HW#1: P2.1 (a), (c), (e), (g); P2.4; P2.24
HW#2: P2.5; P2.18; P2.29 (a), (c), (e)
HW#3: P2.40; P2.41; P3.27 (a), (c)
HW#4: P3.6 (d), (e); P3.20; P4.1; P4.3
HW#5: P4.5; P4.7; P5.2; P5.3
HW#6: P5.10; P5.12; P5.15
HW#7: P6.7; P6.8; P6.11; P6.25; P7.15

HW#1: P2.1 (a), (c), (e), (g); P2.4; P2.24

P2.1
(a) \(T(x[n]) = g[n]x[n]\);
   - Stable if \(g[n]\) is bounded;
   - Causal – output is not decided by future input;
   - Linear – \(T(ax[n] + by[n]) = ag[n]x[n] + bg[n]y[n] = aT(x[n]) + bT(y[n])\);
   - Time variant – \(T(x[n-m]) = g[n]x[n-m]\);
   - Memoryless – output only depends on \(x[n]\) with same \(n\);

(c) \(T(x[n]) = \sum_{k=n-n_0}^{n+n_0} x[k]\);
   - Stable;
   - Causal only if \(n_0 = 0\), else non-causal;
   - Linear \(T(ax[n] + by[n]) = aT(x[n]) + bT(y[n])\);
   - Time Invariant \(T(x[n-m]) = \sum_{k=n-m-n_0}^{n-m+n_0} x[k] = \sum_{k=n-n_0}^{n+n_0} x[k-m]\);
   - Memoryless only if \(n_0 = 0\);

(e) \(T(x[n]) = e^{x[n]}\);
   - Stable;
   - Causal;
   - Nonlinear – \(T(ax[n] + by[n]) \neq aT(x[n]) + bT(y[n])\);
   - Time Invariant \(T(x[n-m]) = e^{x[n-m]}\);
   - Memoryless – output only depends on \(x[n]\) with same \(n\);

(g) \(T(x[n]) = x[-n]\);
Stable;
Non-Causal – output depends on future input;
Linear – \( T(ax[n] + by[n]) = aT(x[n]) + bT(y[n]) \);
Time Variant;
Not Memoryless;

**P2.4**

As \( y[n] = -\frac{3}{4}y[n-1] + \frac{1}{8}y[n-2] = 2x[n-1] \), the Fourier transform is

\[
Y(e^{j\omega}) - \frac{3}{4}Y(e^{j\omega})e^{-j\omega} + \frac{1}{8}Y(e^{j\omega})e^{-2j\omega} = 2X(e^{j\omega})e^{-j\omega}
\]

When \( x[n] = \delta[n] \Rightarrow X(e^{j\omega}) = 1 \), thus

\[
Y(e^{j\omega}) = \frac{2e^{-j\omega}}{1 - (3/4)e^{-j\omega} + (1/8)e^{-2j\omega}} = \frac{1}{1 - (1/2)e^{-j\omega}} - \frac{1}{1 - (1/4)e^{-j\omega}}
\]

\[
y[n] = 8 \left( \frac{1}{2} \right)^n - \left( \frac{1}{4} \right)^n u[n]
\]

**P2.24**

As \( h[n] = [1 \ 1 \ 1 \ -2 \ -2] \) for \( n \) from 0 to 5 and \( x[n] = u[n-4] \), the system response is:

\( y[n] = x[n] * h[n] = [0 \ 0 \ 0 \ 0 \ 1 \ 2 \ 3 \ 4 \ 2 \ 0 \ ...] \); The sketch is shown as follows:

**HW#2: P2.5; P2.18; P2.29 (a), (c), (e)**

**P2.5**

(a) The roots for polynomial \( 1 - 5z^{-1} + 6z^{-2} = 0 \) are 2 and 3, so the homogeneous response for the system is:

\[
y[n] = A_1 2^n + A_2 3^n
\]
(b) As \( y[n] - 5y[n-1] + 6y[n-2] = 2x[n-1] \) and \( x[n] = \delta[n] \), the impulse response of the system is:

\[
H(e^{j\omega}) = \frac{2e^{-j\omega}}{1-5e^{-j\omega} + 6e^{-2j\omega}} = 2 \left( \frac{1}{1-3e^{-j\omega}} - \frac{1}{1-2e^{-j\omega}} \right)
\]

\[\Rightarrow h[n] = 2(3^n - 2^n)u[n]\]

(c) As \( y[n] - 5y[n-1] + 6y[n-2] = 2x[n-1] \) and \( x[n] = u[n] \), the step response of the system is:

\[
Y(z) = H(z)X(z) = \frac{2z^{-1}}{1-5z^{-1} + 6z^{-2}} \left( \frac{1}{1-z^{-1}} \right) = \frac{1}{1-z^{-1}} - \frac{4}{1-2z^{-1}} + \frac{3}{1-3z^{-1}}, |z| > 3
\]

\[\Rightarrow y[n] = (3^{n+1} - 2^{n+2} + 1)u[n]\]

**P2.18**

(a) \( h[n] = (1/2)^nu[n] \)

Causal, the output of the system does not depend on future input;

(b) \( h[n] = (1/2)^n u[n-1] \)

Causal, the output of the system does not depend on future input;

(c) \( h[n] = (1/2)^{|n|} \)

Non-Causal, the output of the system depends on future input;

(d) \( h[n] = u[n+2] - u[n-2] \)

Non-Causal, the output of the system depends on future input;

(e) \( h[n] = (1/3)^nu[n] + 3^n u[-n-1] \)

Non-Causal, the output of the system depends on future input;

**P2.29**

As \( x[n] = [1 1 1 1 1 1/2] \) for \( n \) from –1 to 4,

(a) \( x[n-2] = [1 1 1 1 1 1/2] \) for \( n \) from 1 to 6; The sketch is:
(c) $x[2n] = [1 \ 1 \ 1/2]$ for $n$ from 0 to 2; The sketch is:

(e) $x[n-1] \delta[n-3] = x[2]$; The sketch is:

HW#3: P2.40, P2.41, P3.27 (a), (c)

P2.40
\( x[n] = \cos(\pi n)u[n] = (-1)^nu[n] \)

\( h[n] = \left(\frac{j}{2}\right)^nu[n] \)

\( y[n] = h[n] \ast x[n] = \sum_{k=\infty}^{\infty} h[k]x[n-k] = \sum_{k=\infty}^{\infty} \left(\frac{j}{2}\right)^k u[k](-1)^{n-k} u[n-k] = (-1)^n \sum_{k=0}^{\infty} \left(\frac{j}{2}\right)^k \)

\[= (-1)^n \frac{1 - \left(-\frac{j}{2}\right)^{n+1}}{1 - \left(-\frac{j}{2}\right)} \]

Since \( \lim_{n \to \infty} \frac{1 - \left(-\frac{j}{2}\right)^{n+1}}{1 - \left(-\frac{j}{2}\right)} = \frac{1}{1 + j/2} \)

The steady state response to the excitation \( x[n] = (-1)^n u[n] \) is

\[(-1)^n \frac{1}{1 + j/2} = \frac{\cos(\pi n)}{1 + j/2} \]

**P2.41**

Given a periodic impulse train \( x[n] = \sum_{k=-\infty}^{\infty} \delta[n + kN] \), we can write its Fourier transform as

\[X(e^{j\omega}) = \frac{2\pi}{N} \sum_{k=-\infty}^{\infty} \delta(\omega + 2\pi k / N) \] (1)

(Refer to *Signal and Systems*, 2nd edition by A.V. Oppenheim and A.S. Willsky, Page 371 for its proof)

In problem 2.41, \( N=16 \), so its Fourier transform is

\[X(e^{j\omega}) = \frac{2\pi}{16} \sum_{k=-\infty}^{\infty} \delta(\omega + 2\pi k / 16) \] (2)

Let \( Y(e^{j\omega}) \) denotes the output of the system, then

\[Y(e^{j\omega}) = X(e^{j\omega})H(e^{j\omega}) \] (3)

If \( |\omega| < 3\pi / 8 \), \( H(e^{j\omega}) = e^{-j3\omega} \)

\[e^{-j3\omega} \frac{2\pi}{16} \sum_{k=-\infty}^{\infty} \delta(\omega + 2\pi k / 16) = \frac{2\pi}{16} [\delta(\omega) + e^{j3\pi/8} \delta(\omega + \pi / 8) + e^{-j3\pi/8} \delta(\omega - \pi / 8)] \] (4)

If \( |\omega| \geq 3\pi / 8 \), \( H(e^{j\omega}) = 0 \), thus \( Y(e^{j\omega}) = 0 \) ,

So \( Y(e^{j\omega}) = \frac{2\pi}{16} [\delta(\omega) + e^{j3\pi/8} \delta(\omega + \pi / 8) + e^{-j3\pi/8} \delta(\omega - \pi / 8)] \)

(5)

(6)

Take the inverse Fourier transform, we can get

\[y[n] = \frac{1}{16} \left[1 + e^{j3\pi/8} e^{-n\pi/8} + e^{-j3\pi/8} e^{n\pi/8}\right] = \frac{1}{16} \left[1 + e^{-(n-3)\pi/8} + e^{(n-3)\pi/8}\right] \]

\[= \frac{1}{16} \left[1 + 2 \cos\left(\frac{\pi}{8}(n-3)\right)\right] = \frac{1}{16} + \frac{1}{8} \cos\left(\frac{\pi}{8}(n-3)\right) \]

(7)

Note: Take a look at (3), \( H(e^{j\omega}) \) is band limited, \( X(e^{j\omega}) \) is infinite pulse train. If we multiply them together, we can only consider those pulses falling into the band \((-3\pi / 8, 3\pi / 8)\). The rest pulses are cancelled due to the multiplication with 0. There are
three pulses falling into the band \( \frac{2\pi}{16} \delta(\omega), \frac{2\pi}{16} \delta(\omega + \pi/8), \frac{2\pi}{16} \delta(\omega - \pi/8) \), so we get (6)

\[
P3.27
\]

\( (a) \)

\[X(z) = \frac{1}{(1 + \frac{1}{2} z^{-1})^2 (1 - 2z^{-1})(1 - 3z^{-1})} = A + B + C + D\]

\( X(z) \)'s poles are \( z = -1/2, 2, 3 \), if it is stable, the ROC is \( |z| \in (1/2, 2) \)

\[A = X(z)(1 + \frac{1}{2} z^{-1})^2 |_{z=-1/2} = \frac{1}{(1 - 2z^{-1})(1 - 3z^{-1})} |_{z=-1/2} = \frac{1}{35}\]

\[C = X(z)(1 - 2z^{-1}) |_{z=2} = \frac{1}{(1 + \frac{1}{2} z^{-1})^2 (1 - 3z^{-1})} |_{z=2} = -\frac{1568}{1225}\]

\[D = X(z)(1 - 3z^{-1}) |_{z=3} = \frac{1}{(1 + \frac{1}{2} z^{-1})^2 (1 - 2z^{-1})} |_{z=3} = \frac{2700}{1225}\]

Also, let \( z^{-1} = 0 \) at both sides,

\[X(z) |_{z^{-1}=0} = 1 = A + B + C + D\]

Thus, \( B = 1 - A - C - D = \frac{58}{1225}\)

\[X(z) = \frac{1}{(1 + \frac{1}{2} z^{-1})^2 (1 - 2z^{-1})(1 - 3z^{-1})} = \frac{1/35}{(1 + \frac{1}{2} z^{-1})^2} + \frac{58/1225}{1 + \frac{1}{2} z^{-1}} - \frac{1568/1225}{1 - 2z^{-1}} + \frac{2700/1225}{1 - 3z^{-1}}\]

Since the ROC is \( |z| \in (1/2, 2) \),

\[x[n] = \frac{1}{35} (n+1)(-\frac{1}{2})^n u[n+1] + \frac{58}{1225} (-\frac{1}{2})^n u[n] + \frac{1568}{1225} 2^n u[-n-1] - \frac{2700}{1225} 3^n u[-n-1]\]

Note: To get the inverse Z-Transform of second-order term or multiple order term, we can use the differentiation property \( nx[n] \leftrightarrow -z^n \frac{dX(z)}{dz} \) (Refer to page 122 of textbook for its proof)

E.g. right side sequence \( x[n] = a^n u[n] \leftrightarrow \frac{1}{1 - az^{-1}} \) (ROC: \( |z| > |a| \))

\[na^n[n] \leftrightarrow -z^n \left[ \frac{d(\frac{1}{1 - az^{-1}})}{dz} \right] = \frac{az^{-1}}{(1 - az^{-1})^2} , \quad \text{So} \quad na^{n-1}[n] \leftrightarrow \frac{z^{-1}}{(1 - az^{-1})^2}\]

\( (c) \)

\[x[n] \leftrightarrow X(z) = \frac{z^3 - 2z}{z - 2} = z^2 + 2z + \frac{2}{1 - 2z^{-1}}\]
X(z) has its only pole at z=2. If x[n] is a left-sided sequence, the ROC is |z| < 2
\[ x[n] = \delta[n + 2] + 2\delta[n + 1] - 2(2)^nu[-n - 1] \]
or \[ x[n] = \delta[n + 2] + 2\delta[n + 1] - 2^{n+1}u[-n - 1] \]

**P3.1**

(g)
\[ \left( \frac{1}{2} \right)^n (u[n] - u[n - 10]) = \sum_{n=0}^{9} \left( \frac{1}{2} \right)^n \delta[n] \]
is a finite length sequence, so its ROC is |z| ≠ 0. The solution in the textbook is right.

**HW#4: P3.6 (d), (e); P3.20; P4.1; P4.3**

**P3.6**

(d) \[ X(z) = \frac{1 - (1/2)z^{-1}}{1 - (1/4)z^{-2}} \quad |z| > 1/2 \]

- Partial Fraction Expansion:
\[ X(z) = \frac{1 - (1/2)z^{-1}}{1 - (1/4)z^{-2}} = \frac{1 - (1/2)z^{-1}}{(1 - (1/2)z^{-1})(1 + (1/2)z^{-1})} = \frac{1}{1 + (1/2)z^{-1}} \]
\[ \Rightarrow x[n] = \left( -\frac{1}{2} \right)^n u[n] \]

- Power Series Expansion:
\[ X(z) = \frac{1 - (1/2)z^{-1}}{1 - (1/4)z^{-2}} = 1 - (1/2)z^{-1} + (1/4)z^{-2} - (1/8)z^{-3} + (1/16)z^{-4} + \cdots \]
\[ \Rightarrow x[n] = \left( -\frac{1}{2} \right)^n u[n] \]

- Fourier Transform exists as the ROC including unit circle.

(e) \[ X(z) = \frac{1 - az^{-1}}{z^{-1} - a} \quad |z| > |1/a| \]

- Partial Fraction Expansion:
\[ X(z) = \frac{1 - az^{-1}}{z^{-1} - a} = \frac{1}{z^{-1} - a} - \frac{az^{-1}}{z^{-1} - a} = \frac{1}{z^{-1} - a} - a \frac{a^2}{z^{-1} - a} = \frac{-1/a}{1 - (1/a)z^{-1}} - a + \frac{a}{1 - (1/a)z^{-1}} \]
\[ \Rightarrow x[n] = -\left( \frac{1}{a} \right)^{n+1} u[n] - a\delta[n] + \left( \frac{1}{a} \right)^{n-1} u[n] = -\left( \frac{1}{a} \right)^{n+1} u[n] + \left( \frac{1}{a} \right)^{n-1} u[n-1] \]

- Power Series Expansion:
\[ X(z) = \frac{1 - az^{-1}}{z^{-1} - a} = \frac{1}{z^{-1} - a} - \frac{az^{-1}}{z^{-1} - a} \]

\[ \frac{1}{z^{-1} - a} = \left(-\frac{1}{a}\right) \left(1 + \frac{1}{a} z^{-1} + \frac{1}{a^2} z^{-2} + \frac{1}{a^3} z^{-3} + \cdots\right) \Rightarrow x_1[n] = \left(-\frac{1}{a}\right) \left(\frac{1}{a}\right)^n u[n] = \left(-\frac{1}{a}\right)^{n+1} u[n] \]

\[ -\frac{az^{-1}}{z^{-1} - a} = z^{-1} + \frac{a}{a^2} z^{-2} + \frac{a^2}{a^3} z^{-3} + \cdots \Rightarrow x_2[n] = \left(\frac{1}{a}\right)^{n+1} u[n-1] \]

\[ \Rightarrow x[n] = x_1[n] + x_2[n] = -\left(\frac{1}{a}\right)^{n+1} u[n] + \left(\frac{1}{a}\right)^{n-1} u[n-1] \]

- Fourier Transform exists when the ROC including unit circle, which means \(|a| < 1|\).

**P3.20**
(a) As the ROC of \(X(z)\) is \(|z| > 3/4\), and the ROC of \(Y(z)\) is \(|z| > 2/3\), the ROC of \(H(z)\) should be \(|z| > 2/3|\);
(b) As the ROC of \(X(z)\) is \(|z| < 1/3\), and the ROC of \(Y(z)\) is \(1/6 < |z| < 1/3\), the ROC of \(H(z)\) should be \(|z| > 1/6|\);

**P4.1**
As \(x_c(t) = \sin[2\pi(100t)]\) and \(T = 1/400\) sec,

\[ x[n] = x_c(nT) = \sin[2\pi(100nT)] = \sin\left(\frac{n\pi}{2}\right) \]

**P4.3**
As \(x_c(t) = \cos[4000\pi t]\) and \(x[n] = \cos\left(\frac{n\pi}{3}\right)\),

(a) Let \(x[n] = x_c(nT) \Rightarrow T = \frac{1}{12,000}\)

(b) \(T\) is not unique, for example, \(T = \frac{5}{12,000}\)

**HW#5: P4.5; P4.7; P5.2; P5.3**

**P4.5**
(a) From Nyquist Sampling theorem, to avoid aliasing in the C/D converter, the sampling frequency \(\Omega_s = \frac{1}{T_s} \geq 2\Omega_m = 2 * 5000Hz = 10^4\) Hz, so \(T_s \leq 10^{-4}\) s
(b) \( \Omega_{\text{cutoff}} = \frac{f_{\text{cutoff}}}{2\pi} \Omega_s = \frac{\pi/8}{2\pi} 10^4 = 625\text{Hz} \)

(c) \( \Omega_{\text{cutoff}} = \frac{f_{\text{cutoff}}}{2\pi} \Omega_s = \frac{\pi/8}{2\pi} 2 \times 10^4 = 1250\text{Hz} = 1.25\text{kHz} \)

Note: The relation between digital frequency \( f \) and analog frequency \( \Omega \) is \( \frac{\Omega}{\Omega_s} = \frac{f}{2\pi} \), where \( \Omega_s \) is the sampling frequency, \( f \) is in radians.

P4.7

(a) \( x_c(t) = s_c(t) + \alpha s_c(t - \tau_d) \)
\( X_c(j\Omega) = S_c(j\Omega)(1 + \alpha e^{-j\Omega \tau_d}) \)

Consider sampling, \( x[n] = x_c(nT) \), in frequency domain (refer to Eq.4.19 in textbook, P147),
\[
X(e^{j\Omega T}) = \frac{1}{T} \sum_{k=-\infty}^{\infty} X_c(j(\Omega - \frac{2\pi k}{T}))
\]
\[
X(e^{j\omega}) = X(e^{j\Omega T}) = \frac{1}{T} \sum_{k=-\infty}^{\infty} X_c(j(\Omega - \frac{2\pi k}{T})) = \frac{1}{T} (1 + \alpha e^{-j\Omega \tau_d}) \sum_{k=-\infty}^{\infty} S_c(j(\frac{\Omega}{T} - \frac{2\pi k}{T}))
\]
\[
= \frac{1}{T} (1 + \alpha e^{-j\Omega \tau_d / T}) \sum_{k=-\infty}^{\infty} S_c(j(\Omega / T - \frac{2\pi k}{T}))
\]

(b) \( H(e^{j\omega}) = 1 + \alpha e^{-j\Omega \tau_d / T} \)

(c) \[
h[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} H(e^{j\omega})e^{j\omega n} d\omega = \frac{1}{2\pi} \int_{-\pi}^{\pi} (1 + \alpha e^{-j\Omega \tau_d / T}) e^{j\omega n} d\omega = \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{j\omega n} d\omega + \frac{1}{2\pi} \int_{-\pi}^{\pi} \alpha e^{-j\omega(n - \tau_d / T)} d\omega
\]
\[
= \frac{\sin(n\pi)}{n\pi} + \alpha \frac{\sin[(n - \tau_d / T)\pi]}{(n - \tau_d / T)\pi}
\]

if \( \tau_d = T \), \( h[n] = \frac{\sin(n\pi)}{n\pi} + \alpha \frac{\sin[(n-1)\pi]}{(n-1)\pi} = \delta[n] + \alpha \delta[n-1] \)

if \( \tau_d = T/2 \), \( h[n] = \frac{\sin(n\pi)}{n\pi} + \alpha \frac{\sin[(n-1/2)\pi]}{(n-1/2)\pi} = \delta[n] + \alpha \frac{\sin[(n-1/2)\pi]}{(n-1)\pi} \)

P5.2

\( y[n-1] - \frac{10}{3} y[n] + y[n+1] = x[n] \)

\( z^{-1} Y(z) - \frac{10}{3} Y(z) + zY(z) = X(z) \)
\[ H(z) = \frac{Y(z)}{X(z)} = \frac{1}{z^{-1} - \frac{10}{3} + z} = \frac{z^{-1}}{(1-(1/3)z^{-1})(1-3z^{-1})} = \frac{-3/8}{1-(1/3)z^{-1}} + \frac{3/8}{1-3z^{-1}} \]

(a) \( H(z) \) has two zeroes: 0, \( \infty \); two poles: 1/3, 3

(b) The system is stable, so the ROC includes the unit circle. The ROC is \( 1/3 < |z| < 3 \)

\[ h[n] = -\frac{3}{8} \left(\frac{1}{3}\right)^n u[n] - \frac{3}{8} 3^n u[-n-1] \]

**P5.3**

\[ y[n-1] + \frac{1}{3} y[n-2] = x[n] \]

\[ z^{-1}Y(z) + \frac{1}{3} z^{-2} Y(z) = X(z) \]

\[ H(z) = \frac{Y(z)}{X(z)} = \frac{1}{z^{-1} + (1/3)z^{-2}} = \frac{1}{z^{-1}(1+1/3)z^{-1}} = \frac{z}{1+1/3z^{-1}} \]

The poles are: 1/3, there are two ROC, \( 0 < |z| < 1/3, |z| > 1/3 \)

(1) \( 0 < |z| < 1/3 \): \( h[n] = (-\frac{1}{3})^{n+1} u[-n-2] \), choose (d)

(2) \( |z| > 1/3 \): \( h[n] = (-\frac{1}{3})^{n+1} u[n+1] \), choose (a)

**HW#6: P5.10; P5.12; P5.15**

**P5.10**

As one of the zeros of \( H(z) \) is at \( z = \infty \), the corresponding pole of \( H(z) \) will be also at infinity. The existence of a pole at \( z = \infty \) implies that the system is not causal.
As the poles of $H(z)$ are 0.9, -0.9, ROC includes the unit circle, the system is stable.

(b) 

\[
H(z) = \frac{(1 + 0.2z^{-1})(1 + 3z^{-1})(1 - 3z^{-1})}{(1 + j0.9z^{-1})(1 - j0.9z^{-1})} 
- \frac{-9(1 + 0.2z^{-1})(1 + (1/3)z^{-1})(1 - (1/3)z^{-1})(z^{-1} + 1/3)(z^{-1} - 1/3)}{(1 + j0.9z^{-1})(1 - j0.9z^{-1})(1 + (1/3)z^{-1})(1 - (1/3)z^{-1})} 
= H_1(z)H_{ap}(z)
\]

P5.15

Generalized Linear Phase – GLP; Linear Phase – LP;

(a) As $h[n] = 2\delta[n] + \delta[n-1] + 2\delta[n-2] \Rightarrow H(jw) = (1 + 4\cos w)e^{-jw}$, $A(jw) = 1 + 4\cos w$ and $\alpha = 1, \beta = 0$, it is a GLP, but not LP as $A(jw)$ is not always nonnegative for all $w$;

(b) It is not a GLP or LP as it is not a symmetric filter;

(c) As $h[n] = \delta[n] + 3\delta[n-1] + \delta[n-2] \Rightarrow H(jw) = (3 + 2\cos w)e^{-jw}$, $A(jw) = 3 + 2\cos w$ and $\alpha = 1, \beta = 0$, it is a GLP and also LP as $A(jw)$ is always nonnegative for all $w$;

(d) As $h[n] = \delta[n] + \delta[n-1] \Rightarrow H(jw) = 2\cos(w/2)e^{-j(w/2)}$, 
\( A(jw) = 2 \cos(w/2) \) and \( \alpha = 1/2, \beta = 0 \), it is a GLP, but not LP as \( A(jw) \) is not always nonnegative for all \( w \);

(e) As \( h[n] = \delta[n] - \delta[n - 2] \Rightarrow H(jw) = 2 \sin \left( w e^{-j(w-z/2)} \right) \), \( A(jw) = 2 \sin w \) and \( \alpha = 1, \beta = \pi/2 \), it is a GLP, but not LP as \( A(jw) \) is not always nonnegative for all \( w \);

HW#7: P6.7; P6.8; P6.11; P6.25; P7.15

P6.7

The difference equation is: \( y[n] - \frac{1}{4} y[n - 2] = x[n - 2] - \frac{1}{4} x[n] \)

Z-Transform: \( Y(z) - \frac{1}{4} Y(z)z^{-2} = X(z)z^{-2} - \frac{1}{4} X(z) \)

Transfer function: \( H(z) = \frac{Y(z)}{X(z)} = \frac{-\frac{1}{4} + z^{-2}}{1 - \frac{1}{4} z^{-2}} \)

P6.8

\( y[n] - 2y[n - 2] = 3x[n - 1] + x[n - 2] \)

P6.11

\( H(z) = \frac{z^{-1}(1 - 2z^{-1})(1 - 4z^{-1})}{1 - \frac{1}{2} z^{-1}} = \frac{z^{-1} - 6z^{-2} + 8z^{-3}}{1 - \frac{1}{2}} \)

(a)
P6.25

(a) \( H(z) = [H_1(z) + H_2(z)]H_3(z) = \left[ \frac{1 - \frac{1}{2} z^{-1}}{1 - \frac{3}{8} z^{-1} + \frac{7}{8} z^{-2}} \right] \left( \frac{1}{1 - z^{-1}} \right) \)

The Z-Transfer function:
\[
H(z) = \frac{2 + \frac{9}{8} z^{-1} + \frac{11}{8} z^{-3} + \frac{7}{8} z^{-4}}{1 - \frac{11}{8} z^{-1} + \frac{5}{4} z^{-2} - \frac{7}{8} z^{-3}}
\]

(b) The difference equation:
\[
y[n] - \frac{11}{8} y[n-1] + \frac{5}{4} y[n-2] - \frac{7}{8} y[n-3] = 2x[n] + \frac{9}{8} x[n-1] + \frac{9}{8} x[n-2] + \frac{11}{8} x[n-3] + \frac{7}{8} x[n-4]
\]

P7.15

Specifications:

(a) Pass band ripple: \( \delta_p = 0.05 \), \( A_p = 20 \log \delta_p = -26.02dB \)
Stop band ripple: \( \delta_s = 0.1 \), \( A_s = 20 \log \delta_s = -20dB \)
Pass band edge: \( \omega_p = 0.25\pi \)
Stop band edge: \( \omega_s = 0.35\pi \)
Cutoff: $\omega_c = 0.3\pi$

The peak approximate error $20\log_{10} \delta < -26.02 dB$

Among the windows in Table 7.1 (Page 471), Hanning, Hamming, Blackman can be used (b)

Hanning: $0.1\pi = \frac{8\pi}{M}$, $M = 80$, $L = M + 1 = 81$

Hamming: $0.1\pi = \frac{8\pi}{M}$, $M = 80$, $L = M + 1 = 81$

Blackman: $0.1\pi = \frac{12\pi}{M}$, $M = 120$, $L = M + 1 = 121$

Note that the estimation is not accurate. We can use MATLAB to find the minimum of filter order to meet the requirements.