Structures of Discrete-Time Systems

- For a given transfer function, there could be multiple choices of structures.
- With finite numerical precision, different structures for a given *ideal* transfer function may result in different *actual* transfer functions.
- The basic elements:
  
  \( \text{sum, multiplication, and delay} \)

- The difference equation of a causal LTI system:

  \[
  y(n) = \sum_{k=1}^{N} a_k y(n-k) + \sum_{k=0}^{M} b_k x(n-k)
  \]

- The corresponding transfer function:

  \[
  H(z) = \frac{\sum_{k=0}^{M} b_k z^{-k}}{1 - \sum_{k=1}^{M} a_k z^{-k}}
  \]
• Block diagram of the first direct form:

Define: \( v(n) = \sum_{k=0}^{M} b_k x(n-k) \)

Then: \( y(n) = \sum_{k=1}^{N} a_k y(n-k) + v(n) \)

*Can you sketch it?!*

• Block diagram of the second direct form:

Define: \( w(n) = \sum_{k=1}^{N} a_k w(n-k) + x(n) \)

Then: \( y(n) = \sum_{k=0}^{M} b_k w(n-k) \)

*Can you sketch it?!*

• The second direct form is also referred to as

*canonic direct form* which has the minimum number of delay elements.
• Signal flow graph representation:

Using “nodes and branches” for “sum, multiplication and delay”

• Signal flow graph of the direct form I

• Signal flow graph of the direct form II

• Cascade form

\[
H(z) = A \prod_{k=1}^{M_1} \left(1 - f_k z^{-1}\right) \prod_{k=1}^{M_2} \left(1 - g_k z^{-1}\right) \left(1 - g_k^* z^{-1}\right)
\]

\[
\prod_{k=1}^{N_1} \left(1 - c_k z^{-1}\right) \prod_{k=1}^{N_2} \left(1 - d_k z^{-1}\right) \left(1 - d_k^* z^{-1}\right)
\]

• Parallel form

\[
H(z) = \prod_{k=1}^{N_s} \frac{b_{0k} + b_{1k} z^{-1} + b_{2k} z^{-2}}{1 - a_{1k} z^{-1} - a_{2k} z^{-2}}
\]

• Transposed form

By reversing the flow of all signals, we obtain an equivalent system.
• Structure for linear phase FIR system

\[ y(n) = \sum_{k=0}^{M} h(k)x(n-k) \]

where \( h(M-n) = h(n) \) or \( h(M-n) = -h(n) \)

Can you sketch a block diagram that preserves the linear phase property even with finite numerical precision?