Constrained CMA-Based Multiuser Detection under Unknown Multipath

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Abstract— **In this paper, a constant modulus algorithm (CMA) based multiuser detector is proposed under linear constraints. Different from previous work, we explicitly consider multipath effect. Therefore multiple constraints are incorporated into the optimization problem to mitigate multipath distortions. The algorithm will work well if constraint parameters are properly pre-selected. However, the detector's performance highly depends on the constraints. Thus we jointly update the detector and the constraint vector by minimizing the Godard's cost function with respect to both the detector and the constraint vector. It is shown that under certain conditions in the absence of noise, the algorithm guarantees global convergence. Numerical examples are presented to demonstrate the results.**

I. Introduction

It has been well recognized that when the communication channel suffers from multipath distortions, an equalizer is required to recover the transmitted symbols. The constant modulus algorithm (CMA) has shown its effectiveness in combating intersymbol interference (ISI) [2]. It is usually implemented adaptively due to difficulty in obtaining a closed-form solution and low cost consideration.

Recently, there emerges great interest in applying the CMA to multiuser detection due to the prevalence of code division multiple access (CDMA) technique. Two different methods exist among those approaches. One is to design a bank of detectors with each one detecting one user [6]. Thus all users can be detected at the same time. This multiuser detection scheme can be implemented in the base station which is capable of processing large amount of data in parallel. But in the mobile station, it is neither possible nor necessary to obtain all detectors. Moreover, there is an embedded permutation ambiguity to be further removed in order to differentiate users. The other method is to consider only the desired user [5], [10]. With given spreading codes of that user, the detector was forced to satisfy a linear constraint when the CMA was applied. The algorithm was improved by introducing a parameter into the CMA cost function in order to guarantee global convergence [11]. It turns out that the power of the desired user determines the convergence status of the algorithm. Only the flat fading channel was considered in these methods. However, this may not be the case in a practical communication system. Typically, the receiver captures several copies of signals of different power from the same user. The first arrived signal could be too weak to be counted on. Unfortunately the multipath effect was not addressed in any of those methods. It motivates us to derive a corresponding method in order to combine all signals which are deemed equally useful in detection.

The previous CMA-based multiuser detection methods exploited linearly constrained adaptive detection technique for a non-frequency selective channel. The constrained optimization idea is not a new one. Instead of minimizing the Godard's cost function, blind adaptive multiuser detection was proposed by [3] for the same scenario by minimizing the mean output energy (MOE) of the detector. This second order statistic (SOS) based method offers some advantages in computation and convergence rate. If the code sequence and timing of the desired user are perfectly known, then the MOE detector is equivalent to the minimum mean square error (MMSE) detector.

The success of the MOE method is not surprising if we review the linearly constrained minimum variance (LCMV) beamforming technique from array signal processing [1]. Many relevant algorithms were later developed to mitigate multipath distortions [4], [8] by pre-selecting a set of constant constraints. However, corresponding to different constraints, a bank of detectors followed by a coherent combiner are normally required. Channel estimation appears as a prerequisite for better multiuser detection performance. Moreover, complex implementation is not favorable in most applications. This inspires another simple structure recently proposed by [9] where only one detector was needed to detect the desired user in the presence of multipath. To combat multipath distortion, multiple constraints were still employed but treated as unknown parameters. They were arranged in a vector for convenience. The method recursively minimizes the output variance with respect to the detector and meanwhile maximizes it with respect to the constraint vector. Thus this min/max approach jointly updates all these parameters. In such a way, channel variation is captured and some optimality is achieved. The detector exhibits performance comparable to the MMSE detector.

Motivated by these facts, we propose a CMA-based multiuser detection method suitable for frequency selective fading communication environment. We only adopt the Godard's cost function although other merit functions such as [7] are also possible candidates. The problem is formulated as minimizing the CMA cost function subject to multiple linear constraints. The constraints can be properly pre-selected or jointly updated based on gradient descent method. We also investigate the convergence property of the algorithm and show that when the constraint vector is properly initialized, a global minimum solution is guaranteed. However, without channel fading information, it is difficult to choose favorable initial values for the constraint parameters. To solve initialization problem, we may resort to the MOE method [9] due to its excellent convergence nature. After some iterations, the constraint vector is taken as the initialization for our proposed recursion. Different numerical examples are presented for such demonstration.

II. Signal Model

Consider a DS-CDMA system with J users. User j is assigned a periodic spreading sequence $c_j(k)$, $k = 0, \ldots, P-1$ of period P and transmits P chips per information symbol. Let the chip sequence be transmitted through a linear multipath channel $g_j(n)$. Then the received discrete-time signal $y(n)$ at the chip rate receiver is a superposition of the signals from all users plus noise $v(n)$ (see [9])

$$y(n) = \sum_{j=1}^{J} \sum_{l=-\infty}^{\infty} w_j(l) h_j(n - d_j - lP) + v(n) \qquad (1)$$

where

$$h_j(n) = \sum_{m=-\infty}^{\infty} g_j(m) c_j(n-m)$$

$w_j(n)$ is zero-mean, i.i.d. information bearing sequence of user j ($j = 1, \ldots, J$) with variance $\sigma_{w_j}^2 = E\{w_j^2(n)\}$, $h_j(n)$ is its signature, d_j is the delay of user j in chip periods, and $v(n)$ is assumed to be AWGN with zero-mean and variance $\sigma_v^2 = E\{v^2(n)\}$. Without loss of generality we may assume that the delay $0 \le d_j < P$. We will also assume that $g_j(n)$ has finite impulse response of maximum order q (typically $q \ll P$ in many applications).

For simplicity, we assume the receiver is synchronized to our desired user - user 1. Collect L measurements of $y(n)$ in a vector $\boldsymbol{y}_n = [y(nP), \ldots, y(nP+L-1)]^T$. Then the received signal has the form

$$\boldsymbol{y}_n = \boldsymbol{C}\boldsymbol{g}_1 w_1(n) + \boldsymbol{H}_{int}\boldsymbol{w}_{int}(n) + \boldsymbol{v}(n) \qquad (2)$$

where $\boldsymbol{w}_{int}(n)$ is the interference vector including ISI and multiple access interference (MAI), $\boldsymbol{H}_{int}$ is the corresponding signature matrix, $\boldsymbol{v}(n)$ is the noise vector, $\boldsymbol{C}$ and $\boldsymbol{g}_1$ are code matrix and channel vector respectively

$$\boldsymbol{C} = \begin{bmatrix} c_1(0) & & 0 \\ \vdots & \ddots & c_1(0) \\ c_1(P-1) & & \vdots \\ 0 & \ddots & c_1(P-1) \end{bmatrix}, \boldsymbol{g}_1 = \begin{bmatrix} g_1(0) \\ \vdots \\ g_1(q) \end{bmatrix} \qquad (3)$$

The structure of the user's signature will be exploited to derive a blind adaptive multiuser detector with capable of combating multipath distortions. The method will be based on the constrained CMA technique as explained next.

III. Constrained CMA-Based Multiuser Detection with Multipath

The CMA algorithm is to seek a linear detector $\boldsymbol{f}$ to minimize the cost function

$$\mathcal{J} = E\{(z^2 - \gamma)^2\}$$

where z is the detector's output $z = \boldsymbol{f}^T \boldsymbol{y}_n$, γ is a constant $\gamma = \frac{E\{w_1^4(n)\}}{E\{w_1^2(n)\}}$. In the case of BPSK modulation with binary inputs, $\gamma = 1$. In order to detect $w_1(n)$ in a multiuser communication system, we constrain the detector as follows: $\boldsymbol{C}^T\boldsymbol{f} = \boldsymbol{g}$. Such constraints ensure no cancellation of the desired signal in z if $\boldsymbol{g}^T\boldsymbol{g}_1 \neq 0$. If there is no multipath, then $\boldsymbol{C}$ degrades to a vector and $\boldsymbol{g}$ becomes a scalar (unity). The method becomes same as the existing ones [5], [10]. With multipath distortions, multiple constraints are required to capture total energy of signals from different paths. The vector $\boldsymbol{g}$ can be pre-selected as a constant. For each given $\boldsymbol{g}$, the residual of the cost function after minimization is then related to the interference power. In order to achieve some optimality in the performance of the detector, we treat $\boldsymbol{g}$ as a parameterized vector and further minimize the minimum of the cost function

$$\min_{\boldsymbol{g}} \min_{\boldsymbol{f}} \mathcal{J} = E\{(z^2-\gamma)^2\}, \text{ subject to } \boldsymbol{C}^T\boldsymbol{f} = \boldsymbol{g} \quad (4)$$

Since $\mathcal{J}$ is a fourth order function of the detector, a closed-form solution to this constrained optimization problem is difficult to obtain. To seek its optimum, we construct a Lagrangian cost function parameterized by $\boldsymbol{f}$ and $\boldsymbol{g}$

$$\mathcal{J}_1 = E\{(z^2-\gamma)^2\} + \boldsymbol{\lambda}^T(\boldsymbol{C}^T\boldsymbol{f} - \boldsymbol{g}) \quad (5)$$

where $\boldsymbol{\lambda}$ is a Lagrange multiplier corresponding to constraints for our detector $\boldsymbol{f}$. Then two update equations for $\boldsymbol{f}$ and $\boldsymbol{g}$ can be formed as

$$\boldsymbol{f}_{n+1} = \boldsymbol{f}_n - \mu_f \nabla_{\boldsymbol{f}} \mathcal{J}_1 \quad (6)$$

$$\boldsymbol{g}_{n+1} = \boldsymbol{g}_n - \mu_g(\boldsymbol{I} - \frac{\boldsymbol{g}_n\boldsymbol{g}_n^T}{\boldsymbol{g}_n^T\boldsymbol{g}_n})\nabla_{\boldsymbol{g}}\mathcal{J}_1 \quad (7)$$

which will recursively minimize $\mathcal{J}_1$ with respect to $\boldsymbol{f}$ and $\boldsymbol{g}$. Projection of $\nabla_{\boldsymbol{g}}\mathcal{J}_1$ onto the space orthogonal to $\boldsymbol{g}$ has been made in (7) in order to update only the orthogonal component. According to (5), the gradients in (6) and (7) can be easily derived. Therefore recursions become

$$\boldsymbol{f}_{n+1} = \boldsymbol{f}_n - \mu_f\left[4E\{z(z^2-\gamma)\boldsymbol{y}_n\} + \boldsymbol{C}\boldsymbol{\lambda}_n\right] \quad (8)$$

$$\boldsymbol{g}_{n+1} = \boldsymbol{g}_n + \mu_g(\boldsymbol{I} - \frac{\boldsymbol{g}_n\boldsymbol{g}_n^T}{\boldsymbol{g}_n^T\boldsymbol{g}_n})\boldsymbol{\lambda}_n \quad (9)$$

The Lagrange multiplier $\boldsymbol{\lambda}_n$ is obtained by enforcing the constraint $\boldsymbol{C}^T\boldsymbol{f}_{n+1} = \boldsymbol{g}_n$ (e.g., [9]). The result is

$$\boldsymbol{\lambda}_n = \frac{1}{\mu_f}\boldsymbol{A}\left[\boldsymbol{C}^T\boldsymbol{f}_n - 4\mu_f E\{z(z^2-\gamma)\boldsymbol{C}^T\boldsymbol{y}_n\} - \boldsymbol{g}_n\right] \quad (10)$$

where

$$\boldsymbol{A} = (\boldsymbol{C}^T\boldsymbol{C})^{-1} \quad (11)$$

By substituting (10) into (8) and (9), and using instantaneous approximation for the expected values, we obtain

$$\boldsymbol{f}_{n+1} = \boldsymbol{\Pi}_{\boldsymbol{C}}^{\perp}\left[\boldsymbol{f}_n - 4\mu_f z(z^2-\gamma)\boldsymbol{y}_n\right] + \boldsymbol{C}\boldsymbol{A}\boldsymbol{g}_n \quad (12)$$

$$\begin{aligned}\boldsymbol{g}_{n+1} &= \frac{\mu_g}{\mu_f}(\boldsymbol{I} - \frac{\boldsymbol{g}_n\boldsymbol{g}_n^T}{\boldsymbol{g}_n^T\boldsymbol{g}_n})\boldsymbol{A}\boldsymbol{C}^T\left[\boldsymbol{f}_n - 4\mu_f z(z^2-\gamma)\boldsymbol{y}_n\right] \\ &- \frac{\mu_g}{\mu_f}(\boldsymbol{I} - \frac{\boldsymbol{g}_n\boldsymbol{g}_n^T}{\boldsymbol{g}_n^T\boldsymbol{g}_n})\boldsymbol{A}\boldsymbol{g}_n + \boldsymbol{g}_n \quad (13)\end{aligned}$$

where

$$\boldsymbol{\Pi}_{\boldsymbol{C}}^{\perp} = \boldsymbol{I} - \boldsymbol{C}\boldsymbol{A}\boldsymbol{C}^T \quad (14)$$

Equations (12) and (13) provide a joint update rule for $\boldsymbol{f}$ and $\boldsymbol{g}$. The behavior of this algorithm turns out to be very difficult to track, because the cost function is highly nonlinear with respect to (w.r.t.) the detector. Even when the output power was used as the cost function in [9], the analysis was much complicated. Next we will only consider the case where $\boldsymbol{g}$ is updated slowly in the neighborhood of the desired solution, and thus $\boldsymbol{g}^T\boldsymbol{g}_1$ could be approximated as a constant.

IV. Convergence Analysis

Due to the previous difficulty in the analysis of the proposed detector and in order to gain some insight into the property of the proposed algorithm, we will simplify our analysis in two steps: first minimize $\mathcal{J}$ w.r.t. $\boldsymbol{f}$ by treating $\boldsymbol{g}$ as a constant, and then minimize $\mathcal{J}_{min}$ w.r.t. $\boldsymbol{g}$. For simplicity, it is also assumed that all users are synchronized, channel is real and noise is negligible.

As in [10], we define $\boldsymbol{H} = [\boldsymbol{C}\boldsymbol{g}_1 \ \boldsymbol{H}_{int}]$ and $\boldsymbol{u}^T = \boldsymbol{f}^T\boldsymbol{H} = [u_1, ..., u_J]$. Since

$$\begin{aligned} z &= \boldsymbol{f}^T\boldsymbol{C}\boldsymbol{g}_1 w_1(n) + \boldsymbol{f}^T\boldsymbol{H}_{int}\boldsymbol{w}_{int}(n) \\ &= \boldsymbol{u}^T\boldsymbol{w}(n) \quad (15)\end{aligned}$$

and the linear constraints $\boldsymbol{C}^T\boldsymbol{f} = \boldsymbol{g}$ are equivalent to $u_1 = \boldsymbol{g}^T\boldsymbol{g}_1$, the constrained optimization problem in (4) reduces to

$$\begin{aligned}\min_{u_1 = \boldsymbol{g}^T\boldsymbol{g}_1} \quad \mathcal{J} &= E\{(z^2-1)^2\} \\ &= 3(\boldsymbol{u}^T\boldsymbol{u})^2 - 2\sum u_k^4 - 2\boldsymbol{u}^T\boldsymbol{u} + 1 \quad (16)\end{aligned}$$

When the constraint vector $\boldsymbol{g}$ is approximately a constant, the analysis is similar to that of [10] with $\boldsymbol{g}^T\boldsymbol{g}_1$ equivalent to A_1 in [10]. Therefore, we directly apply the proposition from [10] and conclude: when $\boldsymbol{g}$ is chosen or updated such that $(\boldsymbol{g}^T\boldsymbol{g}_1)^2 \geq \frac{1}{3}$ always holds, the detector converges to its unique global minimum point with zero MAI; otherwise, when $(\boldsymbol{g}^T\boldsymbol{g}_1)^2 < \frac{1}{3}$, the detector converges to one of its $2J-2$ local minimum points which correspond to the case where only one undesired user has nonzero interference power of $1 - 3(\boldsymbol{g}^T\boldsymbol{g}_1)^2$. Next, we will consider the convergence of $\boldsymbol{g}$ by examining $\mathcal{J}_{min}$ at these two sets of minimum points.

In the first case where $(\boldsymbol{g}^T\boldsymbol{g}_1)^2 \geq \frac{1}{3}$, the MAI becomes zero. We have

$$\mathcal{J}_{min} = [(\boldsymbol{g}^T\boldsymbol{g}_1)^2 - 1]^2$$

Then minimization of $\mathcal{J}_{min}$ will make $(\boldsymbol{g}^T\boldsymbol{g}_1)^2$ converge to 1. If $|\boldsymbol{g}_1| = 1$ and $|\boldsymbol{g}| = 1$, then $\boldsymbol{g}$ converges to $\boldsymbol{g}_1$. Especially, since $(\boldsymbol{g}^T\boldsymbol{g}_1)^2$ is the power of the desired user, this minimum point corresponds to zero MAI and maximum output power of the desired user.

In the second case where $(\boldsymbol{g}^T\boldsymbol{g}_1)^2 < \frac{1}{3}$, when the detector converges to its local minimum point for a specific $\boldsymbol{g}$, we have the interference power as $1 - 3(\boldsymbol{g}^T\boldsymbol{g}_1)^2$, then

$$\begin{aligned} \mathcal{J}_{min} &= [(\boldsymbol{g}^T\boldsymbol{g}_1)^2 + (1 - 3(\boldsymbol{g}^T\boldsymbol{g}_1)^2) - 1]^2 \\ &= 4(\boldsymbol{g}^T\boldsymbol{g}_1)^4 \end{aligned} \quad (17)$$

Minimizing $\mathcal{J}_{min}$ w.r.t. $\boldsymbol{g}$ will clearly make $\boldsymbol{g}$ converge to $\mathbf{0}$. In this case, the output power of the desired user will be zero, while one of the undesired user will have the maximum power of 1. This case leads the algorithm to converge to another user while giving the worst detection performance w.r.t. the desired user.

We thus conclude this section by the following statement:

(1) When the constraint $\boldsymbol{g}$ is initialized with $(\boldsymbol{g}^T\boldsymbol{g}_1)^2 \geq \frac{1}{3}$, and $\boldsymbol{g}$ is updated slowly enough, the proposed detector converges to the desired global minimum point with zero MAI and maximum desired user power.

(2) Otherwise, when $\boldsymbol{g}$ is initialized with $(\boldsymbol{g}^T\boldsymbol{g}_1)^2 < \frac{1}{3}$ and $\boldsymbol{g}$ is updated slowly enough, the proposed detector will converge to one of its $2J - 2$ local minimum points which are undesirable for the detection of the user of interest.

However, without training sequence, the channel $\boldsymbol{g}_1$ is not a priori-known. The question arises as how to choose initial value for the constraint vector $\boldsymbol{g}$. A good (although not optimal) solution is to use the updated result for $\boldsymbol{g}$ from [9] after $\boldsymbol{g}$ converges, since $\boldsymbol{g}$ was shown in [9] to converge to $\boldsymbol{g}_1$ with very small error. The error is due to the AWGN. Then we take that value to initialize $\boldsymbol{g}$ and switch the recursion for $\boldsymbol{g}$ to the proposed one (13).

V. Simulations

In this section, we provide some simulation examples to illustrate the capability of the proposed detector to suppress MAI in a multipath environment. A system with 10 equal power users and spreading factor $P = 31$ was considered. Gold codes of length 31 were used as spreading codes. Each user transmits BPSK signals through a (different) randomly generated multipath channel of length equal to 4 chips. All users were synchronized at the receiver, with the first user assumed to be the desired user. The $15dB$ AWGN was assumed.

In Fig.1, we compare the proposed detector with the trained MMSE, MOE and CMA (under the constant constraints) receivers in terms of the signal to interference plus noise ratio (SINR). The constraint vector $\boldsymbol{g}$ was initialized with $|\boldsymbol{g}^T\boldsymbol{g}_1|^2 = 0.58$. It is seen that the receiver converges to the desired global minimum with the output SINR level very close to that of MMSE receiver, higher than that of the MOE receiver, and much better than that of the constant-constrained CMA receiver.

In the second experiment, we tested the initialization effect. If $\boldsymbol{g}$ was initialized with $|\boldsymbol{g}^T\boldsymbol{g}_1|^2 = 0.14$, it can be observed from Fig. 2 that the detector converges to an undesired local minimum with a negative SINR. However, if we resort to MOE for satisfactory initial constraints after 200 iterations and take the corresponding constrained vector as the initialization for our algorithm, then the proposed detector converges to the desired global minimum point with a higher SINR level than the MOE receiver.

The last experiment investigated the behavior of the proposed detector in a near-far communication environment. We considered the situation that user 1 was assigned the power $0dB$, $2dB$, $4dB$ and $6dB$ weaker than each of other users. The constraint vector was initialized as in the first experiment. The corresponding output SINRs are plotted in Fig.3. Clearly, the SINRs converge to a very similar level, indicating that the proposed detector is near-far resistant.

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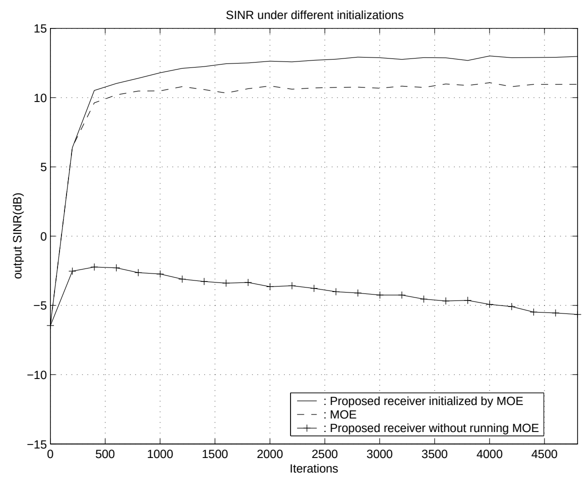

Fig. 2. Effect of the initialization.

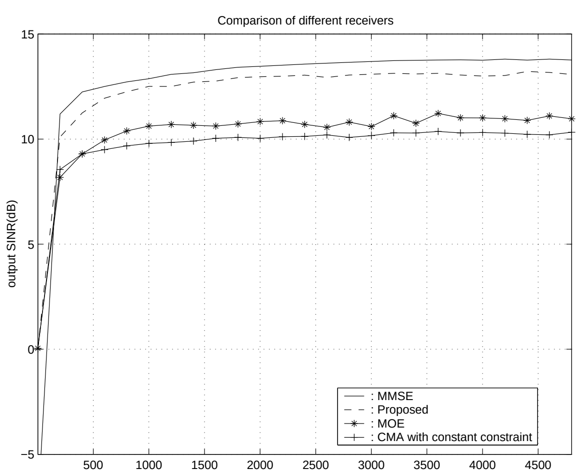

Fig. 1. Comparison of different receivers,SIR=0dB, SNR=15dB.

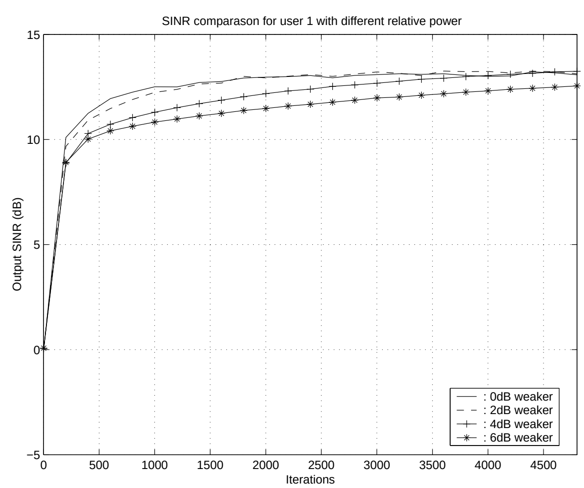

Fig. 3. Near-far effect on the output SINR.