Statistical Performance of Correlation-Matching Based Channel Estimators for Aperiodic Time Hopping UWB Systems

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***Abstract*— In a time hopping ultra-wideband (UWB) impulse radio system, each user is assigned a unique time-hopping sequence. If aperiodic codes are used for time-hopping sequences, some benefits, such as smoothed spectrum and better performance, can be achieved. Recently, a correlation matching method is proposed to estimate channels in aperiodic time-hopping systems. Performance of corresponding multiuser and single user channel estimators is studied in this paper. Closed-form analytical results are provided and verified by computer simulation.**

I. Introduction

Ultra-wideband impulse radio is shown to be an attractive spread spectrum technique in short range wireless communications [1]. A UWB system transmits trains of ultra-short pulses modulated by pulse position modulation (PPM) or pulse amplitude modulation (PAM). Multiple access is enabled by assigning each user a distinct pseudorandom (PN) time hopping (TH) sequence, which adds an additional time shift to each pulse in the pulse train. The sub-nanosecond pulses can provide high multipath resolution and significantly reduce fading effect in an indoor environment. Meanwhile, the low power spectral density of transmitted signals and low probability of interception make it suitable for secure communication applications.

To achieve reliable detection performance, UWB receivers, including single user receivers [2] and multiuser receivers [3], [4], need to know channel parameters. However, in a dense multipath wireless environment, channel information is not known *a priori*. Single-user maximum likelihood (ML) methods are first proposed to estimate UWB channels [5], [6]. Subsequently, several blind channel estimation methods, which use only up to the second order statistics of channel outputs are developed to improve estimation performance for a multiuser communication system [7], [8]. Those methods are suitable for UWB systems with periodic TH sequences. Recently, aperiodic time hopping sequences generated by using symbolic dynamics are proposed for UWB impulse radio systems to enhance spread-spectrum characteristics [9]. Some studies on power spectral density properties have also shown the signal spectrum can be smoothed when the length and randomness of TH sequence increase [10], [11]. TH sequence of longer period can reduce the probability of interception by unintended parties as well. Furthermore, because performance of multiple access UWB communication systems is dependent on specific realization of a periodic TH sequence, aperiodic sequence can help to improve system performance by avoiding periodic use of bad codes or exploring channel diversity in a fading channel. Despite these benefits, channel estimation becomes a more complicated task if aperiodic TH codes are used since hopping codes vary from symbol to symbol causing signature waveforms change dramatically. Fortunately, difficulties are overcome by our recently proposed correlation matching methods [12] for either single-user or multiuser channel estimation. The single-user method yields only the desired user's channel estimate without knowing hopping codes of interfering users while the multiuser method provides channel estimates for all users simultaneously given their hopping codes. It is the purpose of this paper to analyze statistical performance of those methods. All analytical results will be verified by computer simulations.

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Notations: Following common practice, we denote Kronecker product by $\otimes$, Hadamard (element-wise) product by $\odot$, transpose by ^{T}, inverse by $^{-1}$. $E\{\cdot\}$ represents expectation of a random variable, $\boldsymbol{I}_a$ an identity matrix of degree a whose ith column is denoted by $\boldsymbol{e}_{a,i}$. $\mathbf{1}_a$ is a vector of length a with all elements equal to one. An estimate of a quantity (scalar, vector or matrix) is denoted by putting a $\widehat{}$ over it, and correspondingly, the estimation error by preceding the quantity with a δ, such as $\widehat{\boldsymbol{x}}$ and $\delta\boldsymbol{x}$ for vector $\boldsymbol{x}$ respectively.

II. System Model

Consider a multiple access (MA) TH UWB system with K users. The transmitted baseband UWB signal from user k can be described by [1]

$$\alpha_k(t) = \sqrt{\mathcal{P}_k} \sum_{i=-\infty}^{\infty} w(t - iT_f - c_k(i)T_c - \tau_{I_k(\lfloor i/N_f \rfloor)}) \quad (1)$$

where $\mathcal{P}_k$ is the kth user's transmission power, $w(t)$ is the baseband monopulse with duration T_w, T_f is the frame duration, N_f is the number of frames over which an M-ary PPM symbol repeats, $c_k(i) \in [0, N_c - 1]$ is a non-periodic time hopping sequence. Each chip has duration T_c. $I_k(\lfloor i/N_f \rfloor) \in [0, M-1]$ is the kth user's information bearing symbol during the ith frame, $\tau_{I_k(\lfloor i/N_f \rfloor)} = I_k(\lfloor i/N_f \rfloor)T_w$ is the corresponding modulation delay in multiples of T_w seconds. Assume $T_f = N_c T_c$ and $T_c = MT_w$. Eq. (1) shows a nonlinear relationship between $\alpha_k(t)$ and the transmitted information symbol. However, a linear relationship can be obtained as shown in [3]. Let us define M virtual inputs for user k with the mth one as $s_{k,m}(\lfloor i/MN_cN_f \rfloor) = \delta\Big(I_k(\lfloor i/MN_cN_f \rfloor) - m\Big)$. Here i is the pulse rate index. Clearly, only one of the M inputs will be nonzero each time. Also, we can define a new pulse-rate code sequence with the ith element $\widetilde{c}_k(i) = \delta\Big(\lfloor i/MN_c \rfloor MN_c + c_k(\lfloor i/MN_c \rfloor)M - i\Big)$. The mth delayed version of this sequence $\widetilde{\boldsymbol{c}}_{k,m}$ $(m = 0, \cdots, M-1)$ corresponding to the mth virtual input can be generated by $\widetilde{c}_{k,m}(i) = \widetilde{c}_k(i-m)$. Then, (1) can be equivalently expressed as

$$\alpha_k(t) = \sqrt{\mathcal{P}_k} \sum_{i=-\infty}^{\infty} \sum_{m=0}^{M-1} u_{k,m}(i) w(t - iT_w) \tag{2}$$

where $u_{k,m}(i) = s_{k,m}(\lfloor i/MN_cN_f \rfloor)\widetilde{c}_{k,m}(i)$, and $\widetilde{c}_{k,m}(i)$ is the ith element of $\widetilde{\boldsymbol{c}}_{k,m}$. If we define an effective channel including effects from modulated pulse at the transmitter, propagation channel and matched filter at the receiver by $g_k(t) = \sqrt{\mathcal{P}_k} w(t) \star \bar{g}_k(t) \star w(-t)$ where $\star$ denotes convolution. Assume each effective channel spans $(q-1)$ pulse durations. Considering additive white Gaussian noise (AWGN) and propagation delay d_k for user k, the discrete-time output of the matched filter sampled at pulse-rate becomes

$$y(n) = \sum_{k,m} \sum_{i=1}^{q} u_{k,m}(n - i - d_k) g_k(i) + v(n). \tag{3}$$

If we collect $\nu = MN_cN_f$ samples from $y(n\nu + 1), \ldots, y(n\nu + \nu)$ in a vector, then the received data vector follows

$$\boldsymbol{y}_n = \sum_{k,m,l} \boldsymbol{C}_{n,k,m,l} \boldsymbol{g}_k s_{k,m}(n+l) + \boldsymbol{v}_n \tag{4}$$

where l takes all integers $-\lceil q/\nu \rceil, \cdots, 0$, $\boldsymbol{g}_k$ is an unknown channel vector for user k which contains channel coefficients at the pulse rate and power factor $\sqrt{\mathcal{P}_k}$. $\boldsymbol{C}_{n,k,m,l}$ is a code filtering matrix constructed from $\widetilde{\boldsymbol{c}}_{k,m}$ and is uniquely determined by the original TH sequence. Because TH sequence is aperiodic, $\boldsymbol{C}_{n,k,m,l}$ is dependent on symbol index n.

This model can be compactly expressed in another form

$$\boldsymbol{y}_n = \sum_{k,l} \boldsymbol{H}_{n,k,l} \boldsymbol{s}_{n,k,l} + \boldsymbol{v}_n = \boldsymbol{H}_n \boldsymbol{s}_n + \boldsymbol{v}_n \tag{5}$$

after collecting M inputs in a vector

$$\boldsymbol{s}_{n,k,l} = [s_{k,0}(n+l), \cdots, s_{k,M-1}(n+l)]^T,$$

defining a corresponding effective channel matrix $\boldsymbol{H}_{n,k,l} = [\boldsymbol{C}_{n,k,0,l}\boldsymbol{g}_k, \cdots, \boldsymbol{C}_{n,k,M-1,l}\boldsymbol{g}_k]$, and successively stacking such matrices (or vectors) in $\boldsymbol{H}_n$ (or $\boldsymbol{s}_n$). By employing data model (4), all channels can be estimated based on correlation of $\boldsymbol{y}_n$.

III. Blind Channel Estimation

It is observed that all $\boldsymbol{g}_k$ are embedded in the statistics of the data vector. However, because TH sequence is aperiodic and $C_{n,k,m,l}$ is dependent on n, the received signal (5) is not cyclostationary. Its correlation cannot be easily estimated. But if we treat $\boldsymbol{y}_n \boldsymbol{y}_n^T$ as an instantaneous estimate of the correlation matrix and choose a proper cost function, channel estimate can still be obtained. If all users' hopping codes are known, then their channel parameters can be estimated simultaneously, yielding a multiuser correlation matching (MU-CM) channel estimator. In a case that only the desired user's hopping codes are given, its channel can still be estimated, resulting in a single user correlation matching (SU-CM) channel estimator. Both estimators have been proposed in [12]. They are reviewed next in order to study their statistical performance.

A. MU-CM Channel Estimator

Since $\boldsymbol{s}_{k,n,l}$ has non-zero mean, the time-dependent autocorrelation matrix of $\boldsymbol{y}_n$ includes cross terms $\boldsymbol{G}_{k_1,k_2} = \boldsymbol{g}_{k_1}\boldsymbol{g}_{k_2}^T$ of two generic users k_1 and k_2:

$$\boldsymbol{R}_n = \sum \boldsymbol{C}_{k_1,n,m_1,l_1} \boldsymbol{G}_{k_1,k_2} \boldsymbol{C}_{k_2,n,m_2,l_2}^T \gamma_{k_1,k_2,m_1,m_2,l_1,l_2} + \sigma_v^2 \boldsymbol{I}. \tag{6}$$

Here, summation is over $k_1, k_2, m_1, m_2, l_1, l_2$ and $\gamma_{k_1,k_2,m_1,m_2,l_1,l_2}$ is the correlation of inputs $s_{k_1,m_1}(n+l_1)$ and $s_{k_2,m_2}(n+l_2)$

$$\begin{aligned} \gamma_{k_1,k_2,m_1,m_2,l_1,l_2} &= \frac{1}{M}\delta(m_1 - m_2)\delta(l_1 - l_2)\delta(k_1 - k_2) \\ &+ \frac{1}{M^2}\left[1 - \delta(l_1 - l_2)\delta(k_1 - k_2)\right]. \end{aligned}$$

As in [13], vectored form is convenient to handle in the CM context. Define a new vector $\boldsymbol{x}$ which has entries of all possible $\boldsymbol{x}_{k_1,k_2} = vec(\boldsymbol{G}_{k_1,k_2})$ and σ_v^2 as follows

$$\boldsymbol{x} = [\boldsymbol{x}_{1,1}^T, \boldsymbol{x}_{1,2}^T, \cdots, \boldsymbol{x}_{K,K}^T, \sigma_v^2]^T. \tag{7}$$

Using the property of vec [14], we obtain vectorized correlation $\boldsymbol{r}_n = vec(\boldsymbol{R}_n)$

$$\begin{aligned} &\boldsymbol{r}_n = \boldsymbol{S}_n \boldsymbol{x}, \quad \boldsymbol{S}_n = [\boldsymbol{S}_{n,1,1}, \boldsymbol{S}_{n,1,2}, \cdots, \boldsymbol{S}_{n,K,K}, vec(\boldsymbol{I}_\nu)], \\ &\boldsymbol{S}_{n,k_1,k_2} = \sum_{m_1,m_2,l_1,l_2} \gamma_{k_1,k_2,m_1,m_2,l_1,l_2} \boldsymbol{C}_{n,k_2,m_2,l_2} \otimes \boldsymbol{C}_{n,k_1,m_1,l_1}. \end{aligned} \tag{8}$$

Therefore, the following matching errors between $\boldsymbol{r}_n$ and its instantaneous estimate $\widehat{\boldsymbol{r}}_n = vec(\boldsymbol{y}_n \boldsymbol{y}_n^T)$ for N symbol periods can be minimized to obtain an estimate of $\boldsymbol{x}$

$$\mathcal{J} = \frac{1}{N} \sum_n ||\boldsymbol{r}_n - \widehat{\boldsymbol{r}}_n||^2. \tag{9}$$

Considering (8), the solution to this optimization problem becomes

$$\widehat{\boldsymbol{x}} = (\frac{1}{N} \sum_n \boldsymbol{S}_n^T \boldsymbol{S}_n)^{-1} (\frac{1}{N} \sum_n \boldsymbol{S}_n^T \widehat{\boldsymbol{r}}_n). \tag{10}$$

Once $\boldsymbol{x}$ is estimated, entries corresponding to $\boldsymbol{G}_{k,k}$ can be extracted. Then $\boldsymbol{G}_{k,k}$ is reconstructed by the reverse vec operation. These operations can be described by

$$\widehat{\boldsymbol{G}}_{k,k} = [(\boldsymbol{e}_{q,1}^T \otimes \boldsymbol{I}_q)\widehat{\boldsymbol{x}}_{k,k}, \cdots, (\boldsymbol{e}_{q,q}^T \otimes \boldsymbol{I}_q)\widehat{\boldsymbol{x}}_{k,k}],$$

$$\widehat{\boldsymbol{x}}_{k,k} = [\boldsymbol{e}^T_{K^2,(k-1)K+k} \otimes \boldsymbol{I}_{q^2}, \boldsymbol{0}_{q^2\times 1}]\widehat{\boldsymbol{x}}. \quad (11)$$

Using (10), we can relate $\widehat{\boldsymbol{G}}_{k,k}$ to $\widehat{\boldsymbol{r}}_n$,

$$\widehat{\boldsymbol{G}}_{k,k} = \left[\boldsymbol{\Phi}_{k,1}(\frac{1}{N}\sum_n \boldsymbol{S}^T_n \widehat{\boldsymbol{r}}_n), \cdots, \boldsymbol{\Phi}_{k,q}(\frac{1}{N}\sum_n \boldsymbol{S}^T_n \widehat{\boldsymbol{r}}_n)\right] \quad (12)$$

where

$$\boldsymbol{\Phi}_{k,i} = (\boldsymbol{e}^T_{q,i} \otimes \boldsymbol{I}_q)[\boldsymbol{e}^T_{K^2,(k-1)K+k} \otimes \boldsymbol{I}_{q^2}, \boldsymbol{0}_{q^2\times 1}](\frac{\sum_n \boldsymbol{S}^T_n \boldsymbol{S}_n}{N})^{-1}$$

for $i = 1, \cdots, q$.

Once $\widehat{\boldsymbol{G}}_{k,k}$ is obtained, channel vector $\boldsymbol{g}_k$ can be estimated from its singular value decomposition (SVD) by finding the singular vector corresponding to its maximum singular value. That singular vector becomes an estimate of $\boldsymbol{g}_k$ up to a multiplicative scalar.

B. SU-CM Channel Estimator

During the above process for obtaining channel parameters, we have to know all users' TH codes. When only the desired user's TH codes are available, a single user channel estimator is desirable. Because more unknowns (cross-products of channel vectors) are introduced in the MU-CM method, significant complexity reduction occurs in the single-user solution.

Without loss of generality, we assume user one is the desired user. Then we can break down the received signal (4) into the desired signal, interference signal and noise:

$$\boldsymbol{y}_n = \sum_{1,m,l} \boldsymbol{C}_{n,1,m,l}\boldsymbol{g}_1 s_{1,m}(n+l) + \boldsymbol{y}_{int,n} + \boldsymbol{v}_n. \quad (13)$$

Here, the total effect of multiple access interference from all other $K-1$ users is lumped into $\boldsymbol{y}_{int,n}$ and approximated as a stationary process with unknown mean $\boldsymbol{b}_{int}$ and unknown autocorrelation $\boldsymbol{R}_{int}$. Then we can rewrite the time-varying autocorrelation matrix (6) as

$$\begin{aligned}\boldsymbol{R}_n &= \sum_{m_1,m_2,l_1,l_2} \boldsymbol{C}_{1,n,m_1,l_1}\boldsymbol{G}_{1,1}\boldsymbol{C}^T_{1,n,m_2,l_2}\gamma_{1,1,m_1,m_2,l_1,l_2} \\ &+ \frac{1}{M}\sum_{m,l}(\boldsymbol{C}_{1,n,m,l}\boldsymbol{Q}_1 + \boldsymbol{Q}^T_1\boldsymbol{C}^T_{1,n,m,l}) + \boldsymbol{R}_{tot} \quad (14)\end{aligned}$$

where $\boldsymbol{Q}_1 = \boldsymbol{g}_1\boldsymbol{b}^T_{int}$, $\boldsymbol{R}_{tot} = \boldsymbol{R}_{int} + \sigma^2_v\boldsymbol{I}$. Let us define nuisance quantities $\boldsymbol{z}_1 = vec(\boldsymbol{Q}_1)$, and $\boldsymbol{r}_{tot} = vec(\boldsymbol{R}_{tot})$. After taking vec operation, (14) becomes

$$\boldsymbol{r}_n = \boldsymbol{S}_{n,1,1}\boldsymbol{x}_1 + \widetilde{\boldsymbol{D}}_{1,n}\boldsymbol{z}_1 + \boldsymbol{r}_{tot} \quad (15)$$

Here, $\widetilde{\boldsymbol{D}}_{1,n} = \boldsymbol{D}_{1,n} + \boldsymbol{\Gamma}\boldsymbol{D}_{1,n}$, $\boldsymbol{D}_{1,n} = 1/M\boldsymbol{I}_\nu \otimes \boldsymbol{C}_{n,1,m,l}$, $\boldsymbol{\Gamma} = [\boldsymbol{I}_\nu \otimes \boldsymbol{e}_{\nu,1}, \cdots, \boldsymbol{I}_\nu \otimes \boldsymbol{e}_{\nu,\nu}]^T$ and we use the property of vec operation that for any matrix $\boldsymbol{A}$, $vec(\boldsymbol{A}^T) = \boldsymbol{\Gamma}vec(\boldsymbol{A})$. Invoking cost function (9), unknowns $\boldsymbol{x}_1, \boldsymbol{z}_1, \boldsymbol{r}_{tot}$ can be estimated. For example, we can solve $\boldsymbol{r}_{tot}$ first

$$\widehat{\boldsymbol{r}}_{tot} = -\frac{1}{N}\sum_n(\boldsymbol{S}_{n,1,1}\boldsymbol{x}_1 + \widetilde{\boldsymbol{D}}_{1,n}\boldsymbol{z}_1 - \widehat{\boldsymbol{r}}_n). \quad (16)$$

After back substitution and optimizing (9), we obtain the estimate of all entries in $\boldsymbol{x}_1$ and $\boldsymbol{z}_1$

$$\widehat{\widetilde{\boldsymbol{x}}} = \left(\frac{1}{N}\sum_n \boldsymbol{W}^T_n\boldsymbol{W}_n\right)^{-1}\left(\frac{1}{N}\sum_n \boldsymbol{W}^T_n\widehat{\widetilde{\boldsymbol{r}}}_n\right) \quad (17)$$

where $\widehat{\widetilde{\boldsymbol{x}}}$ and $\widehat{\widetilde{\boldsymbol{r}}}$ are estimates of

$$\widetilde{\boldsymbol{x}} = [\boldsymbol{x}^T_1, \boldsymbol{z}^T_1]^T, \quad \widetilde{\boldsymbol{r}}_n = \boldsymbol{r}_n - \frac{1}{N}\sum_n \boldsymbol{r}_n$$

respectively,

$$\boldsymbol{W}_n = [\Delta\boldsymbol{S}_{n,1,1}, \Delta\widetilde{\boldsymbol{D}}_{1,n}],$$

$$\Delta\boldsymbol{S}_{n,1,1} = \boldsymbol{S}_{n,1,1} - \frac{1}{N}\sum_n \boldsymbol{S}_{n,1,1},$$

$$\Delta\widetilde{\boldsymbol{D}}_{1,n} = \widetilde{\boldsymbol{D}}_{1,n} - \frac{1}{N}\sum_n \widetilde{\boldsymbol{D}}_{1,n}.$$

Similar as (11), we can extract $\boldsymbol{x}_1$ from $\widetilde{\boldsymbol{x}}$ and construct $\boldsymbol{G}_{1,1}$ as following

$$\widehat{\boldsymbol{G}}_{1,1} = \left[\widetilde{\boldsymbol{\Phi}}_{1,1}(\frac{1}{N}\sum_n \boldsymbol{W}^T_n\widehat{\widetilde{\boldsymbol{r}}}_n), \cdots, \widetilde{\boldsymbol{\Phi}}_{1,q}(\frac{1}{N}\sum_n \boldsymbol{W}^T_n\widehat{\widetilde{\boldsymbol{r}}}_n)\right],$$

$$\widetilde{\boldsymbol{\Phi}}_{1,i} = (\boldsymbol{e}^T_{q,i} \otimes \boldsymbol{I}_q)[\boldsymbol{I}_{q^2}, \boldsymbol{0}_{q^2\times q\nu}]\left(\frac{1}{N}\sum_n \boldsymbol{W}^T_n\boldsymbol{W}_n\right)^{-1} \quad (18)$$

for $i = 1, \cdots, q$. Then we can apply SVD on $\widehat{\boldsymbol{G}}_{1,1}$ to obtain $\widehat{\boldsymbol{g}}_1$.

C. Complexity

From (12) and (18), we can see matrix inversion and corresponding multiplication involved in obtaining $\boldsymbol{\Phi}_{k,i}$ or $\widetilde{\boldsymbol{\Phi}}_{1,i}$ constitute major computational complexity of our proposed estimators. In the MU-CM method, matrix $\boldsymbol{S}_n$ has dimensionality $\nu^2 \times (q^2K^2+1)$. Multiplication in $\boldsymbol{S}^T_n\boldsymbol{S}_n$ is thus about $\nu^2q^4K^4$ and matrix inverse has complexity $O(q^6K^6)$. Total complexity is $O(N\nu^2q^4K^4)$. Similarly, the dimension of $\boldsymbol{W}_n$ in SU-CM method is $\nu^2 \times (\nu q + q^2)$ and computation of $(\frac{1}{N}\sum_n \boldsymbol{W}^T_n\boldsymbol{W}_n)^{-1}$ requires complexity $O(N\nu^4q^2)$. Then we can conclude SU-CM method has less complexity than MU-CM method if $\nu < qK^2$ is satisfied, which is usually the case in a multiuser system.

We want to mention there are several different approaches to estimate $\boldsymbol{x}_1$ from (15) in the SU-CM method and each approach will result in different complexity. If we estimate $\boldsymbol{z}_1$ using $\boldsymbol{x}_1$ first and then solve $\boldsymbol{x}_1$, instead of estimating both $\boldsymbol{x}_1$ and $\boldsymbol{z}_1$ simultaneously as we did above, we can show the complexity can be reduced to $O(N\nu^3q^3)$. Due to limited space, we will discuss this in detail elsewhere.

IV. Performance Analysis

For both MU-CM and SU-CM methods, a major source of channel estimation error is due to an estimation error in the data correlation $\boldsymbol{R}_n$ or equivalently $\boldsymbol{r}_n$. This error is highly dependent on the number of data vectors used in our estimation process. Thus, we analyze the statistical performance of both methods in this section.

A. MU-CM estimator

From our previous discussion, $\boldsymbol{g}_k$ is an eigenvector of $\boldsymbol{G}_{k,k}$ corresponding to its unique non-zero eigenvalue. Because there is an estimation error $\delta \boldsymbol{r}$ between $\boldsymbol{r}_n$ and its instantaneous estimate $\widehat{\boldsymbol{r}}_n$, an error is introduced to $\widehat{\boldsymbol{G}}_{k,k}$. From (10) and (11), $\boldsymbol{G}_{k,k}$ is perturbed by $\delta \boldsymbol{G}_{k,k}$ as

$$\delta \boldsymbol{G}_{k,k} = \left[\boldsymbol{\Phi}_{k,1}(\frac{1}{N}\sum_n \boldsymbol{S}_n^T \delta \boldsymbol{r}_n), \cdots, \boldsymbol{\Phi}_{k,q}(\frac{1}{N}\sum_n \boldsymbol{S}_n^T \delta \boldsymbol{r}_n) \right]. \tag{19}$$

Then the first-order perturbation in its eigenvector $\boldsymbol{g}_k$ becomes [15]

$$\delta \boldsymbol{g}_k \approx \boldsymbol{\Pi}^{\perp}_{\boldsymbol{g}_k} \delta \boldsymbol{G}_{k,k} \boldsymbol{g}_k, \quad \boldsymbol{\Pi}^{\perp}_{\boldsymbol{g}_k} = \boldsymbol{\Sigma}_k \boldsymbol{\Sigma}_k^T \tag{20}$$

where $\boldsymbol{\Sigma}_k$ is in size of $q \times (q-1)$ and spans a $(q-1)$-dimensional subspace orthogonal to $\boldsymbol{g}_k$. Substituting (19) into (20), we obtain

$$\delta \boldsymbol{g}_k \approx \boldsymbol{\Psi}_k(\frac{1}{N}\sum_n \boldsymbol{S}_n^T \delta \boldsymbol{r}_n), \;\; \boldsymbol{\Psi}_k = \boldsymbol{\Pi}^{\perp}_{\boldsymbol{g}_k} \sum_{i=1}^{q} g_k(i) \boldsymbol{\Phi}_{k,i}. \tag{21}$$

Then the auto-covariance of channel estimate becomes

$$\begin{aligned} \mathrm{Cov}(\delta \boldsymbol{g}_\mathrm{k}) &\approx \frac{1}{N^2}\boldsymbol{\Psi}_k \left(\sum_{n_1,n_2} \boldsymbol{S}_{n_1}^T E\{\delta \boldsymbol{r}_{n_1} \delta \boldsymbol{r}_{n_2}^T\} \boldsymbol{S}_{n_2} \right) \boldsymbol{\Psi}_k^T \\ &= \frac{1}{N^2}\boldsymbol{\Psi}_k \left(\sum_{n} \boldsymbol{S}_{n}^T E\{\delta \boldsymbol{r}_{n} \delta \boldsymbol{r}_{n}^T\} \boldsymbol{S}_{n} \right) \boldsymbol{\Psi}_k^T, \end{aligned} \tag{22}$$

where the last equation follows from our assumption that output vectors at different time are independent. This can be always achieved by collecting data free of inter-symbol interference. The term $E\{\delta \boldsymbol{r}_n \delta \boldsymbol{r}_n^T\}$ depends on the data model (5) and estimation method of $\boldsymbol{r}_n$. To facilitate evaluating it, we will define some new variables first.

Notice that the mean of data vector $\bar{\boldsymbol{y}}_n$ is not zero. But it is easier to acquire different statistics of zero-mean random vector. Therefore, we introduce a new data vector $\boldsymbol{z}_n$ by subtracting $\bar{\boldsymbol{y}}_n$ from $\boldsymbol{y}_n$

$$\boldsymbol{z}_n = \boldsymbol{y}_n - \bar{\boldsymbol{y}}_n = \sum_{k,l} \boldsymbol{H}_{n,k,l} \boldsymbol{a}_{n,k,l} + \boldsymbol{v}_n, \tag{23}$$

where $\boldsymbol{a}_{n,k,l} = \boldsymbol{s}_{n,k,l} - \frac{1}{M}\boldsymbol{1}_M$. The correlation matrix of $\boldsymbol{a}_{n,k,l}$ is $\boldsymbol{A} = E\{\boldsymbol{a}_{n,k,l}\boldsymbol{a}_{n,k,l}^T\}$ and that of $\boldsymbol{z}_n$ is $\widetilde{\boldsymbol{R}}_n = E\{\boldsymbol{z}_n \boldsymbol{z}_n^T\}$.

Now, we give the result of $E\{\delta \boldsymbol{r}_n \delta \boldsymbol{r}_n^T\}$ in the following proposition with proof provided elsewhere.

Proposition: If channel model follows (5) and data correlation is estimated from its instantaneous estimate as $\widehat{\boldsymbol{r}}_n = vec(\boldsymbol{y}_n \boldsymbol{y}_n^T)$, then for a binary system ($M = 2$)

$$E\{\delta \boldsymbol{r}_n \delta \boldsymbol{r}_n^T\} = \boldsymbol{K}_z + \widetilde{\boldsymbol{R}}_n \otimes \widetilde{\boldsymbol{R}}_n + \boldsymbol{B}_1 \odot \boldsymbol{B}_1 + \boldsymbol{B}_2, \tag{24}$$

where

$$\boldsymbol{B}_1 = (\boldsymbol{I}_\nu \otimes \boldsymbol{1}_\nu)\widetilde{\boldsymbol{R}}_n(\boldsymbol{1}_\nu^T \otimes \boldsymbol{I}_\nu),$$

$$\boldsymbol{B}_2 = \bar{\boldsymbol{y}}_n \bar{\boldsymbol{y}}_n^T \otimes \widetilde{\boldsymbol{R}}_n + \widetilde{\boldsymbol{R}}_n \otimes \bar{\boldsymbol{y}}_n \bar{\boldsymbol{y}}_n^T + \bar{\boldsymbol{y}}_n \otimes \widetilde{\boldsymbol{R}}_n \otimes \bar{\boldsymbol{y}}_n^T + \bar{\boldsymbol{y}}_n^T \otimes \widetilde{\boldsymbol{R}}_n \otimes \bar{\boldsymbol{y}}_n,$$

$\boldsymbol{K}_z$ is the cumulant matrix of $\boldsymbol{z}_n$ and related to the cumulant matrix $\boldsymbol{K}_a$ of input vector $\boldsymbol{a}_{n,k,l}$ as

$$\boldsymbol{K}_z = \sum_{k,l} (\boldsymbol{H}_{n,k,l} \otimes \boldsymbol{H}_{n,k,l}) \boldsymbol{K}_a (\boldsymbol{H}_{n,k,l} \otimes \boldsymbol{H}_{n,k,l})^T,$$

$$\begin{aligned} \boldsymbol{K}_a &= \frac{1}{M}\sum_{i=1}^{M} (\widetilde{\boldsymbol{e}}_{M,i}\widetilde{\boldsymbol{e}}_{M,i}^T) \otimes (\widetilde{\boldsymbol{e}}_{M,i}\widetilde{\boldsymbol{e}}_{M,i}^T) \\ &- vec(\boldsymbol{A})vec(\boldsymbol{A})^T - \boldsymbol{A} \otimes \boldsymbol{A} - \boldsymbol{B}_3 \odot \boldsymbol{B}_3^T \end{aligned}$$

where $\widetilde{\boldsymbol{e}}_{M,i} = \boldsymbol{e}_{M,i} - \frac{1}{M}\boldsymbol{1}_M$

$$\boldsymbol{B}_3 = (\boldsymbol{I}_M \otimes \boldsymbol{1}_M)\boldsymbol{A}(\boldsymbol{1}_M^T \otimes \boldsymbol{I}_M).$$

□

In (24), $\boldsymbol{B}_2$ accounts for the non-zero mean of output signal $\boldsymbol{y}_n$ in UWB systems. With (22) and (24), channel mean-square-error (MSE) can be found as the trace of $\mathrm{Cov}(\delta \boldsymbol{g}_\mathrm{k})$. We can see the MSE is dependent on the fourth order statistics of input because CM method uses the second-order statistics of the channel output. Moreover, MSE is roughly inversely proportional to data length N.

B. SU-CM estimator

From (18), channel estimation error is introduced by inaccurate instantaneous estimate of $\widetilde{\boldsymbol{r}}_n$:

$$\delta \boldsymbol{g}_1 \approx \boldsymbol{\Pi}^{\perp}_{\boldsymbol{g}_1} \delta \boldsymbol{G}_{1,1} \boldsymbol{g}_1 = \boldsymbol{\Omega}(\frac{1}{N}\sum_n \boldsymbol{W}_n^T \delta \widetilde{\boldsymbol{r}}_n),$$

$$\boldsymbol{\Omega} = \boldsymbol{\Pi}^{\perp}_{\boldsymbol{g}_1} \sum_{i=1}^{q} g_1(i) \widetilde{\boldsymbol{\Phi}}_{1,i}. \tag{25}$$

Then we can obtain the auto-covariance matrix for the user of interest

$$\mathrm{Cov}(\delta \boldsymbol{g}_1) = \frac{1}{N^2}\boldsymbol{\Omega} \sum_{n_1,n_2} \left(\boldsymbol{W}_{n_1}^T E\{\delta \widetilde{\boldsymbol{r}}_{n_1} \delta \widetilde{\boldsymbol{r}}_{n_2}^T\} \boldsymbol{W}_{n_2} \right) \boldsymbol{\Omega}^T. \tag{26}$$

Under the assumption that $\boldsymbol{y}_n$ at different n are independent, it reduces to:

$$\mathrm{Cov}(\delta \boldsymbol{g}_1) = \frac{1}{N^2}\boldsymbol{\Omega} \sum_{n} \left(\boldsymbol{W}_{n}^T E\{\delta \boldsymbol{r}_{n} \delta \boldsymbol{r}_{n}^T\} \boldsymbol{W}_{n} \right) \boldsymbol{\Omega}^T. \tag{27}$$

Here, we have used the fact that $\sum_n \boldsymbol{W}_n = 0$. Comparing (27) to (26), we only need to evaluate the correlation of $\delta \boldsymbol{r}_n$ instead of $\delta \widetilde{\boldsymbol{r}}_n$. As a result, we can directly apply the result in the above proposition for $E\{\delta \boldsymbol{r}_n \delta \boldsymbol{r}_n^T\}$ and evaluate the covariance matrix $\mathrm{Cov}(\delta \boldsymbol{g}_1)$. Channel MSE for the user of interest is then the trace of this matrix.

It is worth to point out that the interfering signal plus noise has been assumed as stationary process with unknown constant mean and constant correlation in obtaining our SU-CM channel estimator. This approximation is not accurate in aperiodic TH systems, yielding additional modelling error in channel estimates compared with MU-CM estimator, but is not included in the above analysis. We will address its effect later by simulation.

V. Simulation

In this section, we present numerical results for our previous analysis. The system in our simulation adopts binary PPM modulation with $N_c = 4$, $N_f = 2$. The monocycle pulse is chosen as normalized second derivative of the Gaussian pulse with pulse duration $T_w = 0.7ns$. Four users in the system have equal transmitting power. A randomly generated

4-path channel has time delay resolution of T_w. Path gains for different users are modelled as independent Gaussian random variables and weighted by linearly decreasing weights [8].

We randomly generate 30 sets of TH codes and perform 20 independent realizations for each set. Averaged channel estimation errors with respect to data length N are illustrated in Fig. 1. Experimental and analytical results for both MU-CM and SU-CM methods are plotted for comparison. Clearly the MU-CM estimator outperforms the SU-CM one due to accurate modeling of the system. All MSEs decrease with increase of N. When N is above 300, MSEs from both estimators are less than 10^{-2}. For the MU-CM estimator, the analytical curve matches with the experimental curve very well. However, an obvious gap occurs for the SU-CM estimator mainly attributed to possible violation of our stationary assumption for the interference signal. For verification, we intentionally assign short-codes to all interfering users and plot the MSE from the SU-CM estimator in Fig. 2. Satisfactory agreement is observed. In Fig. 3, we study the effect of SNR on MSE with $N = 500$. Similar conclusions regarding closeness between analytical and experimental results for both estimators can be made. All MSEs decrease monotonically with increase of SNR. MSE floors at high SNR are due to finite data length.

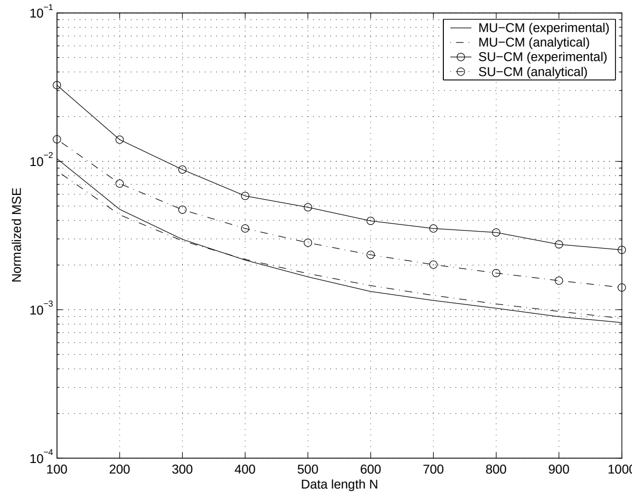

Fig. 1. Channel estimation error under SNR=15dB.

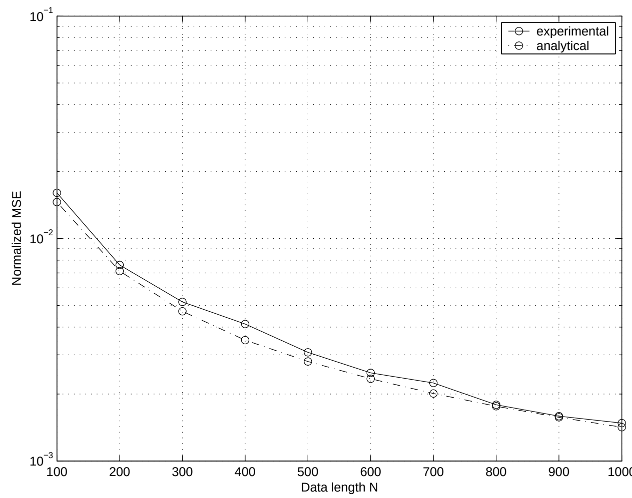

Fig. 2. Performance of SU-CM in a hypothetical system where interfering users use short TH codes.

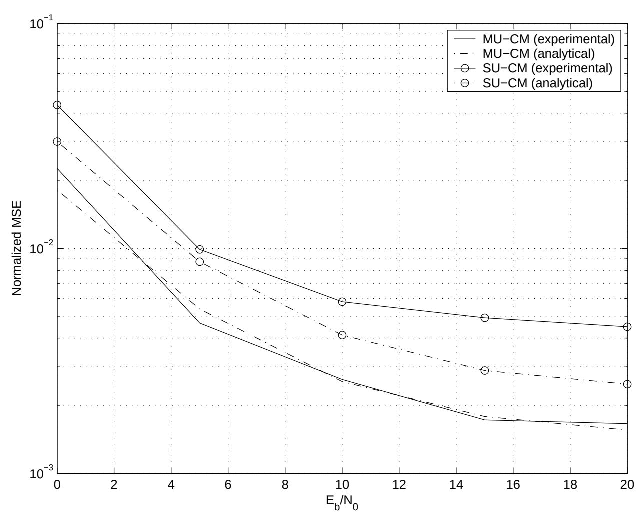

Fig. 3. Channel estimation error with respect to SNR.

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