THROUGHPUT OF LARGE WIRELESS NETWORKS ON SQUARE, HEXAGONAL AND TRIANGULAR GRIDS

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ABSTRACT

Capacity analysis of sensor networks or wireless ad hoc networks has attracted a great attention in the past years. But the research in this area has primarily focused on scaling laws of arbitrary or random network instead of the exact capacity of given topologies. While the insight into how the capacity of an arbitrary or random network scales with the number of nodes in a given area is important, the exact capacity of a network depends on the network topology and can be more desirable in practice. In this paper, we compare the throughput of a large network with three possible topologies: square (rectangular), hexagonal and triangular. With a given topology, the network throughput also depends on the choice of routing protocols. We follow a synchronous array method (SAM) that is known so far to yield the highest throughput of a network on a rectangular grid.

1. INTRODUCTION

Until the recent work [1], capacity analysis of sensor networks or wireless ad hoc networks has primarily focused on a random or arbitrary network. The work by Gupta and Kumer [2] pioneered a series of research activities on random networks as shown in [3], [4], [5], [6], [7]. These works emphasize the scaling laws of the network capacity with respect to the number of nodes in a given area. The exact capacity of a network however is also influenced by the exact topology of the network. To better understand the capacity of wireless ad hoc networks, sufficient attention must also be paid to networks of known topologies. The recent work shown in [1] follows this principle.

Networks of known topologies are directly relevant in some applications. Wireless mesh networks that are receiving an increasing interest in industry are an important example. Sensor networks are another. A large wireless mesh network can also be formed by low flying airplanes in air or by specially equipped mobile nodes on ground. Each node in the mesh network can serve as a router (virtual base station) for a neighborhood of mobile clients on the ground. This tiered architecture of ad hoc network does not have the routing overhead that severely limits the capacity of a pure ad hoc mobile network.

In this paper, we will study the throughput of a large network that are located on a rectangular grid, hexagonal grid or a triangular grid. These grids are illustrated in Figures 1, 2, and 3. We will evaluate and compare the throughput of a network on each of the three grids with a fixed node density. The throughput of a network also depends on the choice of medium access protocols. A study shown in [1] (and our recent research) suggests that a protocol called synchronous array method (SAM) is so far known to yield the highest throughput of a network on a rectangular grid.

For this reason, we will apply the SAM protocol to all other grids considered in this paper. The SAM protocol is simple. During a time interval, all nodes in a subset of the network transmit toward their neighboring nodes in a given direction. During another time interval, all nodes in another subset of the network do the same. This process repeats until all nodes in the network have transmitted to their neighboring nodes in one direction. The above process also repeats for each of all possible directions available in the network. By following the SAM protocol, the signal to interference and noise ratio (SINR) at each receiving node can be explicitly expressed. The dependence of SINR on the sparseness of each subset of the network can be used to optimize the network throughput.

2. NETWORK ON SQUARE GRID

As discussed in [1], a network on square grid is illustrated in Figure 1 where a subset of the network is shown by black and gray nodes. The black nodes are the transmitting nodes and the gray nodes are the receiving nodes. The sparseness of the subset is determined by \( p d_s \) and \( q d_s \) where \( p \) and \( q \) are integers and \( d_s \) is the distance between two adjacent nodes. The capacity of each receiving node is limited by the interference from all transmitting nodes, except the desired one, in a corresponding subset. The receivers at the center of the network are the most interfered. The network throughput per node is lower bounded by the throughput of the node at the center of the network. When large enough, a network may appear to be infinite for the nodes at the network center. For convenience, we will assume that the network is infinite.

Also discussed in [1], the SINR at a receiving node can be expressed as:

\[ SINR = \frac{1}{1/SNR_0 + \delta_s} \]  

where \( SNR_0 = P_t (\sigma^2 d_s^n)^{-1} \), \( P_t \) is the transmitted power from each transmitting node, \( \sigma^2 \) is the noise variance, \( n \) is the power decaying exponent, and \( \delta_s \) is referred to as the interference factor for the square grid. When \( P_t \) is large, \( SINR \) becomes saturated at its upper bound \( \frac{1}{\sigma^2} \). Assuming that the noise and interference are
where the vertical spacing of a transmission pair is \(d_e\) meters and the horizontal spacing is \(qd_d\) meters. The throughput is denoted by \(\gamma_s\) in bits/s/Hz/node or \(\alpha_s\) in bits/m/s/Hz/node.

all Gaussian, the network throughput in bits/s/Hz per node along any of the four possible directions is

\[
\gamma_s = \frac{1}{G_s} \log_2 \left( 1 + \frac{1}{\delta_s} \right) \tag{2}
\]

where \(G_s = pq\) is the number of time slots needed for each of all nodes to transmit once to its neighboring node in one of the four possible directions on the square grid. This is actually an upper bound of the network throughput, and achievable when \(P_s\) is large. Here, each node is assumed to have a single antenna.

The selection of each subset of the network influences SINR. We have chosen each subset in such a way that any two adjacent columns of transmission pairs are maximally offset from each other as shown in Figure 1.

Our analysis shows that for \(p > 1\),

\[
\delta_s = e^2 \sum_{i=0}^{+\infty} \sum_{j=-\infty}^{+\infty} \sum_{q=0}^{1} \left( [(2i+1)q + (-1)^q]^2 + (pj - |p/2|)^2 \right)^{-\frac{2}{\epsilon}}
+ e^2 \sum_{i=0}^{+\infty} \sum_{j=-\infty}^{+\infty} \sum_{q=0}^{1} \left( [2(i+1)q - 1]^2 + (pj)^2 \right)^{-\frac{2}{\epsilon}}
+ e^2 \sum_{i=0}^{+\infty} \sum_{j=-\infty}^{+\infty} \sum_{q=0}^{1} \left( [2(i+1)q + 1]^2 + (pj)^2 \right)^{-\frac{2}{\epsilon}}
+ \sum_{i=0}^{+\infty} \left( [2(i+1)q + 1]^2 \right)^{-\frac{2}{\epsilon}} + 2e^2 \sum_{j=1}^{+\infty} \left( (1 + (pj)^2 \right)^{-\frac{2}{\epsilon}} \tag{3}
\]

where \(1 \geq \epsilon \geq 0\) is the power attenuation factor along a non-line-of-sight of a transmitter or a receiver, \([p/2]\) denotes the largest integer no greater than \(p/2\). The power attenuation factor of a transmission pair becomes \(e^2\) when neither the transmitter nor the receiver is in the line-of-sight with respect to each other. For omni-directional antenna, \(\epsilon = 1\). However, when \(p = 1\), we have

\[
\delta_s = e^2 \sum_{i=0}^{+\infty} \sum_{j=-\infty}^{+\infty} \sum_{q=0}^{1} \left( [(2i+1)q + (-1)^q]^2 + (pj)^2 \right)^{-\frac{2}{\epsilon}}
+ e^2 \sum_{i=0}^{+\infty} \sum_{j=-\infty}^{+\infty} \sum_{q=0}^{1} \left( [2(i+1)q - 1]^2 + (pj)^2 \right)^{-\frac{2}{\epsilon}}
+ \sum_{i=0}^{+\infty} \left( [2(i+1)q + 1]^2 \right)^{-\frac{2}{\epsilon}} + 2e^2 \sum_{j=1}^{+\infty} \left( (1 + (pj)^2 \right)^{-\frac{2}{\epsilon}} \tag{4}
\]

For each given pair of \(n\) and \(\epsilon\), \(\gamma_s\) can be optimized over \((p, q)\). In Table 1, samples of the optimal \(\gamma_s\) and the corresponding optimal \((p, q)\) are given.

| Table 1. Optimized \(\gamma_s\) in bits/s/Hz per node for one of four directions of the network on square grid |
|---|---|---|---|
| \(n, (p, q)_{opt}\) | \(\epsilon = 1\) | \(\epsilon = 0.1\) | \(\epsilon = 0.01\) |
| 3 | 0.2166, (2, 3) | 1.7914, (1, 2) | 2.1668, (1, 2) |
| 4 | 0.4208, (2, 3) | 2.3780, (1, 2) | 3.0442, (1, 2) |
| 5 | 0.6210, (2, 3) | 2.7425, (1, 2) | 3.8689, (1, 2) |

3. NETWORK ON HEXAGONAL GRID

A network on the hexagonal grid is shown in Figure 2 where a subset of the network is marked by the black (transmitting) nodes and the gray (receiving) nodes. The two adjacent columns of transmission pairs in each subset are maximally offset from each other. The vertical spacing of adjacent transmission pairs is denoted by \(\sqrt{3}pd\), and the horizontal spacing is \(qd\). Here, \(p\) takes all natural integers. But \(q\) can be either \(q = 3m\) or \(q = 3m - 1.5\) where \(m\) is any natural integer.

The SAM protocol is applied to a subset during each time slot. In order for each of all nodes in the network to transmit once to its neighboring node in one of the three possible directions (of a given node), we need \(G_h = 2p \cdot (2q/3)\) time slots if \(q = 3m\) or \(G_h = 2p \cdot (2q + 1.5)/3\) time slots if \(q = 3m - 1.5\). Then, the network throughput in bits/s/Hz per node in one of three possible directions under the hexagonal grid is given by \(\gamma_h = \frac{1}{G_h} \log_2 \left( 1 + \frac{1}{\delta_h} \right)\). It can be shown that if \(q = 3m\) and \(p > 1\), then

\[
\delta_h = e^2 \sum_{i=0}^{+\infty} \sum_{j=-\infty}^{+\infty} \sum_{q=0}^{1} \left( [(2i+1)q + (-1)^q]^2 + (\sqrt{3}pj - |p/2|\sqrt{3})^2 \right)^{-\frac{2}{\epsilon}}
+ e^2 \sum_{i=0}^{+\infty} \sum_{j=-\infty}^{+\infty} \sum_{q=0}^{1} \left( [2(i+1)q - 1]^2 + (\sqrt{3}pj)^2 \right)^{-\frac{2}{\epsilon}}
+ e^2 \sum_{i=0}^{+\infty} \sum_{j=-\infty}^{+\infty} \sum_{q=0}^{1} \left( [2(i+1)q + 1]^2 + (\sqrt{3}pj)^2 \right)^{-\frac{2}{\epsilon}}
+ \sum_{i=0}^{+\infty} \left( [2(i+1)q + 1]^2 \right)^{-\frac{2}{\epsilon}} + 2\epsilon^2 \sum_{j=1}^{+\infty} \left( (1 + (\sqrt{3}pj)^2 \right)^{-\frac{2}{\epsilon}} \tag{5}
\]
and if $q = 3m$ and $p = 1$, then
\[
\delta_h(n) = \epsilon^2 \sum_{i=0}^{+\infty} \sum_{j=0}^{+\infty} (\lfloor (i+1)q - 1 \rfloor^2 + (\sqrt{3}pj)^2)^{-\frac{q}{2}} \\
+ \epsilon^2 \sum_{i=0}^{+\infty} \sum_{j=0}^{+\infty} (\lfloor (i+1)q + 1 \rfloor^2 + (\sqrt{3}pj)^2)^{-\frac{q}{2}} \\
+ \sum_{i=0}^{+\infty} \sum_{j=0}^{+\infty} (i^2 + (\sqrt{3}pj)^2)^{-\frac{q}{2}} \\
+ \sum_{i=0}^{+\infty} \sum_{j=0}^{+\infty} (i^2 + (\sqrt{3}pj)^2)^{-\frac{q}{2}} 2 \epsilon^2 \sum_{j=1}^{+\infty} (1 + (\sqrt{3}pj)^2)^{-\frac{q}{2}} \tag{5}
\]
Furthermore, if $q = 3m - 1.5$, then
\[
\delta_h =
\epsilon^2 \sum_{i=0}^{+\infty} \sum_{j=0}^{+\infty} (\lfloor (2i+1)q - 1 \rfloor^2 + (\sqrt{3}pj - 1/2 + |p/2|)\sqrt{3})^2)^{-\frac{q}{2}} \\
+ \epsilon^2 \sum_{i=0}^{+\infty} \sum_{j=0}^{+\infty} (\lfloor (2i+1)q + 1 \rfloor^2 + (\sqrt{3}pj - 1/2 + |p/2|)\sqrt{3})^2)^{-\frac{q}{2}} \\
+ \epsilon^2 \sum_{i=0}^{+\infty} \sum_{j=0}^{+\infty} (2i^2 + (\sqrt{3}pj)^2)^{-\frac{q}{2}} \\
+ \sum_{i=0}^{+\infty} \sum_{j=0}^{+\infty} (2i^2 + (\sqrt{3}pj)^2)^{-\frac{q}{2}} 2 \epsilon^2 \sum_{j=1}^{+\infty} (1 + (\sqrt{3}pj)^2)^{-\frac{q}{2}} \tag{6}
\]

Fig. 2. A network on hexagonal grid. The spacing along the transmission direction is $qd_h$, and the spacing along the direction perpendicular to the transmission direction is $\sqrt{3}pd_h$. The throughput is denoted by $\gamma_h$ in bits/s/Hz/node or $\alpha_h$ in bits-m/s/Hz/node.

Samples of the optimal $\gamma_h$ and the corresponding optimal $(p, q)$ are shown in Table 2.

4. NETWORK ON TRIANGULAR GRID

The triangular grid is shown in Figure 3 where a subset of the network is marked by black and grey nodes. The vertical spacing of transmission pairs is $\sqrt{3}pd_t$, and the horizontal spacing is $qd_t$. Here, $p$ takes any natural integers, but $q$ can be either $m$ or $m - 0.5$ where $m$ is a natural integer. The number of time slots required

\[
\delta_t(n) =
\epsilon^2 \sum_{i=0}^{+\infty} \sum_{j=0}^{+\infty} (\lfloor (2i+1)q + (-1)^n \rfloor^2 + (\sqrt{3}pj - |p/2|\sqrt{3})^2)^{-\frac{q}{2}} \\
+ \epsilon^2 \sum_{i=0}^{+\infty} \sum_{j=0}^{+\infty} (\lfloor (2i+1)q - 1 \rfloor^2 + (\sqrt{3}pj)^2)^{-\frac{q}{2}} \\
+ \epsilon^2 \sum_{i=0}^{+\infty} \sum_{j=0}^{+\infty} (\lfloor (2i+1)q + 1 \rfloor^2 + (\sqrt{3}pj)^2)^{-\frac{q}{2}} \\
+ \sum_{i=0}^{+\infty} \sum_{j=0}^{+\infty} (\lfloor (2i+1)q + 1 \rfloor^2 + (\sqrt{3}pj)^2)^{-\frac{q}{2}} 2 \epsilon^2 \sum_{j=1}^{+\infty} (1 + (\sqrt{3}pj)^2)^{-\frac{q}{2}} \tag{7}
\]

As $p = 1$,

\[
\delta_t(n) =
\epsilon^2 \sum_{i=0}^{+\infty} \sum_{j=0}^{+\infty} (\lfloor (i+1)q - 1 \rfloor^2 + (\sqrt{3}pj)^2)^{-\frac{q}{2}} \\
+ \epsilon^2 \sum_{i=0}^{+\infty} \sum_{j=0}^{+\infty} (\lfloor (i+1)q + 1 \rfloor^2 + (\sqrt{3}pj)^2)^{-\frac{q}{2}} \\
+ \sum_{i=0}^{+\infty} \sum_{j=0}^{+\infty} (\lfloor (i+1)q + 1 \rfloor^2 + (\sqrt{3}pj)^2)^{-\frac{q}{2}} 2 \epsilon^2 \sum_{j=1}^{+\infty} (1 + (\sqrt{3}pj)^2)^{-\frac{q}{2}} \tag{8}
\]

Table 2. Optimized $\gamma_h$ in bits/s/Hz per node for one of three directions of the network on the Hexagonal grid

<table>
<thead>
<tr>
<th>$\gamma_h, \text{opt.}(p, q)_{\text{opt}}$</th>
<th>$\epsilon = 1$</th>
<th>$\epsilon = 0.1$</th>
<th>$\epsilon = 0.01$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n = 3$</td>
<td>0.2794, (1.3)</td>
<td>2.1297, (1.1.5)</td>
<td>2.7976, (1.1.5)</td>
</tr>
<tr>
<td>$n = 4$</td>
<td>0.5430, (1.3)</td>
<td>2.5813, (1.1.5)</td>
<td>3.8645, (1.1.5)</td>
</tr>
<tr>
<td>$n = 5$</td>
<td>0.8040, (1.3)</td>
<td>2.7474, (1.1.5)</td>
<td>4.8132, (1.1.5)</td>
</tr>
</tbody>
</table>

for all nodes in the network to transmit once in one of six possible directions is $G_t = 2pq$ if $q = m$, or $G_t = p[2(q - 0.5) + 1]$ if $q = m - 0.5$. The maximal average throughput in bits/s/Hz per node in one direction is therefore $\gamma_t = \frac{1}{G_t} \log_2(1 + \frac{f}{G_t})$, where the interference factor $\delta_t$ can be derived in a similar way as before. When $q = 1, 2, 3, \ldots$ and $p = 2, 3, 4, \ldots$
When \( q = 0.5, 1.5, 2.5, \ldots \), and \( p = 1, 2, 3, \ldots \),

\[
\delta_i(n) = e^{\frac{1}{2} \sum_{i=0}^{\infty} \sum_{j=-\infty}^{\infty} \left( [(2i + 1)q - 1]^2 + \sqrt{3}pj - (1/2 + [p/2])\sqrt{3}j^2 \right)^{-\frac{1}{2}}}
\]

\[
+ e^{\frac{1}{2} \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \left( [2(i + 1)q - 1]^2 + \sqrt{3}pj \right)^{-\frac{1}{2}}}
\]

\[
+ e^{\frac{1}{2} \sum_{i=0}^{\infty} \sum_{j=-\infty}^{\infty} \left( [2(i + 1)q + 1]^2 + \sqrt{3}pjj^2 \right)^{-\frac{1}{2}}}
\]

\[
+ \sum_{i=0}^{\infty} \left( [2(i + 1)q + 1]^2 \right)^{-\frac{1}{2}} + 2e^{\frac{1}{2} \sum_{j=1}^{\infty} \left( 1 + \sqrt{3}pj^2 \right)^{-\frac{1}{2}}}
\]

(9)

The optimal \( \gamma_i \) and \((p, q)\) are illustrated in Table 3.

**Table 3.** Optimized \( \gamma_i \) in bits/s/Hz per node for a given direction of the network on triangular grid

<table>
<thead>
<tr>
<th>( p, q )</th>
<th>( \epsilon = 1 )</th>
<th>( \epsilon = 0.1 )</th>
<th>( \epsilon = 0.01 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( n = 3 )</td>
<td>0.1863, (1, 3)</td>
<td>1.4198, (1, 1.5)</td>
<td>1.8651, (1, 1.5)</td>
</tr>
<tr>
<td>( n = 4 )</td>
<td>0.3620, (1, 3)</td>
<td>1.7209, (1, 1.5)</td>
<td>2.5763, (1, 1.5)</td>
</tr>
<tr>
<td>( n = 5 )</td>
<td>0.5360, (1, 3)</td>
<td>1.8316, (1, 1.5)</td>
<td>3.2088, (1, 1.5)</td>
</tr>
</tbody>
</table>

An alternative choice of a subset of the network on the triangular grid is shown in Figure 4. This network of nodes also falls on a parallelogram grid. The two adjacent parallel sets of transmission pairs (from upper-right to lower left) can be offset with respect to each other in a fashion like that in Figure 1. The interference factor \( \delta_i \) can be similarly found. The network throughput in bits/s/Hz per node in any given direction is \( \gamma_p = \frac{1}{G_p} \log_2 \left( 1 + \frac{\rho}{\gamma_p} \right) \) where \( G_p = pq \). The optimized \( \gamma_p \) and \((p, q)\) are shown in Table 4.

**Table 4.** Optimized \( \gamma_p \) in bits/s/Hz per node for a given direction of the network on parallelogram grid

<table>
<thead>
<tr>
<th>( p, q )</th>
<th>( \epsilon = 1 )</th>
<th>( \epsilon = 0.1 )</th>
<th>( \epsilon = 0.01 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( n = 3 )</td>
<td>0.1796, (3, 3)</td>
<td>1.6944, (1, 2)</td>
<td>2.1684, (1, 2)</td>
</tr>
<tr>
<td>( n = 4 )</td>
<td>0.3383, (2, 3)</td>
<td>2.1597, (1, 2)</td>
<td>3.0374, (1, 2)</td>
</tr>
<tr>
<td>( n = 5 )</td>
<td>0.4909, (2, 3)</td>
<td>2.3831, (1, 2)</td>
<td>3.8459, (1, 2)</td>
</tr>
</tbody>
</table>

### 5. THROUGHPUT COMPARISON

In the previous sections, we have evaluated the network throughput \( \gamma \) in bits/s/Hz per node. But in order to compare the different topologies, we need to use a metric \( \gamma \) in bits-meters/s/Hz/node. Furthermore, we need to assume that the node density \( \rho \) is the same for all topologies. The smallest square area surrounded by four nodes in the square topology is denoted by \( A_s \). The smallest hexagonal area surrounded by six nodes in the hexagonal topology is denoted by \( A_h \). The smallest triangular area defined by three nodes in the triangular topology is denoted by \( A_t \). Then, a simple analysis shows that

\[
\frac{1}{A_s} = \rho , \quad \frac{2}{A_h} = \rho , \quad \frac{0.5}{A_t} = \rho \tag{10}
\]

and

\[
A_s = \frac{d_s^2}, \quad A_h = \frac{3\sqrt{3}}{2} \frac{d_h^2}, \quad A_t = \frac{\sqrt{3}}{4} \frac{d_t^2} \tag{11}
\]

Therefore,

\[
d_s = \sqrt{\frac{\pi}{\rho}}, \quad d_h = \sqrt{\frac{4}{3\sqrt{3}\rho}}, \quad d_t = \sqrt{\frac{2}{\sqrt{3}\rho}} \tag{12}
\]

On the square grid, the number of hops required for a packet to move over a long distance \( D \) (with \( D \gg d_s \)) in any direction \( \theta \) is given by

\[
N_s = \frac{D \cos (\pi/4 - \theta)}{\sqrt{3}d_s} \times 2 \tag{13}
\]

where \( \theta \in [0, \pi/4] \). So, the average number of hops is given by

\[
\bar{N}_s = \frac{4}{\pi} \int_0^{\pi/4} N_s \cdot d\theta = \frac{4}{\pi} \frac{D}{d_s} \tag{14}
\]

Similarly, we can show that for the hexagonal grid,

\[
N_h = \frac{D \cos \phi}{3d_h} \times 4 , \quad \phi \in [0, \pi/6] \tag{15}
\]

and for the triangular grid,

\[
N_t = \frac{D \cos (\pi/6 - \varphi)}{\sqrt{3}d_t} \times 2 \left( \varphi \in [0, \pi/6] \right) \tag{16}
\]

\[
\bar{N}_t = \frac{6}{\pi} \int_0^{\pi/6} N_t \cdot d\phi = \frac{6}{\pi} \frac{D}{d_t}
\]
Then, the average network throughput in bits-meters/s/Hz per node for the square, hexagonal and triangular grids are given by

\[
\alpha_s = \gamma_s \cdot \frac{D}{N_s} = \frac{\pi}{4} \gamma_s \cdot \sqrt{\frac{T}{\rho}}
\]

\[
\alpha_h = \gamma_h \cdot \frac{D}{N_h} = \frac{\pi}{4} \sqrt{\frac{4}{3\sqrt{3}}} \gamma_h \cdot \sqrt{\frac{T}{\rho}}
\]

\[
\alpha_t = \gamma_t \cdot \frac{D}{N_t} = \frac{\sqrt{3\pi}}{6} \sqrt{\frac{2}{\sqrt{3}}} \gamma_t \cdot \sqrt{\frac{T}{\rho}}
\]

(17)

However, under the parallelogram grid, we can allow each node to transmit in six possible directions as shown in Figure 4 or in only four possible directions as shown in Figure 5. Given six directions, \( N_p = N_t \). But with four directions, we need to recalculate the average number \( N'_p \) of hops required for a long distance \( D \) as follows:

\[
N'_p = \begin{cases} 
\frac{D \cos \left( -\frac{\pi}{6} - \theta \right)}{D \cos \left( \frac{\pi}{3} - \theta \right)} \times 2, & \theta \in \left( -\frac{\pi}{6}, 0 \right) \\
\frac{D \cos \left( \frac{\pi}{3} - \theta \right)}{D \cos \left( -\frac{\pi}{6} - \theta \right)} \times 2, & \theta \in \left( 0, \frac{\pi}{6} \right) 
\end{cases}
\]

(18)

and

\[
N'_p = \frac{2}{\pi} \int_{\theta} \frac{N_p'}{d} \, d\theta = \frac{8}{3\sqrt{3}} \cdot \frac{D}{d_t}
\]

(19)

A comparison of \( \alpha_p \) and \( \alpha'_p \) is

\[
\alpha_p = \frac{\gamma_p \cdot D}{N_p} = \frac{\gamma_p \cdot d_t}{6/\sqrt{3\pi}} \quad > \quad \alpha'_p = \frac{\gamma_p \cdot D}{N'_p} = \frac{\gamma_p \cdot d_t}{8/\sqrt{3\pi}}
\]

We will ignore \( \alpha'_p \) but only consider \( \alpha_p \):

\[
\alpha_p = \frac{\sqrt{3\pi}}{6} \cdot \sqrt{\frac{2}{\sqrt{3}}} \gamma_p \cdot \sqrt{\frac{T}{\rho}}
\]

(20)

Table 5 illustrates \( \alpha_s, \alpha_h, \alpha_t \) and \( \alpha_p \) that are all normalized by the common factor \( \frac{\sqrt{T}}{\rho} \). We can see that \( \alpha_h \) is the largest when omnidirectional antennas are used, and \( \alpha_p \) is the largest when directional antennas are used.

6. CONCLUSION

We have evaluated the throughput in bits-m/s/Hz/node of a large wireless network on three different topological grids - square, hexagonal and triangular. The routing protocol used is called synchronous array method (SAM). Our results suggest that the hexagonal grid is the most efficient when omnidirectional antennas are used, and that the (6-directions) parallelogram variation of the triangular grid is the most efficient when directional antennas are in use. However, the cell partition for each node on a triangular grid is hexagon, and the cell partition for each node on a hexagonal grid is triangle. If the network is used as a virtual network of base stations, the triangular grid may have an advantage.

Another observation from this study is that if the node density of a large network is kept constant, then the network throughput does not seem to change much as the topology changes. A new challenge now is to find the optimal routing protocol for a given topology. So far, we have not been able to work out a protocol to beat the SAM scheme under non-fading channel condition. Further research for fading channel is underway.

7. REFERENCES


