FAQ and Errata Sheet for
The Global Positioning System and Inertial Navigation: Theory and Practice

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Abstract

This document is intended to both list errors reported to the author by readers and to answer certain frequently asked questions. By its nature, this document will evolve over time. Depending on where you received this document, a more recent version may be available at <www.ee.ucr.edu/˜farrell>.

The majority of reported errors are relevant to the first printing of the book. They should be corrected in the second or later printings.

Please contact the first author if you have questions or find additional errors.

1 Chapter 1

p.7 – Eqns. (1.5) and (1.6) are really one equation typographically set with two numbers.

p.8 – Eqn. (1.8) should be three vectors separated by two equal signs. The equal sign between the first two vectors is missing.

$$\begin{bmatrix} \dot{n} \\ \dot{e} \\ \dot{v}_n \\ \dot{v}_e \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} v_n \\ v_e \\ \cos(\psi)a_u - \sin(\psi)a_v \\ \sin(\psi)a_u + \cos(\psi)a_v \\ \omega_r \end{bmatrix} = \begin{bmatrix} v_n \\ v_e \\ a_n \\ a_e \\ \omega_r \end{bmatrix}$$

p.9 – Eqn. (1.9) should read

$$\begin{bmatrix} a_n \\ a_e \end{bmatrix} = \begin{bmatrix} \cos(\psi) & -\sin(\psi) \\ \sin(\psi) & \cos(\psi) \end{bmatrix} \begin{bmatrix} a_u \\ a_v \end{bmatrix}$$

p.9 – The $+\ -sin(\hat{\psi})$ term in eqn. (1.8) and (1.10) is the same as $-sin(\hat{\psi})$.

2 Chapter 2

p. 27– The symbol for longitude in Figure 2.5 should be lower case ϕ, not upper case Φ.

p. 29– On the thirteenth line of Table 2.1 the value in parentheses is too small by a factor of 10.

p. 39– The last sentence of Section 2.4.1.3 containing equations (2.35) and (2.36) should be revised as follows: "The matrix representation of the vector transformation is

$$\begin{aligned}
\mathbf{v}_b &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & \delta\theta_1 \\ 0 & -\delta\theta_1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -\delta\theta_2 \\ 0 & 1 & 0 \\ \delta\theta_2 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & \delta\theta_3 & 0 \\ -\delta\theta_3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \mathbf{v}_a \\
\mathbf{v}_b &= \begin{bmatrix} 1 & \delta\theta_3 & -\delta\theta_2 \\ -\delta\theta_3 & 1 & \delta\theta_1 \\ \delta\theta_2 & -\delta\theta_1 & 1 \end{bmatrix} \mathbf{v}_a \\
\mathbf{v}_b &= (\mathbf{I} - \delta\mathbf{\Theta}^a_{ba})\mathbf{v}_a
\end{aligned}$$

where $\delta\mathbf{\Theta}^a_{ba} = (\delta\theta^a_{ba}\times)$ is the skew symmetric representation of $\delta\theta^a_{ba}$. To first order, the inverse rotation is

$$\mathbf{v}_a = (\mathbf{I} + \delta\mathbf{\Theta}^a_{ba})\mathbf{v}_b.''$$

Note also the correction to Eqn. (A.2) in Appendix A.

p. 46– Eqn. (2.65) should read "$\phi = atan2\,(R_{b2t}[3,2], R_{b2t}[3,3])$" with no space before the "2".

p. 58– In the fourth line, the cross-reference to "Eqs. (3.7)" should be to "Eqs. (3.6)."

3 Chapter 3

p. 69– The second line of Eqn. (3.69) should read

$$\hat{y}(k) = \mathbf{H}\hat{\mathbf{x}}(k).$$

p. 70– Eqn. (3.75) should read

$$\hat{y}^-(k+1) = \mathbf{H}\hat{\mathbf{x}}^-(k+1).$$

p. 83– Equation (3.130) should read

$$\begin{bmatrix} \dot{x} \\ \dot{v} \\ \dot{\phi} \\ \dot{b}_g \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -g & 0 \\ 0 & \frac{1}{R} & 0 & \sqrt{20} \\ 0 & 0 & 0 & -10 \end{bmatrix} \begin{bmatrix} x \\ v \\ \phi \\ b_g \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \omega_g.$$

There is an extra plus sign in the text.

p. 89– There is an extra '(k)' after the term in square brackets.

p. 90– Eqn. (3.173) should read

$$\Gamma\mathbf{Q}(t)\Gamma^T = \begin{bmatrix} 0 & 0 & 0 \\ 0 & R_v & 0 \\ 0 & 0 & Q_b \end{bmatrix} = \begin{bmatrix} 0.0 & 0.0 & 0.0 \\ 0.0 & 2.5\times10^{-3} & 0.0 \\ 0.0 & 0.0 & 1.0\times10^{-6} \end{bmatrix}.$$

4 Chapter 4

p. 124– In eqn. (4.82) there is an inverse matrix symbol (-1 as an exponent) missing after the right hand parenthesis.

5 Chapter 5

p. 165– Due to the ± sign in Eqn. (5.23), Eqn. (5.59) should read either as

$$\begin{aligned}(\tilde{\phi}^{(i)} + N^{(i)})\lambda &= ((X^{(i)} - x)^2 + (Y^{(i)} - y)^2 + (Z^{(i)} - z)^2)^{0.5} \\ &\quad + c\Delta t_r + c\Delta t_{sv}^{(i)} + c\Delta t_a^{(i)} + SA^{(i)} + E^{(i)} + mp^{(i)} + \beta^{(i)} \\ &= \rho^{(i)} + e_{cm}^{(i)} + mp^{(i)} + \beta^{(i)}\end{aligned}$$

or

$$\begin{aligned}(\tilde{\phi}^{(i)} + N^{(i)})\lambda &= ((X^{(i)} - x)^2 + (Y^{(i)} - y)^2 + (Z^{(i)} - z)^2)^{0.5} \\ &\quad + c\Delta t_r + c\Delta t_{sv}^{(i)} + c\Delta t_{trop}^{(i)} - c\Delta t_{ion}^{(i)} + SA^{(i)} + E^{(i)} + mp^{(i)} + \beta^{(i)} \\ &= \rho^{(i)} + e_{cm}^{(i)} + mp^{(i)} + \beta^{(i)}.\end{aligned}$$

p. 171– The derivation of (5.87) is as follows. Please excuse the lack of boldfacing on the symbol font characters that represent vectors and matrices.

After the correction of the secondary measurement of (5.86), the calculated position is

$$\hat{\mathbf{x}} = \mathbf{x}_p + \Delta\mathbf{x}.$$

From the previous equations of Section 5.7, we have

$$\begin{aligned}\mathbf{x}_\phi &= \mathbf{H}_p^{-1}\tilde{\phi}_p \\ \mathbf{B}_p &= \mathbf{H}_p^{-1}\lambda \\ \mathbf{K} &= \left(\mathbf{H}^T\mathbf{H}\right)^{-1}\mathbf{H}_s \\ \mathbf{x}_p &= \mathbf{H}_p^{-1}(\tilde{\phi}_p + \mathbf{N}_p)\lambda\end{aligned}$$

$$\mathbf{H} = \begin{bmatrix}\mathbf{H}_p \\ \mathbf{H}_s\end{bmatrix} \quad \text{and} \quad \tilde{\phi} = \begin{bmatrix}\tilde{\phi}_p \\ \tilde{\phi}_s\end{bmatrix}.$$

Therefore,

$$\mathbf{H}\mathbf{H}_p^{-1} = \begin{bmatrix}\mathbf{I} \\ \mathbf{H}_s\mathbf{H}_p^{-1}\end{bmatrix}.$$

The output residual (in meters) based on $\hat{\mathbf{x}}$ given the hypothesized integer vector $\mathbf{N}^T = [\hat{\mathbf{N}}_p^T, \hat{\mathbf{N}}_s^T]$ is

$$\Delta\mathbf{y} = \left(\tilde{\phi} + \mathbf{N}\right)\lambda - \mathbf{H}\hat{\mathbf{x}}.$$

This residual can be manipulated to produce (5.87) as follows:

$$\begin{aligned}\Delta\mathbf{y} &= \left(\tilde{\phi} + \mathbf{N}\right)\lambda - \mathbf{H}\left(\mathbf{x}_p + \Delta\mathbf{x}\right) \\ &= \left(\tilde{\phi} + \mathbf{N}\right)\lambda - \mathbf{H}\left(\mathbf{H}_p^{-1}(\tilde{\phi}_p + \mathbf{N}_p)\lambda + \Delta\mathbf{x}\right) \\ &= \begin{bmatrix}\tilde{\phi}_p + \mathbf{N}_p \\ \tilde{\phi}_s + \mathbf{N}_s\end{bmatrix}\lambda - \begin{bmatrix}\mathbf{I} \\ \mathbf{H}_s\mathbf{H}_p^{-1}\end{bmatrix}(\tilde{\phi}_p + \mathbf{N}_p)\lambda - \mathbf{H}\Delta\mathbf{x}\end{aligned}$$

$$
\begin{aligned}
&= \begin{bmatrix} \mathbf{0} \\ \left(\tilde{\phi}_s + \mathbf{N}_s\right) - \mathbf{H}_s\mathbf{H}_p^{-1}(\tilde{\phi}_p + \mathbf{N}_p) \end{bmatrix} \lambda - \mathbf{H}\Delta\mathbf{x} \\
&= \begin{bmatrix} \mathbf{0} \\ \left(\tilde{\phi}_s + \mathbf{N}_s\right) - \hat{\phi}_s \end{bmatrix} \lambda - \mathbf{H}\Delta\mathbf{x} \\
&= \left(\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \hat{\mathbf{res}}_{sm} \end{bmatrix} - \mathbf{H}\triangle\mathbf{x} \right)
\end{aligned}
$$

6 Chapter 6

p. 193 – Eqn. (6.21) should read $\mathbf{v} = \dot{\mathbf{R}}^e$.

p. 194 – Eqn. (6.34) should read $\dot{\mathbf{R}}_{b2e} = \mathbf{R}_{b2e}\mathbf{\Omega}^b_{eb}$.

p. 198 – In the first line of eqn. (6.52), $\tilde{f}|_w$ should read $\tilde{f}_w$.

p. 200 – In the first line of eqn. (6.59), the element in the second row and third column should be $-\epsilon_N$.

7 Chapter 7

p. 248 – The caption for Figure 7.2 should read 'GPS position aided INS.'

8 Appendices

p. 292 – The sentence containing Eqns. (A.1) and (A.2) should read: "Given this definition of ω^a_{ba}, it is straightforward (see Appendix B) to show that

$$\omega^a_{ba} \times \mathbf{x} = \mathbf{\Omega}^a_{ba}\mathbf{x} \tag{1}$$

where, if the components of ω^a_{ba} are $(\omega_1, \omega_2, \omega_3)^T$, then

$$\mathbf{\Omega}^a_{ba} = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix}." \tag{2}$$

p. 302 – The two forms of the Matrix Inversion Lemma should read as follows.

Matrix Inversion Lemma 1. Given four matrices $\mathbf{P}_1$, $\mathbf{P}_2, \mathbf{H}$, and $\mathbf{R}$ of compatible dimensions, if $\mathbf{P}_1$, $\mathbf{P}_2$, $\mathbf{R}$, and $\left(\mathbf{H}^T\mathbf{P}_1\mathbf{H} + \mathbf{R}\right)$ are all invertible and

$$\mathbf{P}_2^{-1} = \mathbf{P_1}^{-1} + \mathbf{H}\mathbf{R}^{-1}\mathbf{H}^T, \tag{3}$$

then

$$\mathbf{P}_2 = \mathbf{P}_1 - \mathbf{P}_1\mathbf{H}\left(\mathbf{H}^T\mathbf{P}\mathbf{H} + \mathbf{R}\right)^{-1}\mathbf{H}^T\mathbf{P}_1. \tag{4}$$

Matrix Inversion Lemma 2. Given four matrices $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$, and $\mathbf{D}$ of compatible dimensions, if $\mathbf{A}$, $\mathbf{C}$, and $\mathbf{A} + \mathbf{BCD}$ are invertible, then

$$(\mathbf{A} + \mathbf{BCD})^{-1} = \mathbf{A}^{-1} - \mathbf{A}^{-1}\mathbf{B}\left(\mathbf{D}\mathbf{A}^{-1}\mathbf{B} + \mathbf{C}^{-1}\right)^{-1}\mathbf{D}\mathbf{A}^{-1}. \tag{5}$$

p. 309 – The left hand sides of eqns. (C.22) and (C.23) should be functions of ω not τ.

p. 310 – Eqn. (C.28) should read

$$p(n,e,d) = \frac{1}{(2\pi)^{3/2}\,\sigma_n\sigma_e\sigma_d} exp\left(-\left(\frac{n^2}{2\sigma_n^2}+\frac{e^2}{2\sigma_e^2}+\frac{d^2}{2\sigma_d^2}\right)\right).$$

p. 311 – Eqn. (C.39) should read

$$P\{\rho < R\} = \frac{1}{\sigma^3}\sqrt{\frac{2}{\pi}}\int_0^R r^2 exp\left(\frac{-r^2}{2\sigma^2}\right) dr$$

p. 316 – In the Corrector step, there is an extraneous "f". The equation should read

Corrector Step: $\hat{\mathbf{x}}_c(t_{k+1}) = \hat{\mathbf{x}}(t_k) + g(\hat{\mathbf{x}}_p(t_{k+1}), \tilde{\mathbf{u}}(t_k))\Delta t$

p. 316-317 – Various parentheses in the Runge-Kutta algorithms are out of place. The correct equations should read:

The estimate of the next state by a common version of the three stage, third-order Runge-Kutta algorithm is calculated as

$$\hat{\mathbf{x}}(t_{k+1}) = \hat{\mathbf{x}}(t_k) + \frac{\Delta t}{9}\left(2\mathbf{K}_1 + 3\mathbf{K}_2 + 4\mathbf{K}_3\right) \tag{6}$$

where

$$\mathbf{K}_1 = \mathbf{g}\left(\hat{\mathbf{x}}(t_k), \tilde{\mathbf{u}}(t_k)\right) \tag{7}$$
$$\mathbf{K}_2 = \mathbf{g}\left(\hat{\mathbf{x}}(t_k) + \frac{\Delta t}{2}\mathbf{K}_1, \tilde{\mathbf{u}}(t_k)\right) \tag{8}$$
$$\mathbf{K}_3 = \mathbf{g}\left(\hat{\mathbf{x}}(t_k) + \frac{3\Delta t}{4}\mathbf{K}_2, \tilde{\mathbf{u}}(t_k)\right). \tag{9}$$

The estimate of the next state by the most common version of a four stage, fourth-order Runge-Kutta algorithm is calculated as

$$\hat{\mathbf{x}}(t_{k+1}) = \hat{\mathbf{x}}(t_k) + \frac{\Delta t}{6}\left(\mathbf{K}_1 + 2\mathbf{K}_2 + 2\mathbf{K}_3 + \mathbf{K}_4\right) \tag{10}$$

where

$$\mathbf{K}_1 = \mathbf{g}\left(\hat{\mathbf{x}}(t_k), \tilde{\mathbf{u}}(t_k)\right) \tag{11}$$
$$\mathbf{K}_2 = \mathbf{g}\left(\hat{\mathbf{x}}(t_k) + \frac{\Delta t}{2}\mathbf{K}_1, \tilde{\mathbf{u}}(t_k)\right) \tag{12}$$
$$\mathbf{K}_3 = \mathbf{g}\left(\hat{\mathbf{x}}(t_k) + \frac{\Delta t}{2}\mathbf{K}_2, \tilde{\mathbf{u}}(t_k)\right) \tag{13}$$
$$\mathbf{K}_4 = \mathbf{g}\left(\hat{\mathbf{x}}(t_k) + \Delta t\mathbf{K}_3, \tilde{\mathbf{u}}(t_k)\right). \tag{14}$$

p. 319 – The value of L1 at the top of the page should be 1575.42 MHz.

p. 325 – In eqn. (E.8) all 0.461 terms should be 0.416.