delta)$ and generates a decision variable based on an observation window of duration T_o [13]

$$Z = \int_0^{T_o} m(t) z(t) dt. \quad (9)$$

This "coherent" detector assumes knowledge of the transmitted signal and channel is available to the receiver.

We assume $T_o > T + T_h$, so that the observation window is long enough to collect all the signal energy so as to cover the signal of duration T_h at the largest possible delay of T. After substituting $m(t)$ and $z(t)$, (9) becomes

$$Z = Z_s + Z_v, \quad (10)$$

where Z_s is the signal component, and Z_v is the noise component. These are given by

$$Z_s = \sqrt{E_{\text{rx}}} [R_{b\Delta} - R_{(1-b)\Delta}], \quad Z_v = \int_0^{T_o} m(t) v(t) dt,$$

where R_τ is the autocorrelation of $h(t)$ evaluated at lag τ,

$$R_\tau = \int_0^{T_o} h(t) h(t - \tau) dt = \sum_{l,m=0}^{L-1} \alpha_l \alpha_m \beta_{(m-l)T_t + \tau}, \quad (11)$$

that in turn depends on the transmitted signal autocorrelation

$$\beta_\tau = \int_0^{T_o} p(t) p(t - \tau) dt = \int_0^{\infty} p(t) p(t - \tau) dt. \quad (12)$$

The second equality in (12) is due to $T_o > T + T_p$ with $T_h > T_p$. If this condition does not hold, then β_τ defined by the first equality becomes a truncated correlation, discarding the signal beyond the observation window, and the immediate consequence is the reduced signal energy collected by the receiver. The signal correlation depends on the waveform and duration. For a finite duration signal, $\beta_\tau = 0$ for $|\tau| > T_p$. When a normalized signal is used, then $\beta_0 = 1$.

The decision will be made based on the polarity of Z, in order to decide if $b = 0$ or $b = 1$. Without loss of generality, assume $b = 0$. Then

$$Z_s = \sqrt{E_{\text{rx}}} (R_0 - R_\Delta). \quad (13)$$

An error occurs if $Z < 0$ [13], so the probability of error is given by

$$P_e(\Delta) = \mathrm{P}\{Z < 0\} = \mathrm{P}\{Z_v < -Z_s\}. \quad (14)$$

Since Z_v is a Gaussian random variable with zero mean and variance $\sigma_v^2 = E\{Z_v^2\}$,

$$\begin{aligned} \sigma_v^2 &= \int_0^{T_o}\int_0^{T_o} E\{v(t)v(\tau)m(t)m(\tau)\}dtd\tau \\ &= N_0(R_0 - R_\Delta), \end{aligned}$$

the probability of error can be expressed as

$$P_e(\Delta) = Q(\sqrt{D}), \quad (15)$$

where $Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^\infty \exp(-\frac{t^2}{2})dt$ is the Q-function, related to the complementary error function by $Q(x) = \frac{1}{2}\mathrm{erfc}(\frac{x}{\sqrt{2}})$, D is a function of receiver SNR $\gamma_b = E_{\mathrm{rx}}/N_0$ and the received signal correlation (and thus the channel) through R_τ as

$$D = \gamma_b(R_0 - R_\Delta). \quad (16)$$

Next, we take the expectation over the channel to obtain the average probability of error.

B. Average Probability of Error

The above result is conditioned on the channel realization. Define the random channel gain vector $\boldsymbol{\alpha} = [\alpha_0, \cdots, \alpha_{L-1}]^T$. Then, the average probability of error is given by

$$\overline{P}_e(\Delta) = E\{P_e(\Delta)\} = E\{Q(\sqrt{D})\}, \quad (17)$$

where the expectation is over random gain $\boldsymbol{\alpha} \in \mathcal{R}^L$. The Q-function is difficult to work with, so we turn to the following well known identity [14]

$$Q(x) = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \exp(-\frac{x^2}{2\sin^2\phi})d\phi. \quad (18)$$

Now, (17) becomes

$$\overline{P}_e(\Delta) = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} E\{\exp(-\frac{D}{2\sin^2\phi})\}d\phi. \quad (19)$$

Using (11), D in (16) can be expressed as a quadratic function of $\boldsymbol{\alpha}$. We first obtain

$$R_0 - R_\Delta = \sum_{l,m} \alpha_l\alpha_m[\beta_{(m-l)T_t} - \beta_{\Delta+(m-l)T_t}]. \quad (20)$$

If we introduce an $L \times L$ down-shifting matrix $\boldsymbol{J}$, whose first sub-diagonal elements below the main diagonal are ones while all others are zeros, then D can be expressed as

$$D = \boldsymbol{\alpha}^T\boldsymbol{W}\boldsymbol{\alpha}, \quad (21)$$

where $\boldsymbol{W}$ is a Toeplitz matrix (not necessarily symmetric)

$$\boldsymbol{W} = \gamma_b \sum_{k=-(L-1)}^{L-1} (\beta_{kT_t} - \beta_{\Delta+kT_t})\boldsymbol{J}^k, \quad (22)$$

and $\boldsymbol{J}^{-1} \triangleq \boldsymbol{J}^T$ for concise presentation of the summation. Now, (19) becomes

$$\overline{P}_e(\Delta) = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} E\{\exp(-\frac{\boldsymbol{\alpha}^T\boldsymbol{W}\boldsymbol{\alpha}}{2\sin^2\phi})\}d\phi. \quad (23)$$

In principle, (23) could be evaluated numerically for any given distribution of $\boldsymbol{\alpha}$, although this requires an $(L+1)$-dimensional integration whose complexity is high. Further simplification of (23) can be made by observing that the exponent is in a quadratic form of a Gaussian vector $\boldsymbol{\alpha}$. The error probability (23) is related to the moment generating function (MGF) of $u = \boldsymbol{\alpha}^T\boldsymbol{\Psi}\boldsymbol{\alpha}$ [15], as developed in the following Lemma.

Lemma: Suppose $\boldsymbol{\alpha}$ is an L-dimensional real Gaussian random vector with mean $\boldsymbol{\mu}$, and full rank covariance $\boldsymbol{V}$. For any real matrix $\boldsymbol{\Psi}$, let $\boldsymbol{A} = \boldsymbol{I} - \boldsymbol{V}(\boldsymbol{\Psi} + \boldsymbol{\Psi}^T)$. Then,

$$E\{\exp(\boldsymbol{\alpha}^T\boldsymbol{\Psi}\boldsymbol{\alpha})\} = \exp[\frac{1}{2}\boldsymbol{\mu}^T(\boldsymbol{\Psi} + \boldsymbol{\Psi}^T)\boldsymbol{A}^{-1}\boldsymbol{\mu}]|\boldsymbol{A}|^{-\frac{1}{2}}, \quad (24)$$

if $\boldsymbol{A}$ is a positive definite matrix.

This lemma generalizes a well known scalar version, and the positive definiteness condition is required to obtain a bounded expected value, similar to the scalar case [16].

Invoking the lemma, (23) can be simplified. Letting $\boldsymbol{\Psi} = -\frac{\boldsymbol{W}}{2\sin^2\phi}$ and setting

$$\boldsymbol{A} = \boldsymbol{I} - \boldsymbol{V}(\boldsymbol{\Psi} + \boldsymbol{\Psi}^T) = \boldsymbol{I} + \frac{\gamma_b}{2\sin^2\phi}\boldsymbol{V}\boldsymbol{\Phi}, \quad (25)$$

where

$$\begin{aligned} \boldsymbol{\Phi} &= \frac{1}{\gamma_b}(\boldsymbol{W} + \boldsymbol{W}^T) \\ &= \sum_{k=-(L-1)}^{L-1} (2\beta_{kT_t} - \beta_{\Delta+kT_t} - \beta_{\Delta-kT_t})\boldsymbol{J}^k, \end{aligned} \quad (26)$$

we obtain

$$\overline{P}_e(\Delta) = \frac{1}{\pi}\int_0^{\frac{\pi}{2}}\exp[-\frac{\gamma_b}{4\sin^2\phi}\boldsymbol{\mu}^T\boldsymbol{\Phi}\boldsymbol{A}^{-1}\boldsymbol{\mu}]\ |\boldsymbol{A}|^{-\frac{1}{2}}d\phi. \quad (27)$$

After expressing $\boldsymbol{A}^{-1}$ by $2\sin^2\phi\boldsymbol{B}^{-1}$ according to (25) where

$$\boldsymbol{B} = 2\sin^2\phi\boldsymbol{I} + \gamma_b\boldsymbol{V}\boldsymbol{\Phi}, \quad (28)$$

we have

$$\overline{P}_e(\Delta) = \frac{2^{\frac{L}{2}}}{\pi}\int_0^{\frac{\pi}{2}}\exp[-\frac{\gamma_b}{2}\boldsymbol{\mu}^T\boldsymbol{\Phi}\boldsymbol{B}^{-1}\boldsymbol{\mu}]\ |\boldsymbol{B}|^{-\frac{1}{2}}(\sin\phi)^L d\phi. \quad (29)$$

Evaluation of (29) only requires one-dimensional integration, as compared to the L-dimensional integration required for (23) after expanding the expectation based on the distribution of $\boldsymbol{\alpha}$. For a given value of Δ, the integration involves the following system parameters: mean $\boldsymbol{\mu}$ and variance $\boldsymbol{V}$ of $\boldsymbol{\alpha}$, SNR γ_b, transmitted signal correlation β_τ, channel model tap spacing T_t, and the number of taps L.

C. TDE Bound

We now return to (7), and utilize $\overline{P}_e(\Delta)$ to find the bound

$$\overline{\epsilon^2} \geq \frac{1}{T}\int_0^T \Delta(T-\Delta)\overline{P}_e(\Delta)d\Delta. \quad (30)$$

Substituting (26) and (28) into (29), and then (29) into (30), numerical integration can be readily carried out to obtain the bound for any given system parameters. Evaluation of (30) requires a double integration over ϕ and Δ, using the result in (29). The bound (30) is applicable to many scenarios, such as line-of-sight (LOS) and non-LOS (NLOS) channels, different power delay profiles (PDPs), tap correlation profiles (TCPs), and pulse shaping. (If the random Gaussian channel model were not employed, then a Monte Carlo simulation over channel realizations for some other model could be used.) Note that the bound depends on the prior distribution time delay interval T; as $T \to \infty$, it may become loose [7].

IV. UWB Channel Impact

Next we consider the impact of the channel parameters and pulse shaping for some cases of interest. Our study includes the additive Gaussian noise channel as a benchmark, modeling line of sight and non-line of sight conditions, and the channel power delay profile.

A. AWGN Channel

If $L = 1$ and $\boldsymbol{V} = 0$, then $\mu_0 = 1$. Substituting these into (29) and noting (18), the probability of error becomes

$$\overline{P}_e(\Delta) = Q\Big(\sqrt{\gamma_b(\beta_0 - \beta_\Delta)}\Big). \quad (31)$$

Substituting this result into (30), the resulting ZZB corresponds to the AWGN channel, which we will refer to as the AWGN ZZB.

B. Deterministic Channel

Let $\boldsymbol{V} = \boldsymbol{0}$. Then, (29) reduces to

$$\overline{P}_e(\Delta) = Q\Big(\sqrt{\frac{1}{2}\gamma_b \boldsymbol{\mu}^T\boldsymbol{\Phi}\boldsymbol{\mu}}\Big). \quad (32)$$

After substituting back $\boldsymbol{\Phi}$ and noticing $\boldsymbol{\mu}^T\boldsymbol{W}\boldsymbol{\mu} = \boldsymbol{\mu}^T\boldsymbol{W}^T\boldsymbol{\mu}$, we have

$$\overline{P}_e(\Delta) = Q\Big(\sqrt{\boldsymbol{\mu}^T\boldsymbol{W}\boldsymbol{\mu}}\Big) = Q\Big(\sqrt{\gamma_b(R_0 - R_\Delta)}\Big). \quad (33)$$

Not surprisingly, this result has the same form as (15) and depends on the correlation of $h(t)$.

C. Large Ricean-K Factor

Depending on the specific channel model, some channel taps may have non-zero mean [10]. In the wideband case with line of sight propagation, the first tap may exhibit a large mean. The Ricean-K factor of a tap with mean μ and variance γ is defined as $K = \frac{\mu^2}{\gamma}$. If K is large, then γ is very small, and matrix $\boldsymbol{V}$ is small. Then, we can show that [11]

$$\overline{P}_e(\Delta) = Q\Big(\sqrt{\frac{1}{2}\gamma_b tr(\boldsymbol{C}\boldsymbol{\Phi})}\Big), \quad (34)$$

where $\boldsymbol{C} = \boldsymbol{\mu}\boldsymbol{\mu}^T + \boldsymbol{V}$ is a correlation matrix for $\boldsymbol{\alpha}$. Equation (34) may be applied to (30) to find the large K-factor ZZB.

D. Exponential PDPs

Exponential power delay profiles have been observed in many wireless scenarios, for narrowband/wideband [14] and UWB [10] systems. Consider the case of uncorrelated random channel taps with non-zero mean μ_0 only for the first arrival. The variance matrix $\boldsymbol{V}$ becomes diagonal with $(l+1)$-th diagonal element γ_l ($l = 0, 1\cdots, L-1$). Assume an exponential power delay profile $\gamma_l = a_0\exp(-\frac{lT_t}{\lambda})$ where λ is a decay factor. The Ricean-K factor of the first path is $K = \frac{\mu_0^2}{\gamma_0}$. Imposing a power normalization constraint $\sum_l \gamma_l + \mu_0^2 = 1$, we find that

$$a_0 = \left[K + \frac{1-\exp(-\frac{LT_t}{\lambda})}{1-\exp(-\frac{T_t}{\lambda})}\right]^{-1}. \quad (35)$$

The first path parameters are related to a_0 by

$$\mu_0^2 = Ka_0, \quad \gamma_0 = a_0,$$

and the total power of this path becomes $(K+1)a_0$. As $K \to \infty$, $\mu_0^2 \to 1$ and $\gamma_l \to 0$.

V. Numerical Results

We compare bounds and TDE simulations, adopting the root MSE (RMSE) of the time delay estimate as the performance metric. We use the following UWB parameters unless otherwise stated: $T = 30ns$, $T_t = 1ns$, rectangular pulse duration $T_p = 2ns$. The channel has $L = 5$ independent taps. Based on [10], an exponential power delay profile is used with decay factor $\lambda = 6ns$. The Ricean-K factor for the first path is $K = 50 \approx 17$ dB, and all other paths have a zero Ricean factor (i.e., have zero mean).

Figure 1 illustrates general ZZB behavior over a large SNR range. Curves denoted with "*" and "x" denote high and low SNR approximations, respectively [11]. The dashed-dotted line represents the AWGN ZZB (termed the ZZB-AWGN), and the solid line is the ZZB with the random channel (termed ZZB-random). Also plotted are two horizontal lines representing the low SNR convergence level $T/\sqrt{12}$ and a 3 dB lower MSE level; these indicate the low SNR breakdown point. We find breakdown points at around -8 dB and 35 dB. The asymptotic analyses approximate the corresponding bounds very well for SNR above 40 dB and below 0 dB. The random channel ZZB for this case converges to ZZB-AWGN from 0 up to 20 dB.

In Fig. 2, the ZZB-random of (30), ZZB-AWGN based on (31) and a simulated time delay estimator are shown for a large range of SNRs and K values. The receiver employs a correlator-based time delay estimator, choosing the peak value as the TDE. The ZZB-AWGN is the limiting value for $K = \infty$ of ZZB-random. The simulation parameters are $T = 5ns$, $L = 1$, with curves parameterized by K taking values of $(0, 5, 10, 15, 20)$ dB. The ZZB agrees well with the estimator performance when K is small, or when SNR is low/medium. For larger K, the ZZB and the estimator performance separate as SNR increases. This implies that the ZZB might not be as tight at high SNR.

The channel order L plays an important role as well. Fig. 3 depicts the ZZB over a large range of K and L. The bound decreases as K increases for fixed L. For fixed K, the bound decreases as L increases. When K is large enough, the ZZB-random converges to ZZB-AWGN for all L, indicating that the channel has a single dominant first arrival and therefore acts on average like an AWGN channel. A threshold effect at high SNR is observed for large K.

We also conducted extensive simulation studies on the effects of other system parameters. The time delay interval length T has a big impact on the ZZBs, but the correlation-based estimator is not sensitive to its choice as long as the observation window size is relatively larger. At SNR around 20 dB, the ZZB provides a good approximation to the receiver performance. The power delay profile is determined by the decay factor whose effect is found to be pronounced at very high SNRs. Tap correlation also exhibits a high SNR impact. Rectangular and Gaussian pulses with comparable effective width do not lead to large differences in bound performance.

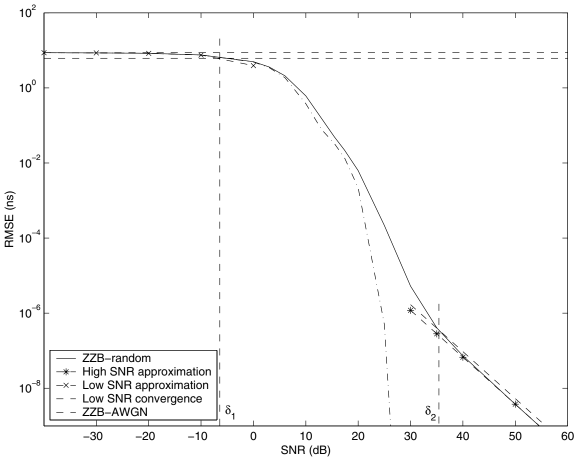

Fig. 1. Typical time delay estimation Ziv-Zakai bound behavior.

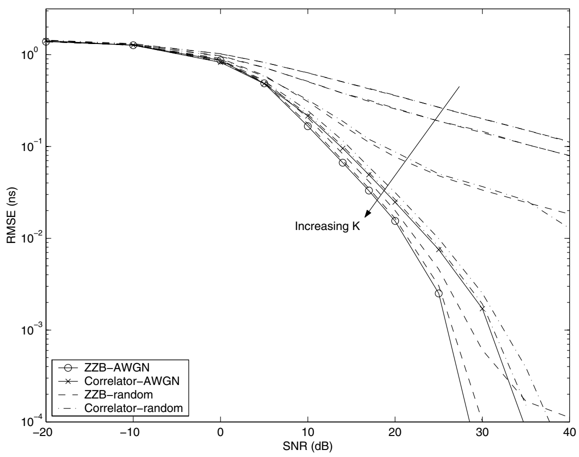

Fig. 2. TDE example comparing ZZB and a simulated receiver employing a correlator-based time delay estimator.

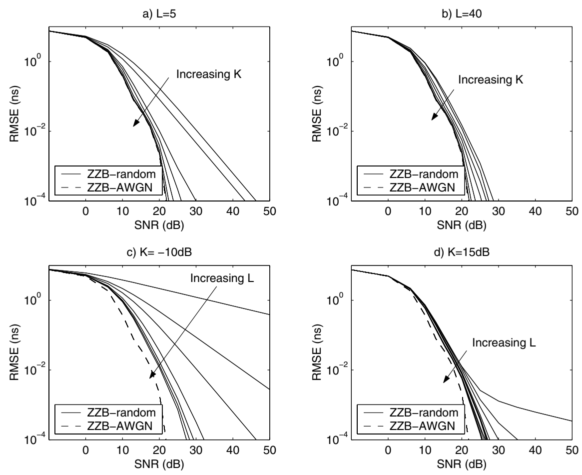

Fig. 3. TDE ZZBs as a function of channel tap length L, and Ricean-K factor in the first arrival.

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